# Documentation for Gar6more2D

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This code compute the quasi-analytical solution of several wave equation in two layered media, using the Cagniard de Hoop method [1, 2, 9, 8, 7, 3, 5, 4]. It produces seismograms at given points.

The equations can be written in the general form.

$$A(y)\frac{\partial^2 U}{\partial t^2} - B(y)U = \delta(\mathbf{x}-\mathbf{x_s})\, f(t), \quad x \in \mathbb{R},\, y \in \mathbb{R} \tag{1}$$

where $A$ and $B$ are operators satisfying

$$\begin{array}{ll} A(y) = A^+,\, B(y) = B^+, & y > 0, \\ A(y) = A^-,\, B(y) = B^-, & y < 0. \end{array}$$

The code analytically compute the Green function $u$ of the problem

$$A(y)\frac{\partial^2 u}{\partial t^2} - B(y)u = \delta(\mathbf{x}-\mathbf{x_s})\, \delta(t), \quad x \in \mathbb{R}, y \in \mathbb{R} \tag{2}$$

and convolves it with the source function $f$. You can modify this function in the subroutine *lib/libgeneral/source.F90*. The convolution is done by a numerical integration, that is why the solution is only "quasi-analytical". You can improve the accuracy of the solution by increasing the number of intervals used for the integration (the variable *Nint* in the data file *cdh2d.dat*)

## 1 Acoustic/acoustic

The code computes a seismogram at points $(x_i, y)_{i=1,Nx}$ of the pressure solution of the equations

$$\frac{\partial^2 P^+}{\partial t^2} - {c^+}^2 \Delta P^+ = \delta(\mathbf{x}-\mathbf{x_s}) f(t), \quad x \in \mathbb{R}, y > 0$$

$$\frac{\partial^2 P^-}{\partial t^2} - {c^-}^2 \Delta P^- = \delta(\mathbf{x}-\mathbf{x_s}) f(t), \quad x \in \mathbb{R}, y < 0$$

We consider $\mathbf{x_s} = (0, h)$ and we use the transmission conditions :

$$\left| \begin{array}{l} P^+ = P^-, \\[2ex] \rho^+ \dfrac{\partial P^-}{\partial y} = \rho^- \dfrac{\partial P^+}{\partial y}. \end{array} \right.$$

on the interface $y = 0$. The code also computes the velocity given by the relation :

$$\frac{\partial \mathbf{V}^{\pm}}{\partial t} = -\frac{1}{\rho^{\pm}}\nabla P^{\pm}.$$

If you want to compute the displacement $U$, it can be easily computed by replacing $f(t)$ by the primitive of the source function you are using. For instance, if you are using a Rickert, you'll have to consider a first derivative of a Gaussian for $f$.

## 2 Acoustic/elastodynamic(isotropic)

The code computes a seismogram at point $(x_i, y)_{i=1,Nx}$ of the pressure (in the fluid) and the velocity (in the solid) solution of the equations

$$\frac{\partial^2 P^+}{\partial t^2} - {c^+}^2 \Delta P^+ = \delta(\mathbf{x} - \mathbf{x_s}) f(t), \quad x \in \mathbb{R}, y > 0, \tag{3}$$

$$\frac{\partial^2 \mathbf{V}^-}{\partial t^2} - (\lambda^- + 2\mu^-)\nabla(\nabla \cdot \mathbf{V}) + \mu^- \nabla \times (\nabla \times \mathbf{V}^-) = 0, \quad x \in \mathbb{R}, y < 0, \tag{4}$$

with $\mathbf{x_s} = (0, h)$ and the transmission conditions

$$\left| \begin{array}{l} \dfrac{\partial V_y^-}{\partial t} = -\dfrac{1}{\rho^+}\dfrac{\partial P^+}{\partial y}, \quad y = 0, \\ (\lambda^- + 2\mu^-)\dfrac{\partial V_y^-}{\partial y} + \lambda^- \dfrac{\partial V_x^-}{\partial x} = \dfrac{\partial P^+}{\partial t} \\ \dfrac{\partial V_x^-}{\partial y} + \dfrac{\partial V_y^-}{\partial x} = 0. \end{array} \right.$$

on the interface $y = 0$. The code also computes the velocity in the fluid by using the relation

$$\frac{\partial V^+}{\partial t} = -\frac{1}{\rho^+}\nabla P^+.$$

Once again, if you want to compute the displacement $U$, it can be easily computed by replacing $f(t)$ by the primitive of the source function you are using.

