

Documentation for Gar6more3D

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This code compute the quasi-analytical solution of several wave equation in two layered media, using the Cagniard de Hoop method [1, 2, 9, 8, 7, 3, 5, 6, 4]. It produces seismograms at given points.

The equations can be written in the general form.

$$A(y) \frac{\partial^2 U}{\partial t^2} - B(z)U = \delta(\mathbf{x} - \mathbf{x}_s) f(t), \quad x \in \mathbb{R}, y \in \mathbb{R}, z \in \mathbb{R} \quad (1)$$

where A and B are operators satisfying

$$\begin{aligned} A(z) &= A^+, B(z) = B^+, & z > 0, \\ A(z) &= A^-, B(z) = B^-, & z < 0. \end{aligned}$$

The code analytically compute the Green function u of the problem

$$A(z) \frac{\partial^2 u}{\partial t^2} - B(z)u = \delta(\mathbf{x} - \mathbf{x}_s) \delta(t), \quad x \in \mathbb{R}, y \in \mathbb{R}, z \in \mathbb{R} \quad (2)$$

and convolves it with the source function f . You can modify this function in the subroutine *lib/libgeneral/source.F90*. The convolution is done by a numerical integration, that is why the solution is only “quasi-analytical”. You can improve the accuracy of the solution by increasing the number of intervals used for the integration (the variable N_{int} in the data file *cdh3d.dat*). Moreover, in 3D, Cagniard-de Hoop requires the computation of another integral (see [6, 4]), the number of intervals for the numerical integration is given by N_{qint}

1 Acoustic/acoustic

The code computes a seismogram at points $(x_i, y_i, z)_{i=1, Nx, j=1, Ny}$ of the pressure solution of the equations

$$\frac{\partial^2 P^+}{\partial t^2} - c^{+2} \Delta P^+ = \delta(\mathbf{x} - \mathbf{x}_s) f(t), \quad x \in \mathbb{R}, z > 0$$

$$\frac{\partial^2 P^-}{\partial t^2} - c^{-2} \Delta P^- = \delta(\mathbf{x} - \mathbf{x}_s) f(t), \quad x \in \mathbb{R}, z < 0$$

We consider $\mathbf{x}_s = (0, h)$ and we use the transmission conditions :

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$$\left| \begin{array}{l} P^+ = P^-, \\ \rho^+ \frac{\partial P^-}{\partial z} = \rho^- \frac{\partial P^+}{\partial z}. \end{array} \right.$$

on the interface $z = 0$. The code also computes the displacement given by the relation :

$$\frac{\partial \mathbf{U}^\pm}{\partial t} = -\frac{1}{\rho^\pm} \nabla P^\pm.$$

Actually, for some reasons related to Cagniard-de Hoop in 3D, the code does not really compute the pressure and the velocity, but the time-integral of the pressure and the displacement. Therefore, to obtain the actual pressure or the velocity, you have to replace $f(t)$ by the source function you are using.

2 Acoustic/elasticdynamic(isotropic)

The code computes a seismogram at point $(x_i, y_j)_{i=1, Nx, j=1, Ny}$ of the pressure (in the fluid) and the velocity (in the solid) solution of the equations

$$\frac{\partial^2 P^+}{\partial t^2} - c^{+2} \Delta P^+ = \delta(\mathbf{x} - \mathbf{x}_s) f(t), \quad x \in \mathbb{R}, z > 0, \quad (3)$$

$$\frac{\partial^2 \mathbf{V}^-}{\partial t^2} - (\lambda^- + 2\mu^-) \nabla(\nabla \cdot \mathbf{V}) + \mu^- \nabla \times (\nabla \times \mathbf{V}^-) = 0, \quad x \in \mathbb{R}, z < 0, \quad (4)$$

with $\mathbf{x}_s = (0, h)$ and the transmission conditions

$$\left| \begin{array}{l} \frac{\partial V_z^-}{\partial t} = -\frac{1}{\rho^+} \frac{\partial P^+}{\partial z}, \quad y = 0, \\ (\lambda^- + 2\mu^-) \frac{\partial V_z^-}{\partial z} + \lambda^- \frac{\partial V_x^-}{\partial x} = \frac{\partial P^+}{\partial t} \\ \frac{\partial V_x^-}{\partial z} + \frac{\partial V_z^-}{\partial x} = 0. \end{array} \right.$$

on the interface $y = 0$. The code also computes the velocity in the fluid by using the relation

$$\frac{\partial V^+}{\partial t} = -\frac{1}{\rho^+} \nabla P^+.$$

Actually, as for the acoustic/acoustic case, the code does not really compute the pressure and the velocity, but the time-integral of the pressure and the displacement. Therefore, to obtain the actual pressure or the velocity, you have to replace $f(t)$ by the source function you are using.

