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Department of Structural Engineering

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Geophysics

THE DYNAMIC STRAIN LOCALISATION: SEM NUMERICAL ANALYSES OF VISCOPLASTIC GEOMATERIALS

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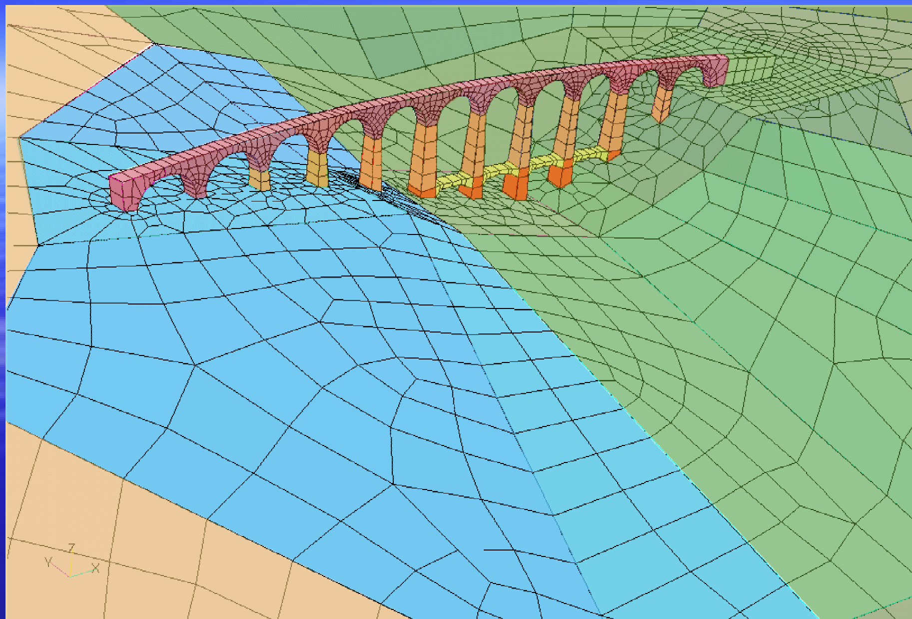
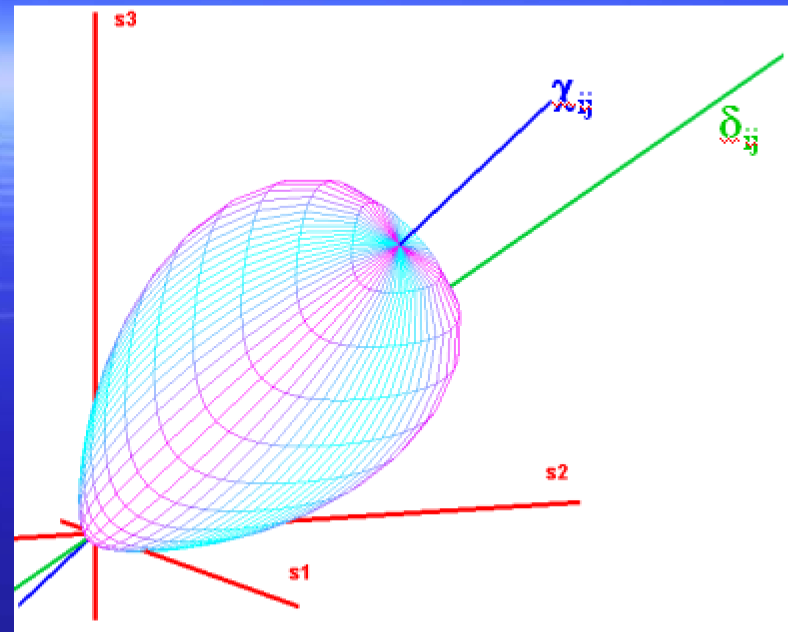
14 June 2005
Chania Crete - Greece

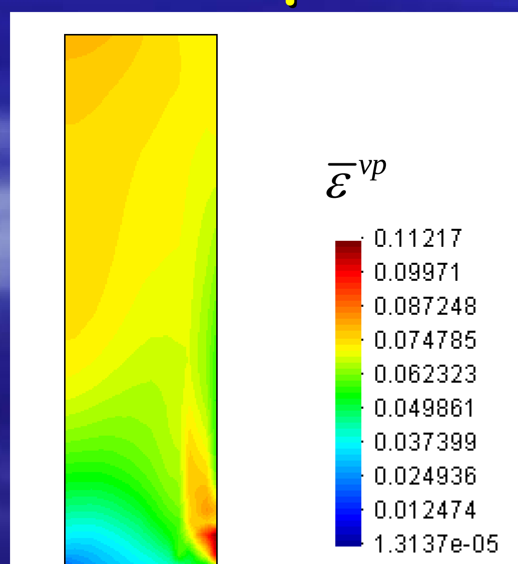
1-Numerical code



2-Constitutive modelling



3-Dynamic response



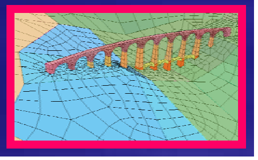
GEO-ELSE
(GEO-ELasticity by Spectral Elements)

- GEO-ELSE is a Spectral Elements code for the study of wave propagation phenomena in 2D or 3D complex domain

- Developers:
  - CRS4 (Center for Advanced, Research and Studies in Sardinia)
  - Politecnico di Milano, DIS (Department of Structural Engineering)

- Native parallel implementation

- Naturally oriented to large scale applications (> at least 10^6 grid points)



Formulation of the elastodynamic problem

Dynamic equilibrium in the weak form:

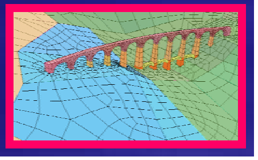
$$\frac{\partial^2}{\partial t^2}\int_\Omega \rho u_i v_i d\Omega + \int_\Omega \sigma_{ij}\varepsilon_{ij} d\Omega = \int_\Gamma t_i v_i + \int_\Omega f_i v_i$$

where u_i = unknown displacement function

v_i = generic admissible displacement function (test function)

t_i = prescribed tractions at the boundary Γ

f_i = prescribed body force distribution in Ω



Time advancing scheme

Finite difference 2nd order (LF2 – LF2)

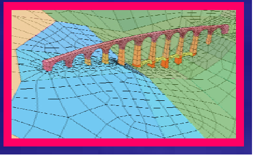
$$\begin{cases} \dfrac{\partial^2 \boldsymbol{u}}{\partial t^2} = \dfrac{\boldsymbol{u}_{n+1} - 2\boldsymbol{u}_n + \boldsymbol{u}_{n-1}}{\Delta t^2} \\ \dfrac{\partial \boldsymbol{u}}{\partial t} = \dfrac{\boldsymbol{u}_{n+1} - \boldsymbol{u}_{n-1}}{2\Delta t} \end{cases}$$

Courant-Friedrichs-Levy (CFL) stability condition

$$\Delta t \leq \eta \left(\frac{\Delta x}{c} \right)_{\min}$$

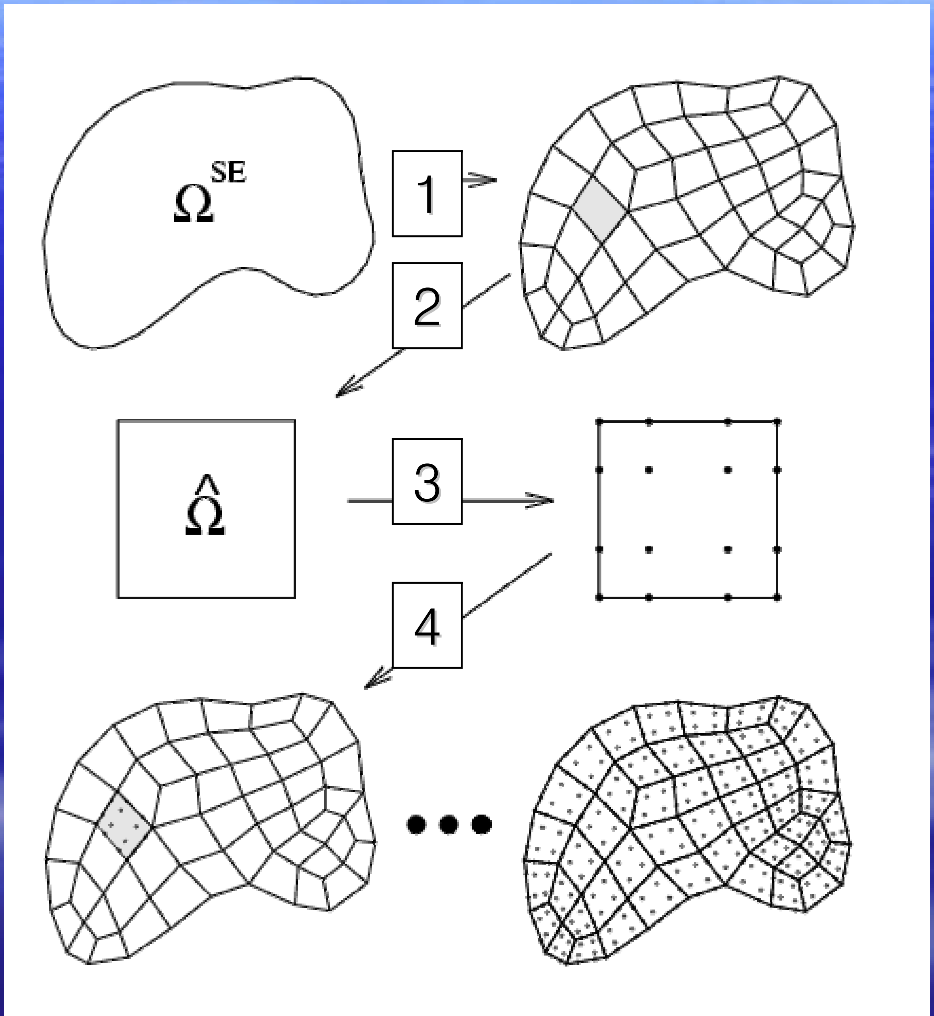
Spatial discretization

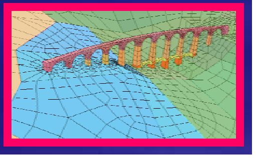
Spectral element method SEM (Faccioli et al., 1997)