## 3 Acoustic/poroelastic(see [5])

The code computes a seismogram at point $(x_i, y)_{i=1,Nx}$ of the potential of velocity $\chi$ and the displacement $U^+$ (in the fluid) and the solid displacement $U_s^-$ (in the poroelastic medium) solution of the equations

$$\left| \begin{array}{l} \dfrac{\partial^2 \chi^+}{\partial t^2} - {c^+}^2 \Delta \chi^+ = \delta(\mathbf{x} - \mathbf{x_s}) f(t), \\ \dfrac{\partial U^+}{\partial t} = -\nabla \chi^+. \end{array} \right.$$

for $x \in \mathbb{R}, y > 0$ and

$$\left|\begin{array}{l}
(1-\phi^-)\rho_s^- \dfrac{\partial^2 \mathbf{U}_s^-}{\partial t^2} + \phi\rho_f^- \dfrac{\partial^2 \mathbf{U}_f^-}{\partial t^2} - (\lambda^- + 2\mu^-)\nabla(\nabla \cdot \mathbf{U}_s^-) + \mu^- \nabla \times (\nabla \times \mathbf{U}_s^-) + \beta \nabla P^- = 0, \\
(1-a^-)\rho_f^- \dfrac{\partial^2 \mathbf{U}_s^-}{\partial t^2} + a^- \rho_f^- \dfrac{\partial^2 \mathbf{U}_f^-}{\partial t^2} + \nabla P^- = 0 \\
\dfrac{1}{m^-} P^- + (\beta^- - \phi^-)\nabla \cdot \mathbf{U}_s^- + \phi^- \nabla \cdot \mathbf{U}_f^- = 0
\end{array}\right.$$

for $x \in \mathbb{R}, y < 0$, either with the open pore transmission conditions (if parameter open is set to 1)

$$\left|\begin{array}{l}
\phi^-(U_{fy}^- - U_{sy}^-) = U_y^+ - U_{sy}^-, \\[2ex]
P^- = \rho^+ \dfrac{\partial \chi^+}{\partial t}, \\[2ex]
(\lambda^- + m^-\beta^-(\beta^- - \phi^-))\, \nabla \cdot \mathbf{U}_s^- + 2\mu^- \dfrac{\partial U_{sy}^-}{\partial y} + m^-\beta^-\phi^- \nabla \cdot \mathbf{U}_f^- = -\rho^+ \dfrac{\partial \chi^+}{\partial t}, \\[2ex]
\dfrac{\partial U_{sx}^-}{\partial y} + \dfrac{\partial U_{sy}^-}{\partial x} = 0,
\end{array}\right.$$

or with the sealed pore transmission conditions (if parameter open is set to 0)

$$\left|\begin{array}{l}
\phi^-(U_{fy}^- - U_{sy}^-) = U_y^+ - U_{sy}^-, \\[2ex]
U_{fy}^- = U_{sy}^-, \\[2ex]
(\lambda^- + m^-\beta^-(\beta^- - \phi^-))\, \nabla \cdot \mathbf{U}_s^- + 2\mu^- \dfrac{\partial U_{sy}^-}{\partial y} + m^-\beta^-\phi^- \nabla \cdot \mathbf{U}_f^- = -\rho^+ \dfrac{\partial \chi^+}{\partial t}, \\[2ex]
\dfrac{\partial U_{sx}^-}{\partial y} + \dfrac{\partial U_{sy}^-}{\partial x} = 0,
\end{array}\right.$$

on the interface $y = 0$. The code does not compute the fluid displacement and the pressure in the poroelastic medium, but there is no particular difficulty to do that.