3 Acoustic/poroelastic (see [6])

The code computes a seismogram at point $(x_i, y_i, z)_{i=1, Nx, j=1, Ny}$ of the pressure P , the displacement U^+ (in the fluid) and the solid displacement U_s^- (in the poroelastic medium)

solution of the equations

$$\left\{ \begin{array}{l} \frac{\partial^2 P^+}{\partial t^2} - c^{+2} \Delta P^+ = \delta(\mathbf{x} - \mathbf{x}_s) f(t), \\ \frac{\partial U^+}{\partial t} = -\nabla P^+. \end{array} \right.$$

for $x \in \mathbb{R}, z > 0$ and

$$\left\{ \begin{array}{l} (1 - \phi^-) \rho_s^- \frac{\partial^2 \mathbf{U}_s^-}{\partial t^2} + \phi \rho_f^- \frac{\partial^2 \mathbf{U}_f^-}{\partial t^2} - (\lambda^- + 2\mu^-) \nabla(\nabla \cdot \mathbf{U}_s^-) + \mu^- \nabla \times (\nabla \times \mathbf{U}_s^-) + \beta \nabla P^- = 0, \\ (1 - a^-) \rho_f^- \frac{\partial^2 \mathbf{U}_s^-}{\partial t^2} + a^- \rho_f^- \frac{\partial^2 \mathbf{U}_f^-}{\partial t^2} + \nabla P^- = 0 \\ \frac{1}{m^-} P^- + (\beta^- - \phi^-) \nabla \cdot \mathbf{U}_s^- + \phi^- \nabla \cdot \mathbf{U}_f^- = 0 \end{array} \right.$$

for $x \in \mathbb{R}, z < 0$, either with the open pore transmission conditions (if parameter open is set to 1)

$$\left\{ \begin{array}{l} \phi^- (U_{fz}^- - U_{sz}^-) = U_z^+ - U_{sz}^-, \\ P^- = \rho^+ \frac{\partial P^+}{\partial t}, \\ (\lambda^- + m^- \beta^- (\beta^- - \phi^-)) \nabla \cdot \mathbf{U}_s^- + 2\mu^- \frac{\partial U_{sz}^-}{\partial z} + m^- \beta^- \phi^- \nabla \cdot \mathbf{U}_f^- = -\rho^+ \frac{\partial P^+}{\partial t}, \\ \frac{\partial U_{sx}^-}{\partial z} + \frac{\partial U_{sz}^-}{\partial x} = 0, \\ \frac{\partial U_{sy}^-}{\partial z} + \frac{\partial U_{sz}^-}{\partial y} = 0. \end{array} \right.$$

or with the sealed pore transmission conditions (if parameter open is set to 0)

$$\left\{ \begin{array}{l} \phi^- (U_{fz}^- - U_{sz}^-) = U_z^+ - U_{sz}^-, \\ U_{fy}^- = U_{sy}^-, \\ (\lambda^- + m^- \beta^- (\beta^- - \phi^-)) \nabla \cdot \mathbf{U}_s^- + 2\mu^- \frac{\partial U_{sz}^-}{\partial z} + m^- \beta^- \phi^- \nabla \cdot \mathbf{U}_f^- = -\rho^+ \frac{\partial P^+}{\partial t}, \\ \frac{\partial U_{sx}^-}{\partial z} + \frac{\partial U_{sz}^-}{\partial x} = 0, \\ \frac{\partial U_{sy}^-}{\partial z} + \frac{\partial U_{sz}^-}{\partial y} = 0. \end{array} \right.$$

on the interface $z = 0$. The code does not compute the fluid displacement and the pressure in the poroelastic medium, but there is no particular difficulty to do that. Once again the code does not really compute the pressure but its time-integral. Therefore, to obtain the actual pressure or the velocity, you have to replace $f(t)$ by the source function you are using.