Spectral discretization of the spatial domain

- The domain is split into quadrilaterals (hexahedra)
- Each subdomain is mapped onto a reference element
- LGL nodes are introduced
- Spectral grid-points are mapped back onto the domain





The Legendre-Gauss-Lobatto quadrature formula

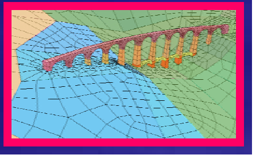
$$\int_{-1}^{1} f(x)dx \cong \sum_{k=0}^{N} \alpha_k f(x_k)$$

where

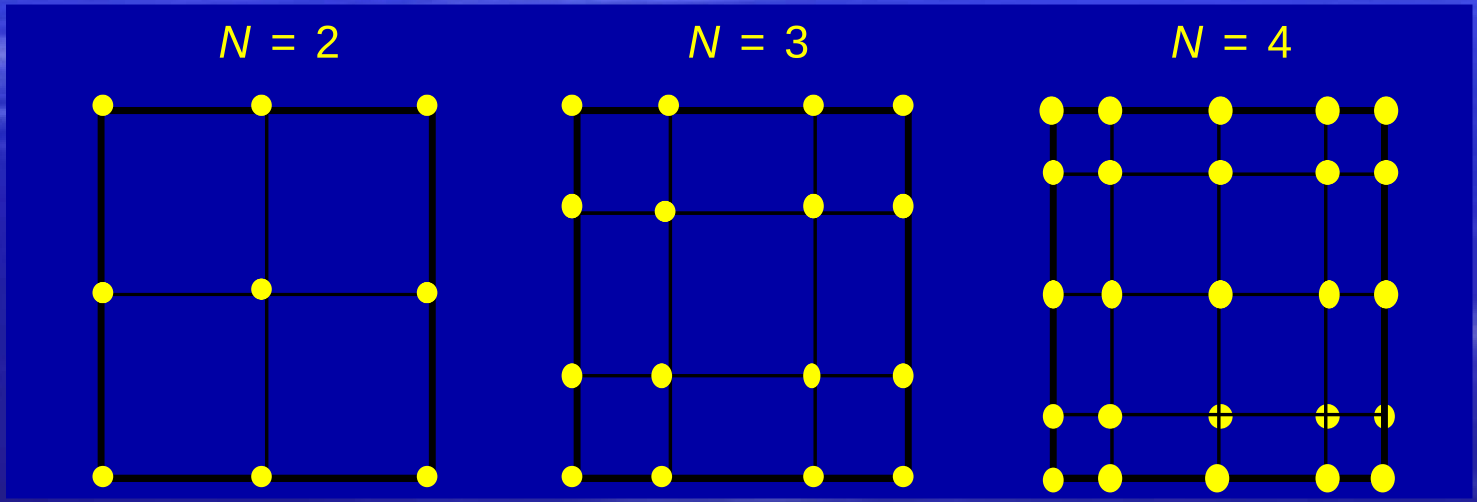
$$\alpha_k = \frac{2}{N(N+1)} \frac{1}{L_N^2(x_k)}$$

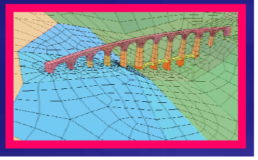
$L_N(x_k)$ being the Legendre orthogonal polynomial of degree N, calculated at the LGL node x_k

LGL nodes: $L'_N(x_k) = 0$ $[x_0 = -1, ..., x_N = 1]$



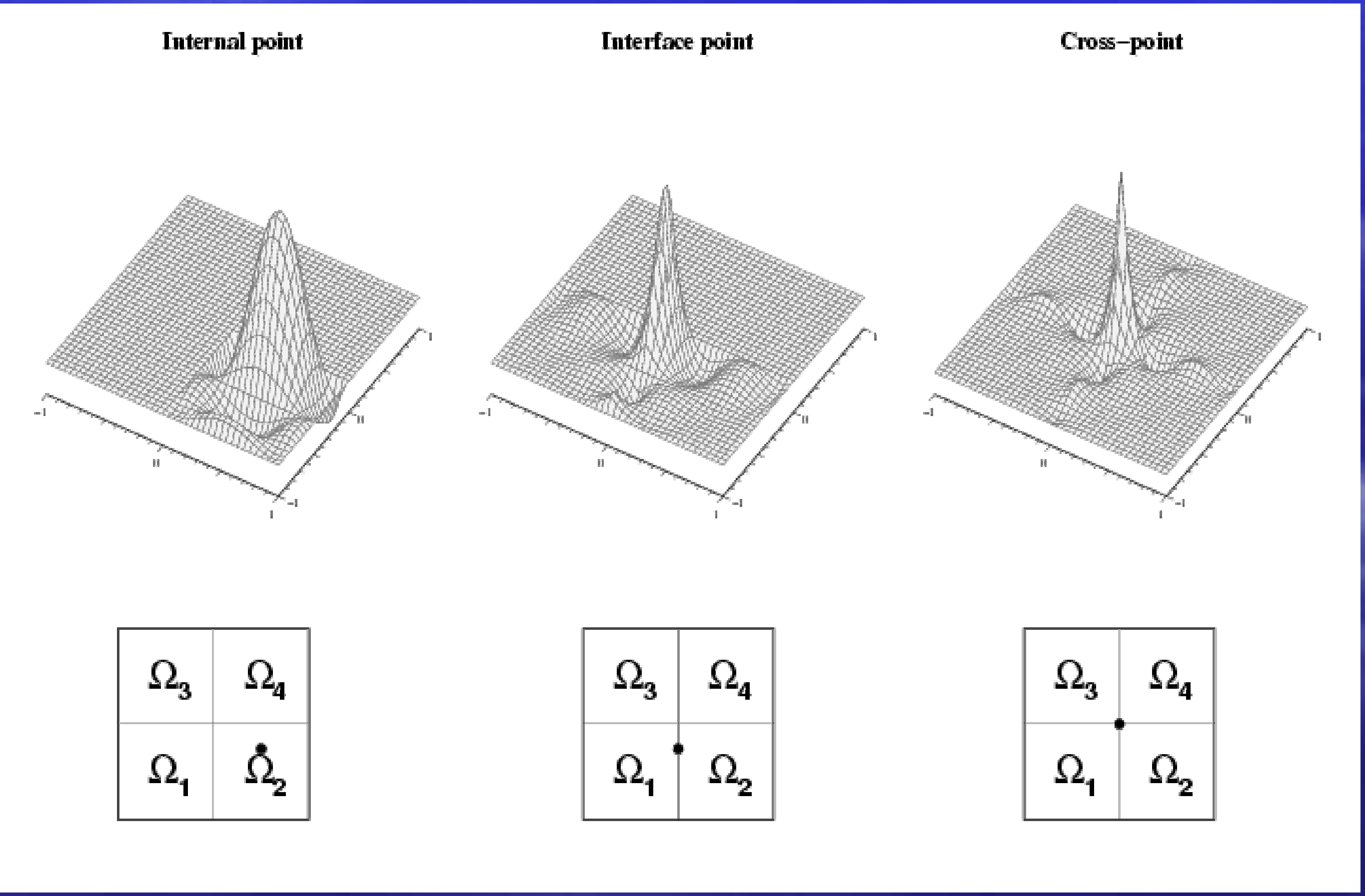
2D spectral elements and LGL nodes for different values of the polynomial degree