**Remark 3.1** *The pressure in the fluid satisfies the relation*

$$P^+ = \rho^+ \frac{\partial \chi}{\partial t}.$$

*Therefore, if you want to compute a seismogram of the pressure and the velocities in the fluid and of the velocities in the solid, you have to replace $f(t)$ by the derivative of the source function you are using. If you want to consider a source of pressure, you'll have to replace $f(t)$ by the integral of the source function you are using.*

## 4 Poroelastic/poroelastic (see [4])

The code computes a seismogram at point $(x_i, y)_{i=1,Nx}$ of the solid displacement $U$ solution of the equations

$$\left|\begin{array}{l} (1-\phi^+)\rho_s^+\dfrac{\partial^2 \mathbf{U}_s^+}{\partial t^2} + \phi^+\rho_f^+\dfrac{\partial^2 \mathbf{U}_f^+}{\partial t^2} - (\lambda^+ + 2\mu^+)\nabla(\nabla\cdot\mathbf{U}_s^+) + \mu^+\nabla\times(\nabla\times\mathbf{U}_s^+) + \beta^+\nabla P^+ = \nabla\delta(\mathbf{x}-\mathbf{x_s})F_s(t), \\ (1-a^+)\rho_f^+\dfrac{\partial^2 \mathbf{U}_s^+}{\partial t^2} + a^+\rho_f^+\dfrac{\partial^2 \mathbf{U}_f^+}{\partial t^2} + \nabla P^+ = \nabla\delta(\mathbf{x}-\mathbf{x_s})F_s(t) \\ \dfrac{1}{m^+}P^+ + (\beta^+ - \phi^+)\nabla\cdot\mathbf{U}_s^+ + \phi^+\nabla\cdot\mathbf{U}_f^+ = \delta(\mathbf{x}-\mathbf{x_s})F_p(t) \end{array}\right.$$

for $x \in \mathbb{R}, y > 0$ and

$$\left|\begin{array}{l} (1-\phi^-)\rho_s^-\dfrac{\partial^2 \mathbf{U}_s^-}{\partial t^2} + \phi^-\rho_f^-\dfrac{\partial^2 \mathbf{U}_f^-}{\partial t^2} - (\lambda^- + 2\mu^-)\nabla(\nabla\cdot\mathbf{U}_s^-) + \mu^-\nabla\times(\nabla\times\mathbf{U}_s^-) + \beta^-\nabla P^- = 0, \\ (1-a^-)\rho_f^-\dfrac{\partial^2 \mathbf{U}_s^-}{\partial t^2} + a^-\rho_f^-\dfrac{\partial^2 \mathbf{U}_f^-}{\partial t^2} + \nabla P^- = 0 \\ \dfrac{1}{m^-}P^- + (\beta^- - \phi^-)\nabla\cdot\mathbf{U}_s^- + \phi^-\nabla\cdot\mathbf{U}_f^- = 0 \end{array}\right.$$

for $x \in \mathbb{R}, y < 0$, with the transmission conditions on the interface $y = 0$

$$\left|\begin{array}{l} \phi^-\,(U_{fy}^- - U_{sy}^-) = \phi^+\,(U_{fy}^+ - U_{sy}^+), \\ \alpha^-\nabla\cdot\mathbf{U}_s^- + 2\mu^-\dfrac{\partial U_{sy}^-}{\partial y} + m^-\beta^-\phi^-\,\nabla\cdot\mathbf{U}_f^- = \alpha^+\nabla\cdot\mathbf{U}_s^+ + 2\mu^+\dfrac{\partial U_{sy}^+}{\partial y} + m^+\beta^+\phi^+\,\nabla\cdot\mathbf{U}_f^+, \\ \mu^-(\dfrac{\partial U_{sx}^-}{\partial y} + \dfrac{\partial U_{sy}}{\partial x}^-) = \mu^+(\dfrac{\partial U_{sy}^+}{\partial y} + \dfrac{\partial U_{sy}^+}{\partial x}), \\ U_{sx}^- = U_{sx}^+, \quad U_{sy}^- = U_{sy}^+, \quad P^- = P^+, \end{array}\right.$$

with

$$\alpha^\pm = \lambda^\pm + m^\pm\beta^\pm(\beta^\pm - \phi^\pm).$$

Actually the code computes the solution for each source $F_s$ and $F_p$ separately. If you want a bulk source ($F_s$), set the parameter *type_source* to 1, if you want a pressure source ($F_p$), set the parameter *type_source* to 2.

**Remark 4.1** *The code does not really compute the displacement, but its derivative (for some reasons related to the Cagniard-de Hoop method, see [6, 4]). Therefore, you have to replace $f(t)$ by the primitive of the source function you are using to compute the displacement.*

## References


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