4 Poroelastic/poroelastic [4]

The code computes a seismogram at point $(x_i, y_j, z)_{i=1, Nx, j=1, Ny}$ of the solid displacement U solution of the equations

$$\left| \begin{aligned} (1 - \phi^+) \rho_s^+ \frac{\partial^2 \mathbf{U}_s^+}{\partial t^2} + \phi^+ \rho_f^+ \frac{\partial^2 \mathbf{U}_f^+}{\partial t^2} - (\lambda^+ + 2\mu^+) \nabla(\nabla \cdot \mathbf{U}_s^+) + \mu^+ \nabla \times (\nabla \times \mathbf{U}_s^+) + \beta^+ \nabla P^+ &= \nabla \delta(\mathbf{x} - \mathbf{x}_s) F_s(t), \\ (1 - a^+) \rho_f^+ \frac{\partial^2 \mathbf{U}_s^+}{\partial t^2} + a^+ \rho_f^+ \frac{\partial^2 \mathbf{U}_f^+}{\partial t^2} + \nabla P^+ &= \nabla \delta(\mathbf{x} - \mathbf{x}_s) F_s(t) \\ \frac{1}{m^+} P^+ + (\beta^+ - \phi^+) \nabla \cdot \mathbf{U}_s^+ + \phi^+ \nabla \cdot \mathbf{U}_f^+ &= \delta(\mathbf{x} - \mathbf{x}_s) F_p(t) \end{aligned} \right|$$

for $x \in \mathbb{R}, z > 0$ and

$$\left| \begin{aligned} (1 - \phi^-) \rho_s^- \frac{\partial^2 \mathbf{U}_s^-}{\partial t^2} + \phi^- \rho_f^- \frac{\partial^2 \mathbf{U}_f^-}{\partial t^2} - (\lambda^- + 2\mu^-) \nabla(\nabla \cdot \mathbf{U}_s^-) + \mu^- \nabla \times (\nabla \times \mathbf{U}_s^-) + \beta^- \nabla P^- &= 0, \\ (1 - a^-) \rho_f^- \frac{\partial^2 \mathbf{U}_s^-}{\partial t^2} + a^- \rho_f^- \frac{\partial^2 \mathbf{U}_f^-}{\partial t^2} + \nabla P^- &= 0 \\ \frac{1}{m^-} P^- + (\beta^- - \phi^-) \nabla \cdot \mathbf{U}_s^- + \phi^- \nabla \cdot \mathbf{U}_f^- &= 0 \end{aligned} \right|$$

for $x \in \mathbb{R}, z < 0$, with the transmission conditions on the interface $z = 0$

$$\left| \begin{aligned} \phi^- (U_{fz}^- - U_{sz}^-) &= \phi^+ (U_{fz}^+ - U_{sz}^+), \\ \alpha^- \nabla \cdot \mathbf{U}_s^- + 2\mu^- \frac{\partial U_{sz}^-}{\partial z} + m^- \beta^- \phi^- \nabla \cdot \mathbf{U}_f^- &= \alpha^+ \nabla \cdot \mathbf{U}_s^+ + 2\mu^+ \frac{\partial U_{sz}^+}{\partial z} + m^+ \beta^+ \phi^+ \nabla \cdot \mathbf{U}_f^+, \\ \mu^- \left(\frac{\partial U_{sx}^-}{\partial z} + \frac{\partial U_{sz}^-}{\partial x} \right) &= \mu^+ \left(\frac{\partial U_{sx}^+}{\partial z} + \frac{\partial U_{sz}^+}{\partial x} \right), \\ \mu^- \left(\frac{\partial U_{sy}^-}{\partial z} + \frac{\partial U_{sz}^-}{\partial y} \right) &= \mu^+ \left(\frac{\partial U_{sy}^+}{\partial z} + \frac{\partial U_{sz}^+}{\partial y} \right), \\ U_{sx}^- = U_{sx}^+, \quad U_{sy}^- = U_{sy}^+, \quad U_{sz}^- = U_{sz}^+ \quad P^- &= P^+ \end{aligned} \right|$$

with

$$\alpha^\pm = \lambda^\pm + m^\pm \beta^\pm (\beta^\pm - \phi^\pm).$$

Actually the code computes the solution for each source F_s and F_p separately. If you want a bulk source (F_s), set the parameter *type_source* to 1, if you want a pressure source (F_p), set the parameter *type_source* to 2.

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