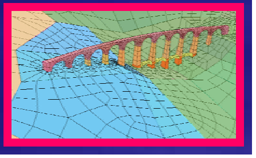




Selection of the test functions

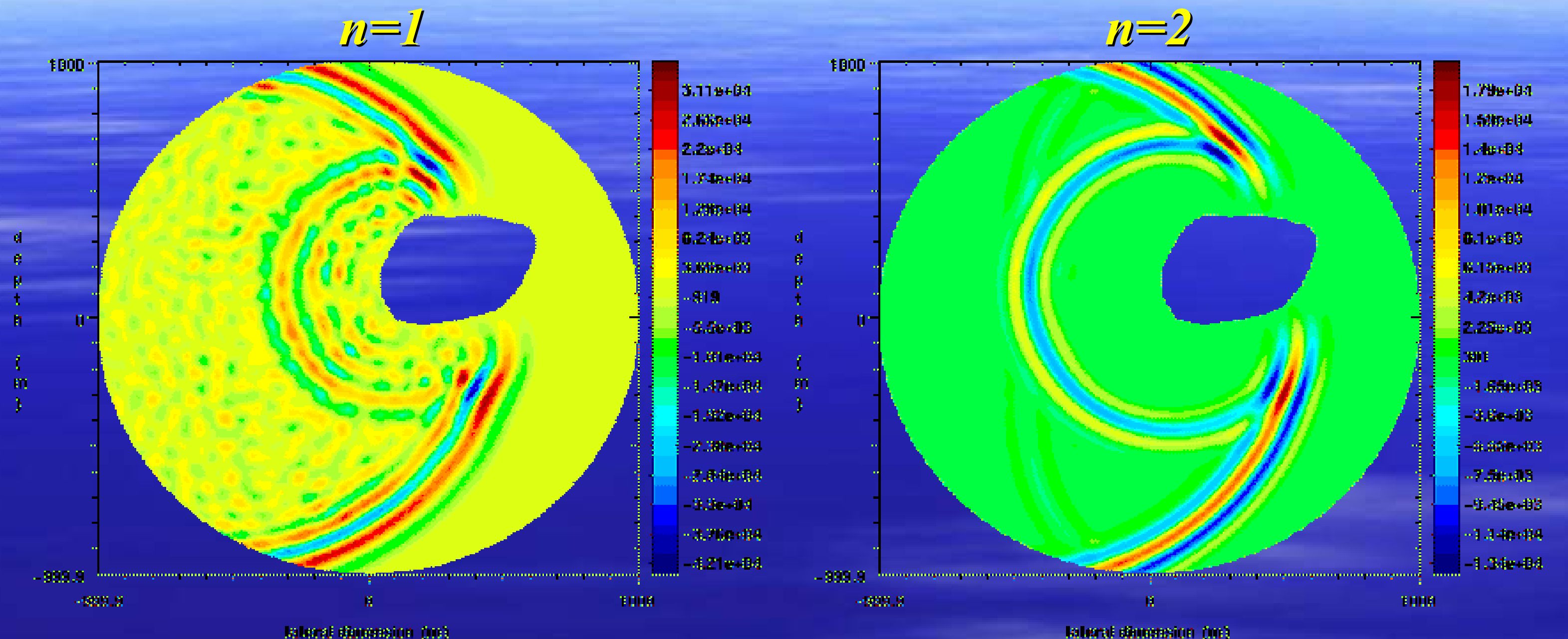
A suitable choice is the Lagrange polynomial of degree N, which is equal to one at the *j* th LGL node and vanishes at all other nodes





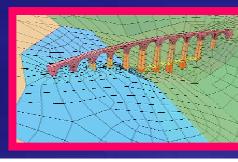
Why using spectral elements ?

acoustic problem



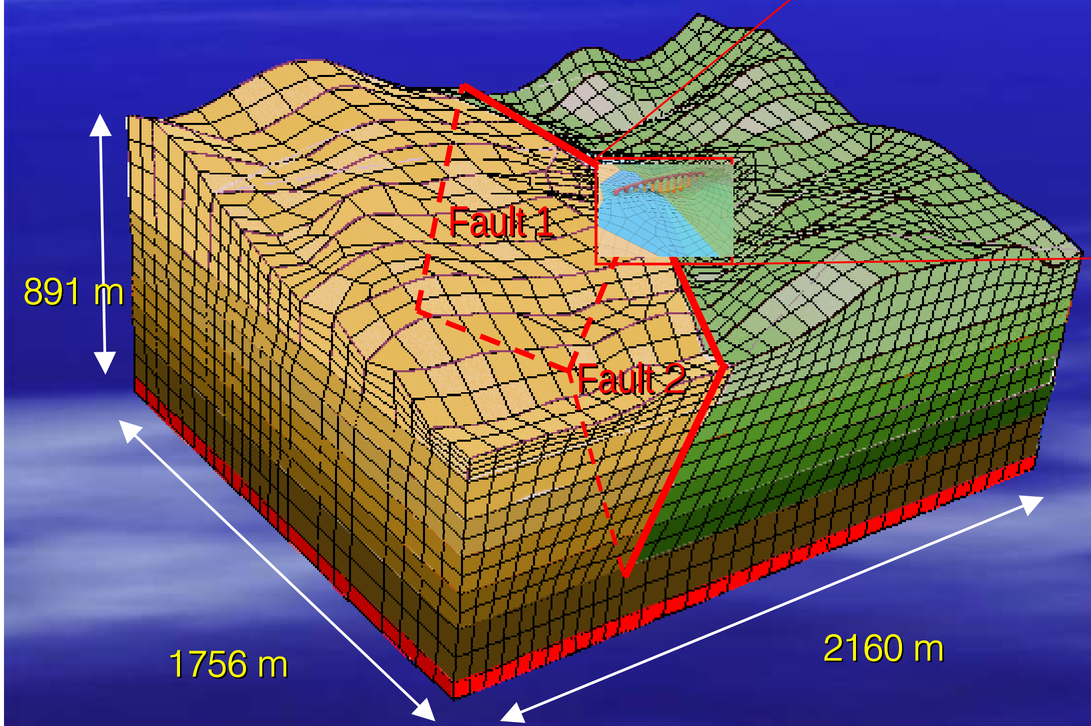
Acoustic wave propagation through an irregular domain. Simulation with spectral degree 1 *(left)* exhibits numerical dispersion due to poor accuracy.

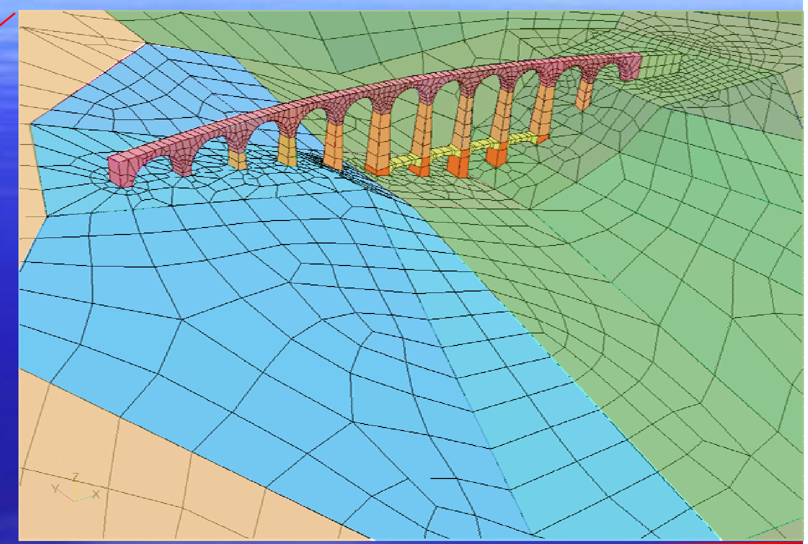
Simulation with spectral degree 2 *(right)* provides better results. Change of spectral degree is done at **run time**.



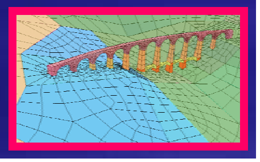
3D Soil-Structure – Acquasanta viaduct

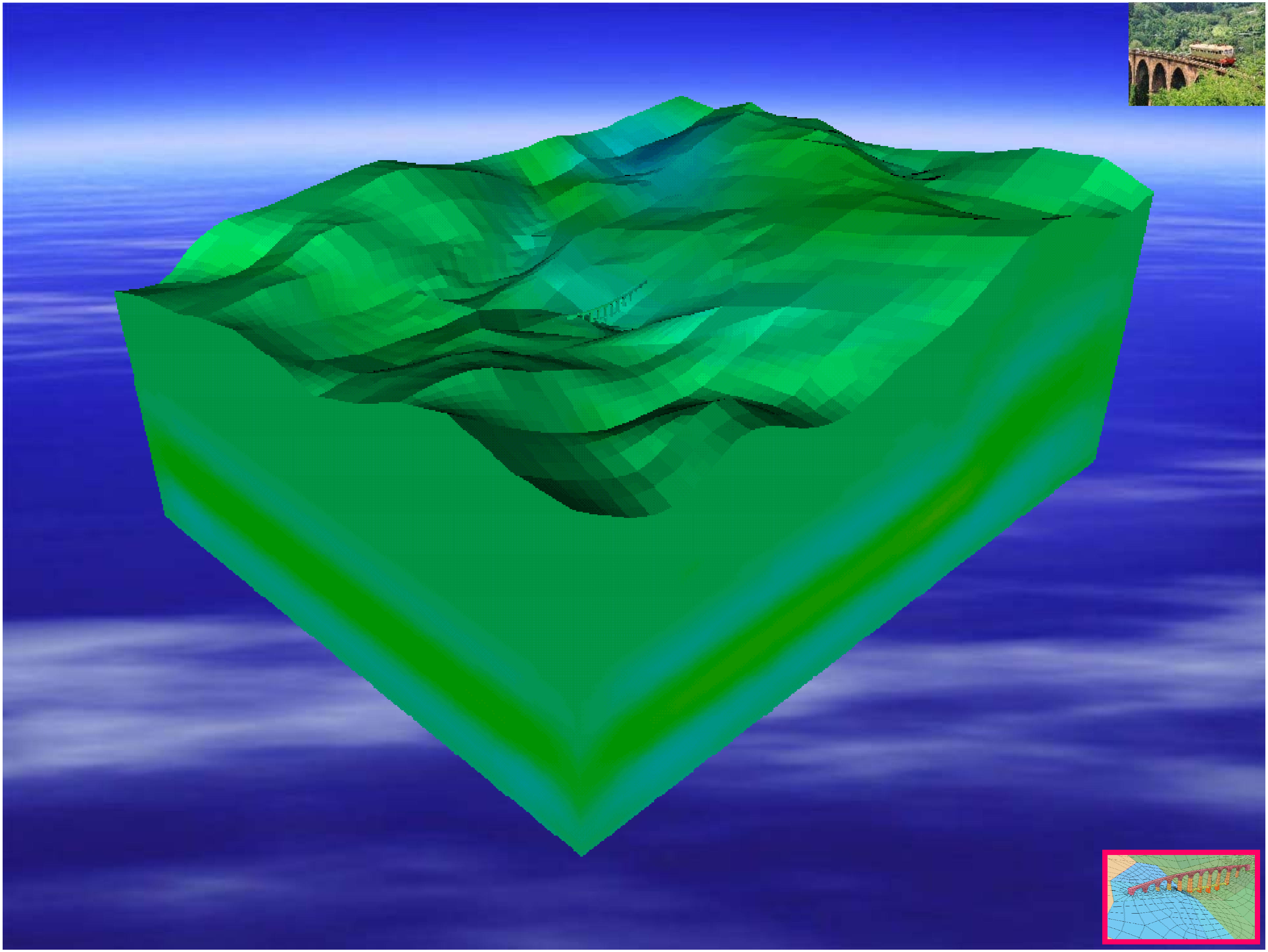
Complex 3D layered structures
with two main faults

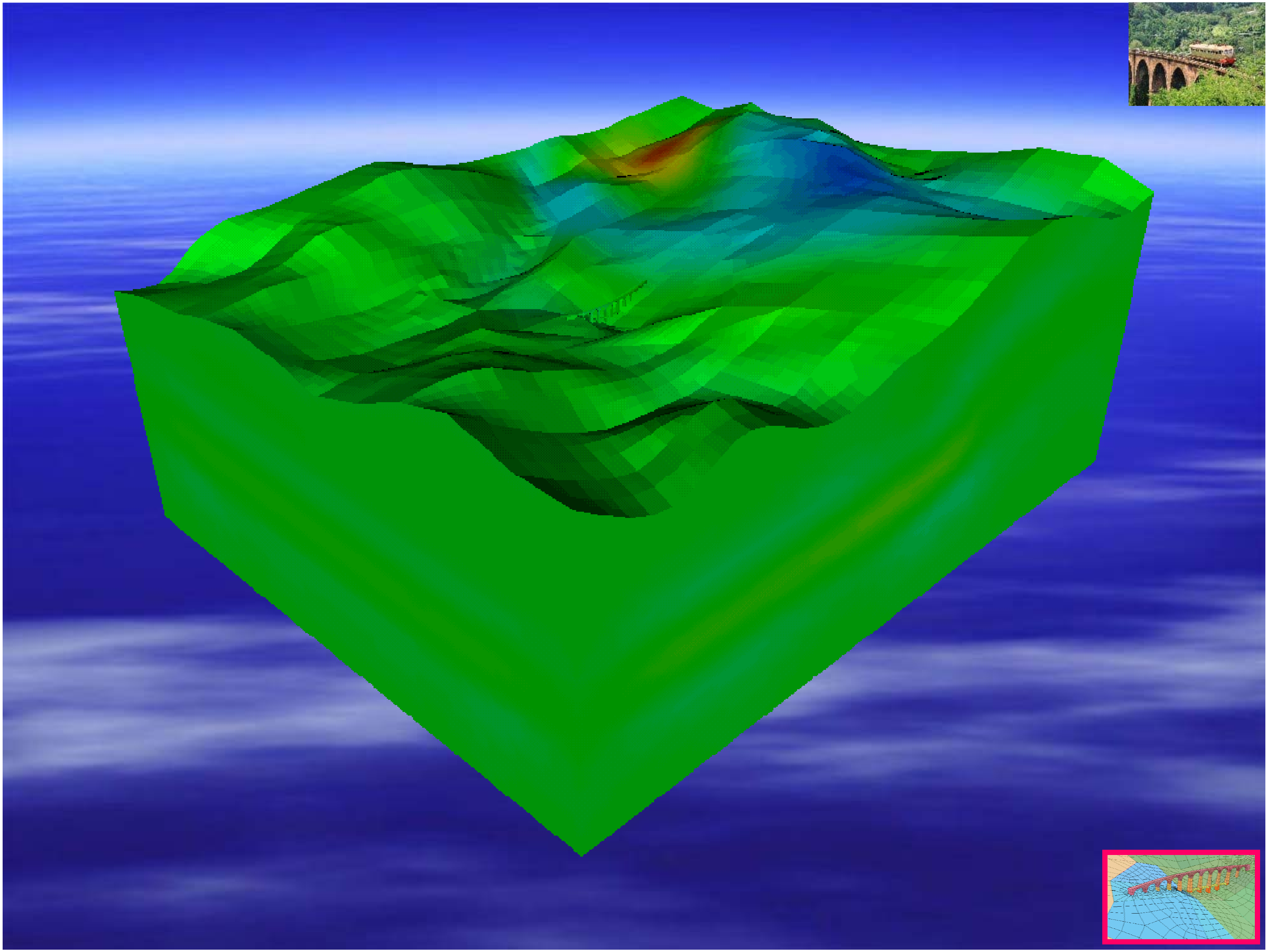


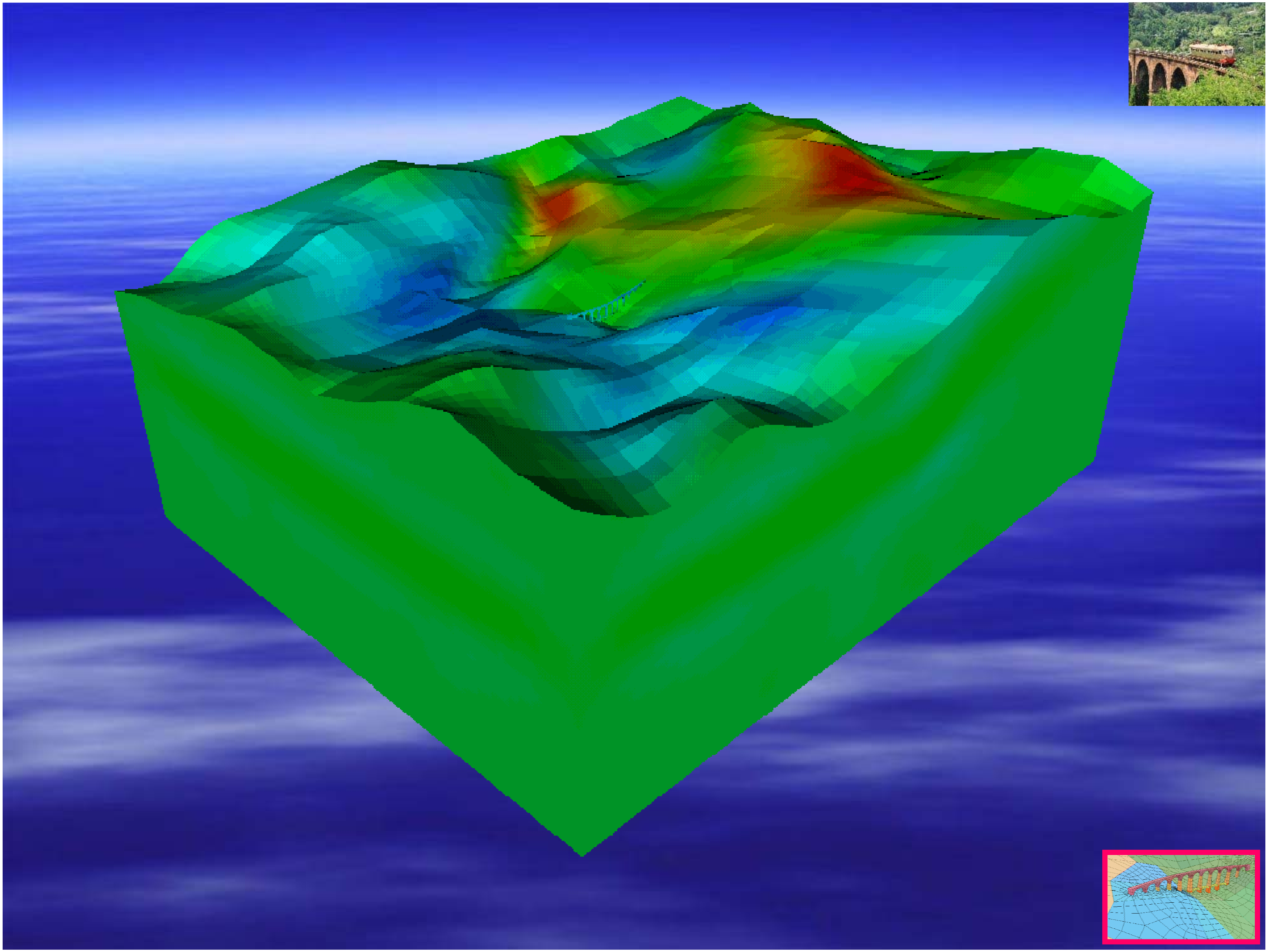


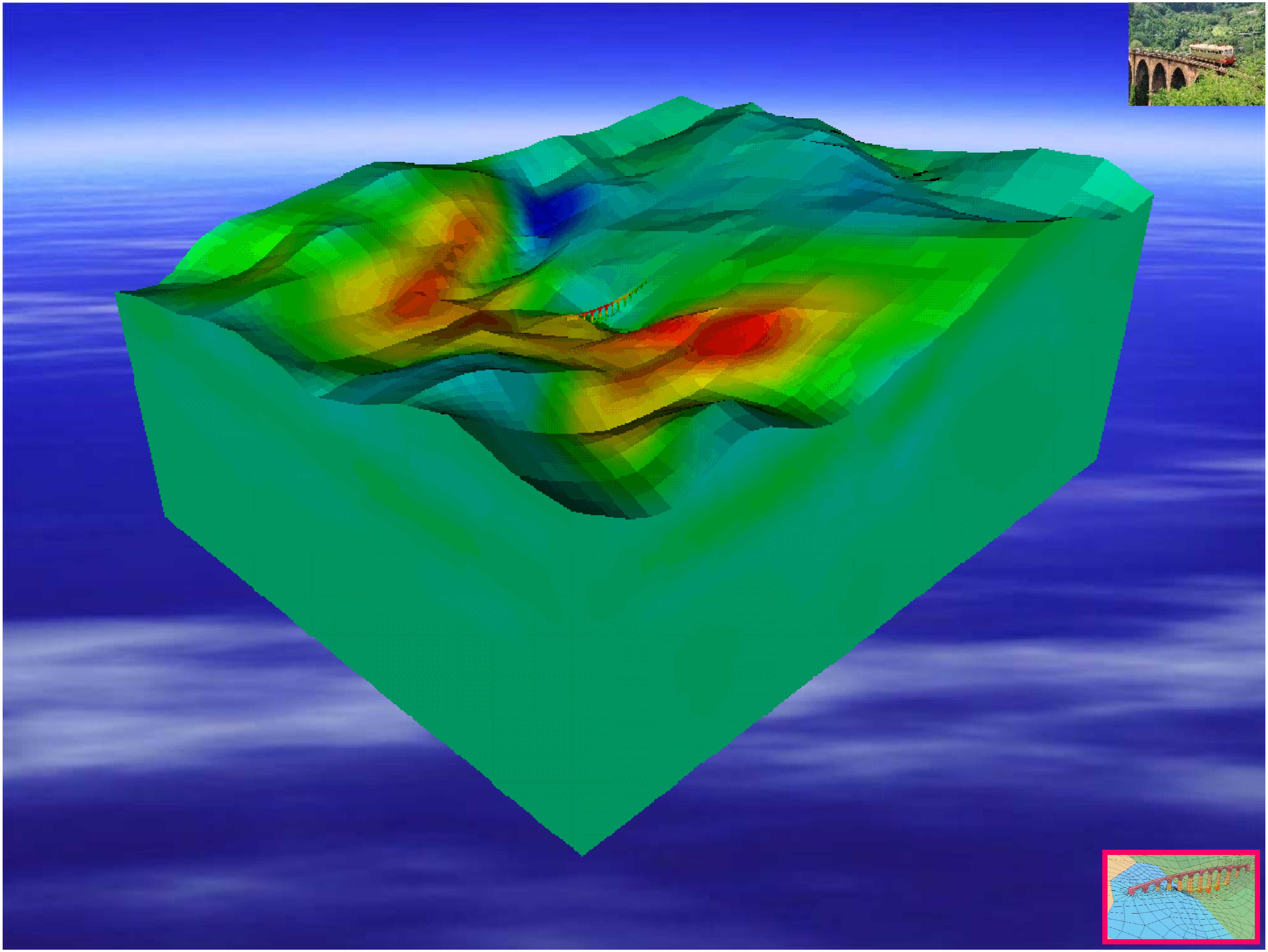
Alluvial Deposits
(max depth 30 m) with a
masonry railway bridge
crossing the valley







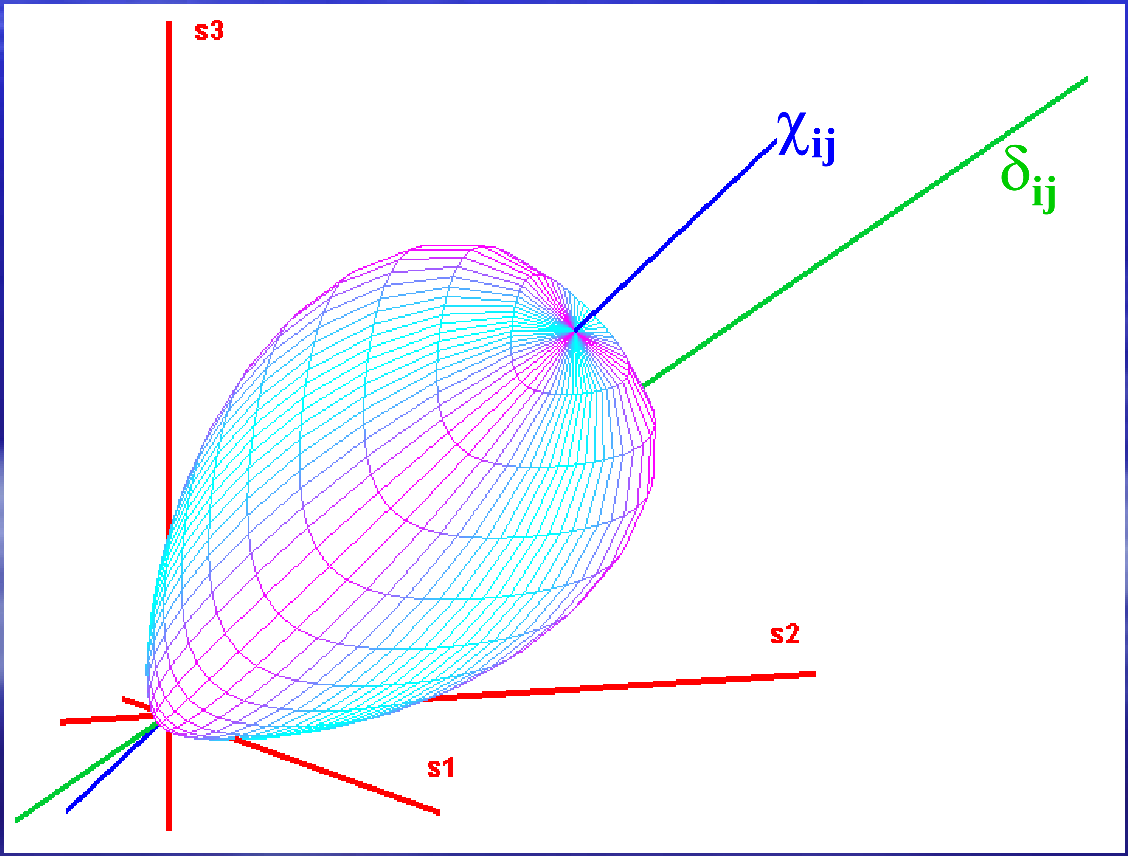




GEO-ELSE Viscoplastic

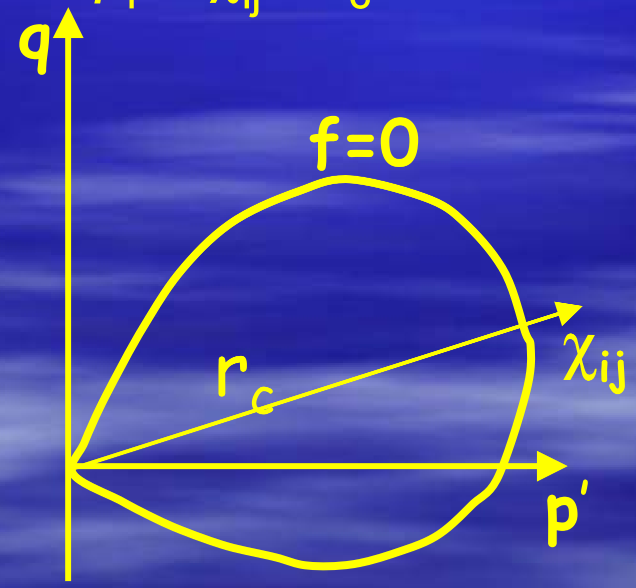
⇒ *Elastoviscoplastic constitutive model*

Single potential, anisotropic hardening, non associated flow rule
[di Prisco et al. 1993]



Hardening Variables

β_f ; χ_{ij} ; r_c



GEO-ELSE Viscoplastic

⇒ Total strain tensor can be written as:

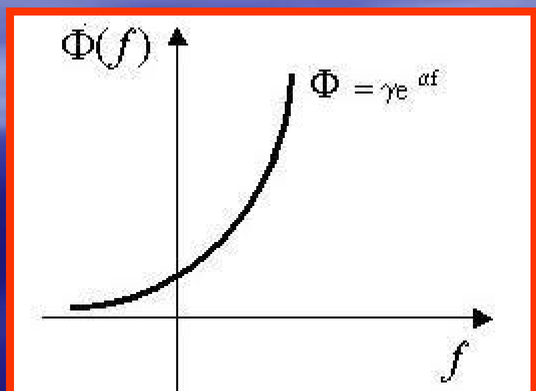
$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^{el} + \dot{\varepsilon}_{ij}^{vp}$$

and constitutive behaviour is given by: $\dot{\sigma}_{hk} = D_{hkij}^{el} : \left(\dot{\varepsilon}_{ij} - \dot{\varepsilon}_{ij}^{vp} \right)$

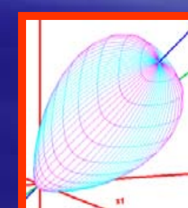
⇒ The flow rule can be written as below

$$\frac{\partial \varepsilon_{ij}^{vp}}{\partial t} = \gamma \, \phi(f) \frac{\partial g}{\partial \sigma_{ij}}$$ [Perzyna 1963]

where f is the yield function, $\phi(f)$ is the viscous nucleus, γ is a parameter which describes the system evolution rate, $\partial g / \partial \sigma_{ij}$ is the gradient of the plastic potential and characterizes the direction of viscoplastic strain.

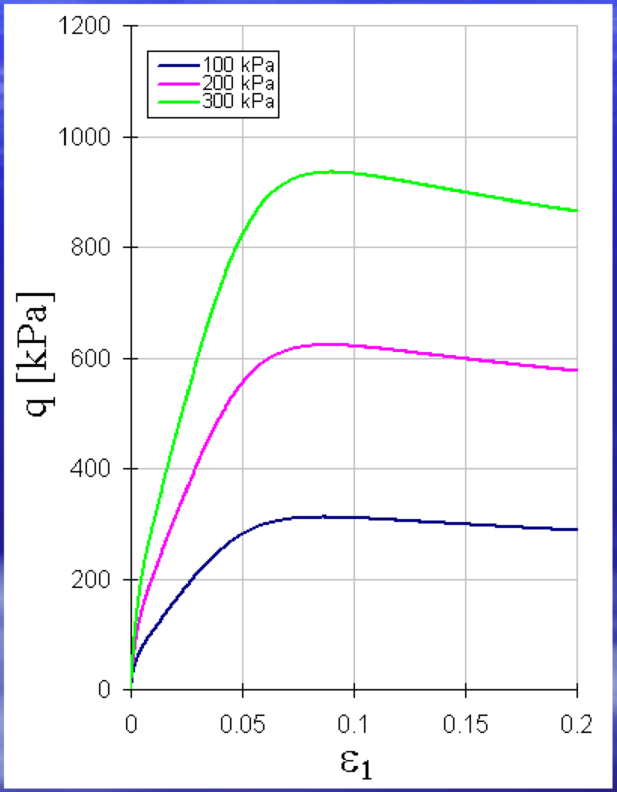


$$\phi(f) = e^{\alpha f}$$ [di Prisco et al.1996]

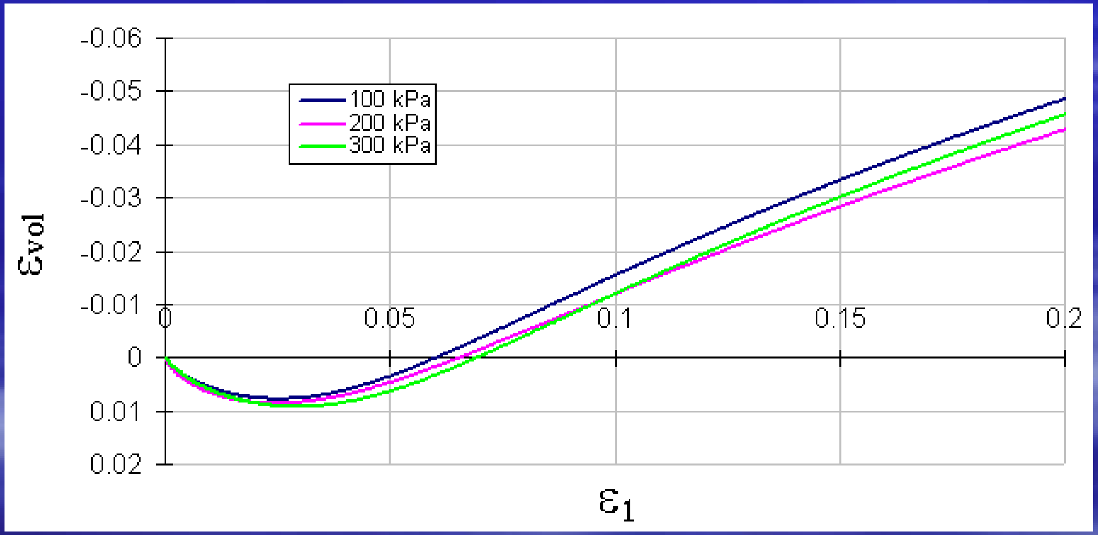


GEO-ELSE Viscoplastic

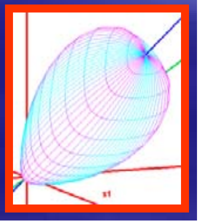
⇒ The constitutive parameters are assumed to be linearly dependent on the current value of Dr



This dependency may be introduced very easily because the consistency rule is absent



It is necessary for capturing the softening regime of dense sands



GEO-ELSE Viscoplastic

⇒ *Nonlocal approach*

It is necessary for numerically studying strain localisation problems in order to avoid mesh dependence of solution

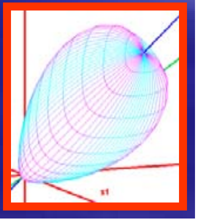
$$\frac{\partial \varepsilon_{ij}^{vp}}{\partial t} = \gamma \, \phi\left(\hat{f}\right) \frac{\partial g}{\partial \sigma_{ij}}$$

the viscous nucleus is dependent on a non-local yield fuction

where $$\hat{f} = \int_{\bar{V}} f\left(x_i\right) \omega\left(x_i - x_{0i}\right) dV$$ [di Prisco et al. 2002]

The size of $\bar{V}$ defines the region whose state influences the microstructural evolution of point taken into consideration. Generally it is associated with the grain size of the material

in one dimension: $$\omega\left(x - x_0\right) = \frac{\left(1/\sqrt{2\pi}\right) e^{-\left(x-x_0\right)^2/2}}{\int_{-\Delta x/2}^{-\Delta x/2} \left(1/\sqrt{2\pi}\right) e^{-\left(x-x_0\right)^2/2} dx}$$



Implementation of the viscoplastic model in GEO-ELSE

For simplicity absence of absorbent conditions and not viscous material

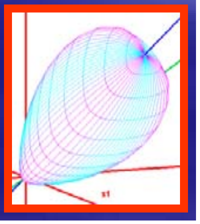
$$[M]\ddot{\boldsymbol{u}}(t)+[K]\boldsymbol{u}(t)=\boldsymbol{F}_{ext}(t)$$

with $[K]\boldsymbol{u}(t)=F_{\text{int}}=\int_{\Omega}\sigma\, d\Omega$

$$\dot{\boldsymbol{\sigma}}=\boldsymbol{D}^{e}:\left(\dot{\boldsymbol{\varepsilon}}-\dot{\boldsymbol{\varepsilon}}^{vp}\right)$$

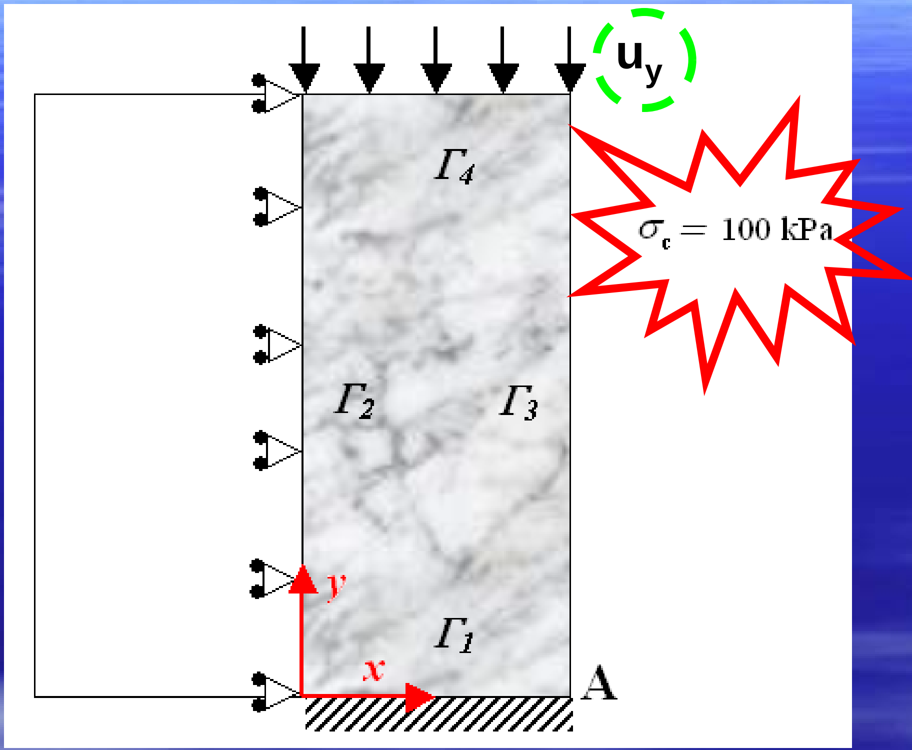
The viscoplastic model introduces an additional right hand side term

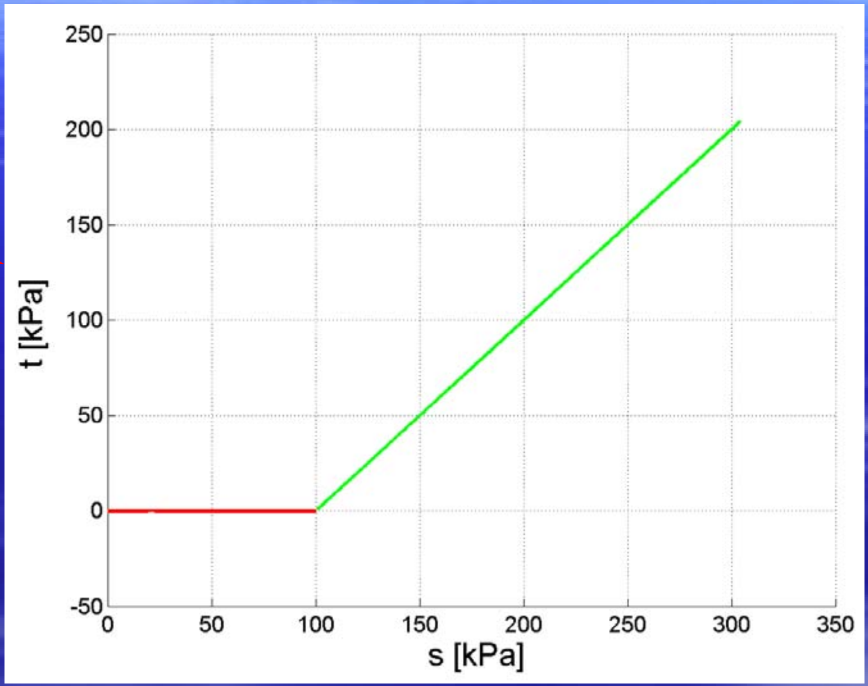
$$\boldsymbol{u}_{n+1}=\left(\boldsymbol{F}_{ext}-\boldsymbol{F}_{int}\right)[M]^{-1}\Delta t^{2}+2\boldsymbol{u}_{n}-\boldsymbol{u}_{n-1}$$



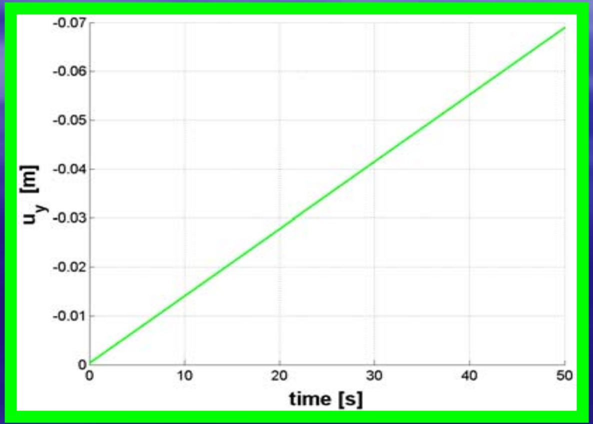
Plane strain biaxial compression test

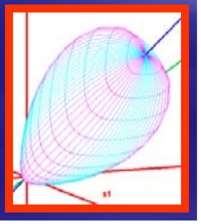
Vertical controlled displacement





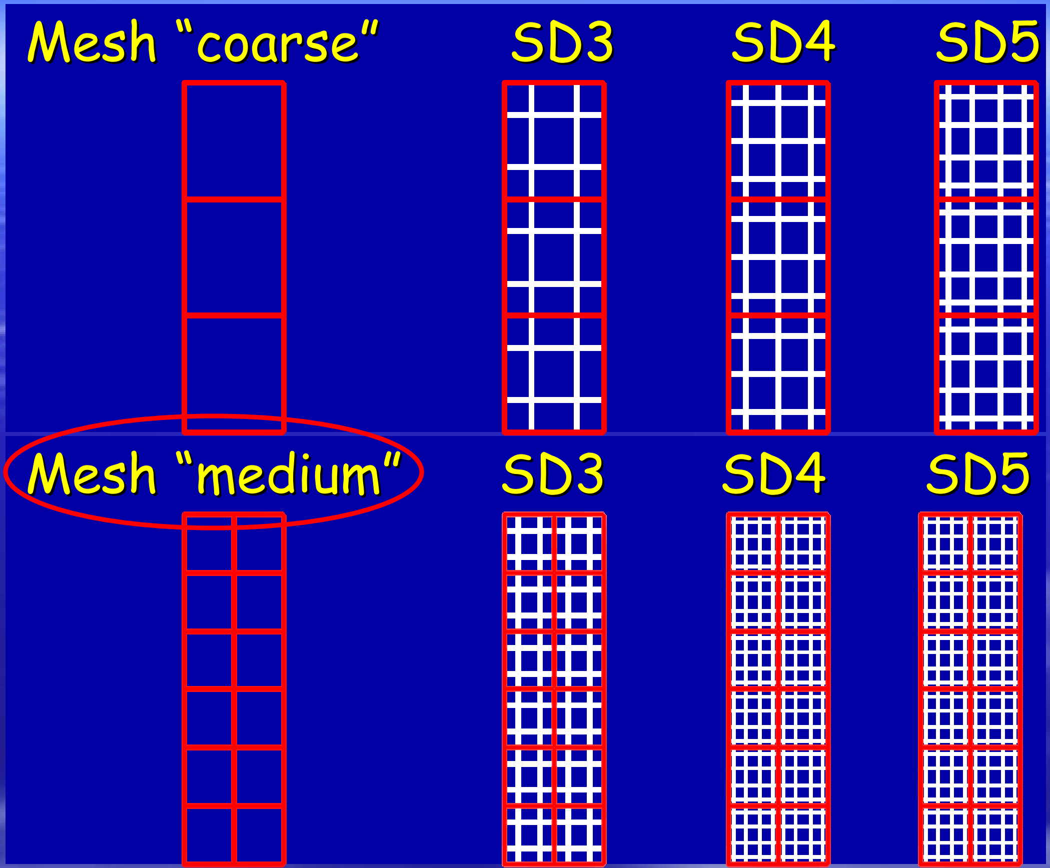
Dense sand homogeneous specimen

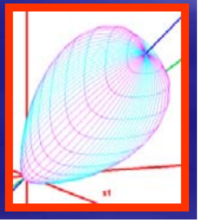




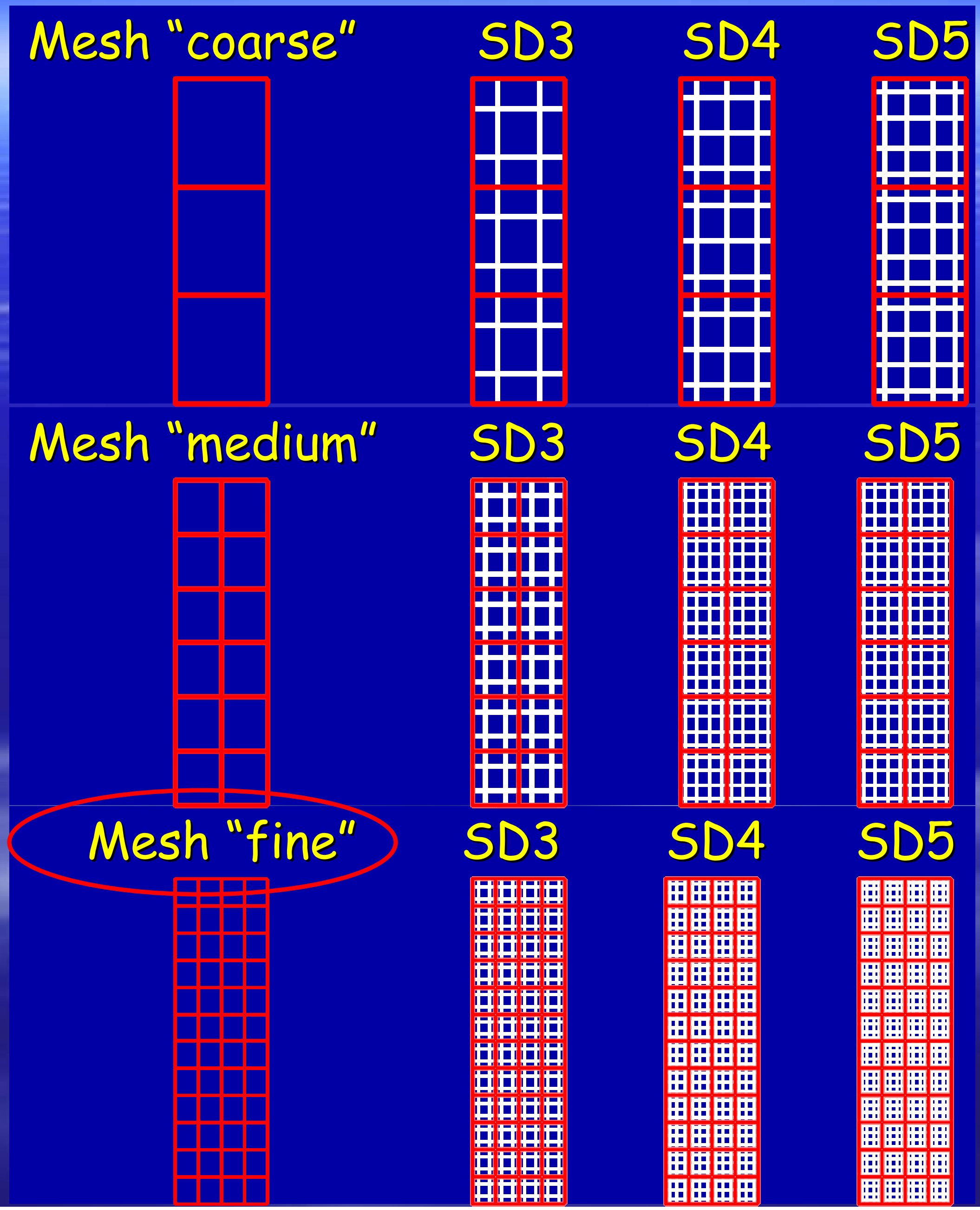
Plane strain biaxial compression test

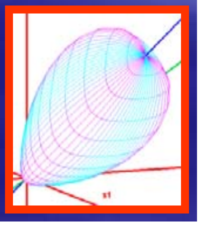
Plane strain biaxial compression test





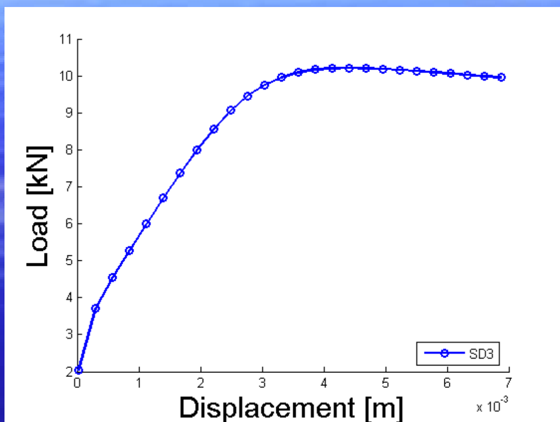
Plane strain biaxial compression test



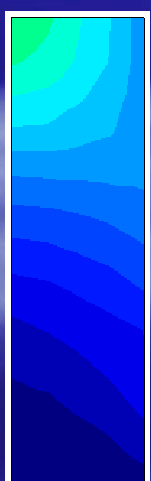


Local Approach

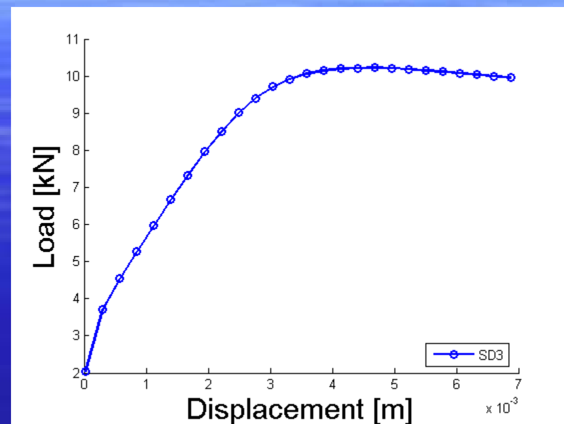
Coarse



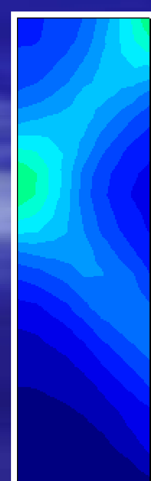
SD3



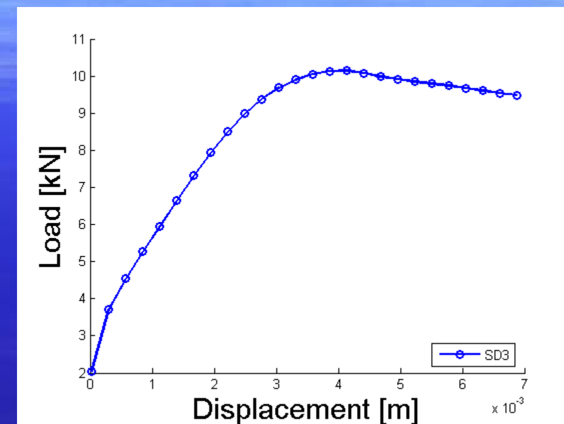
Medium



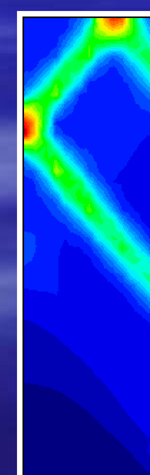
SD3

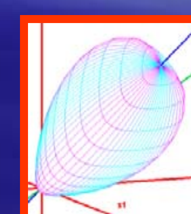


Fine



SD3





Local Approach

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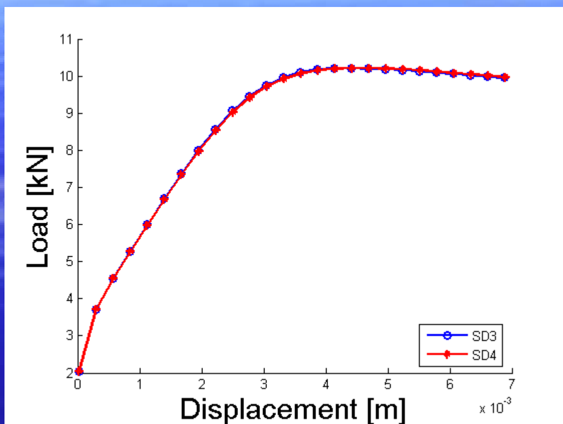
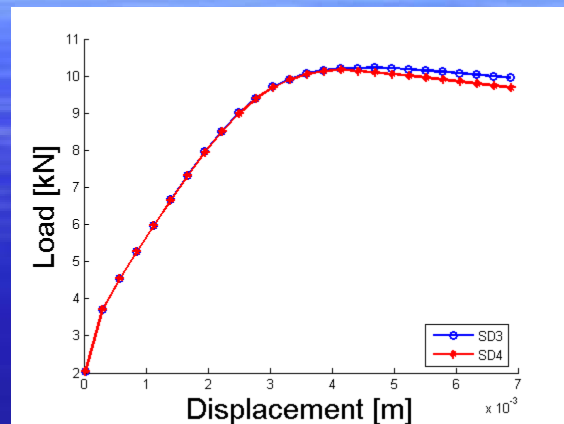
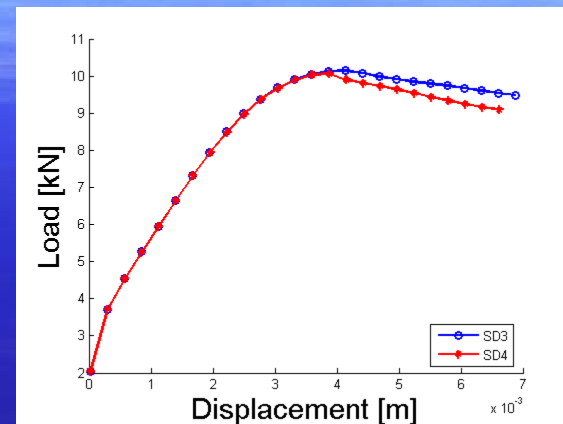


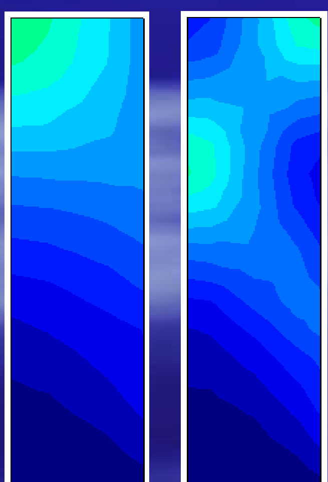
Medium



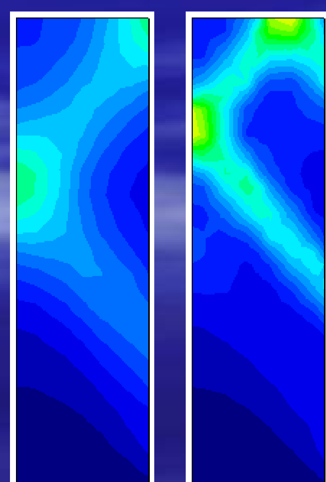
Fine



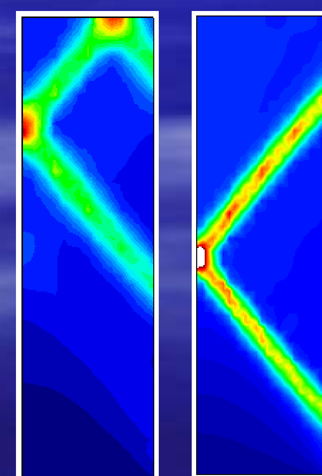
SD3 SD4

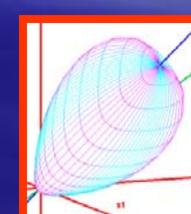


SD3 SD4



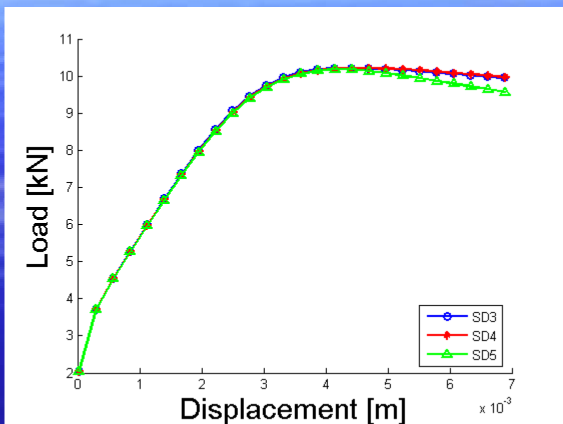
SD3 SD4



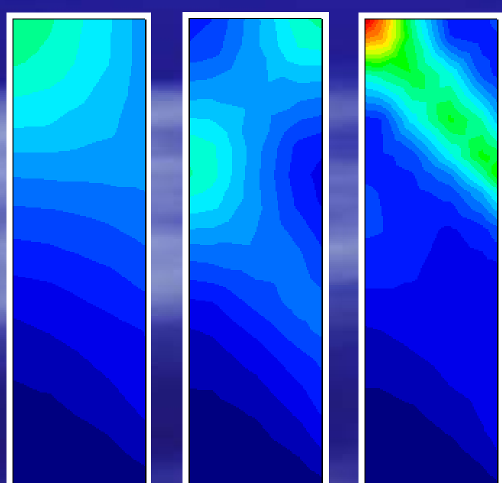


Local Approach

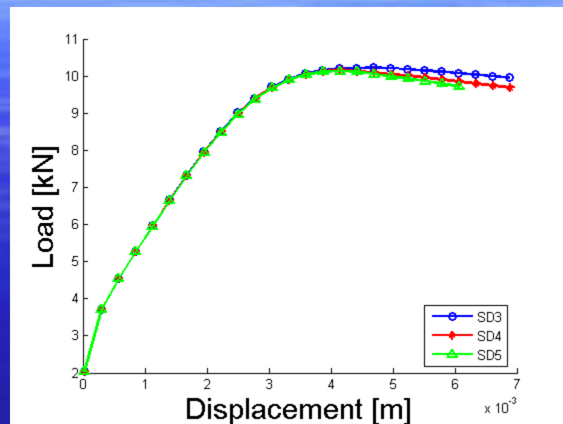
Coarse



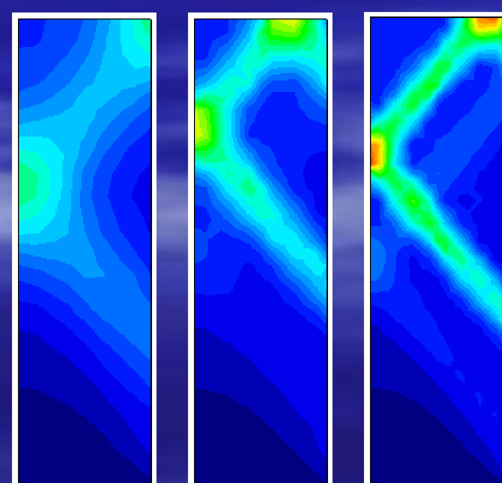
SD3 SD4 SD5



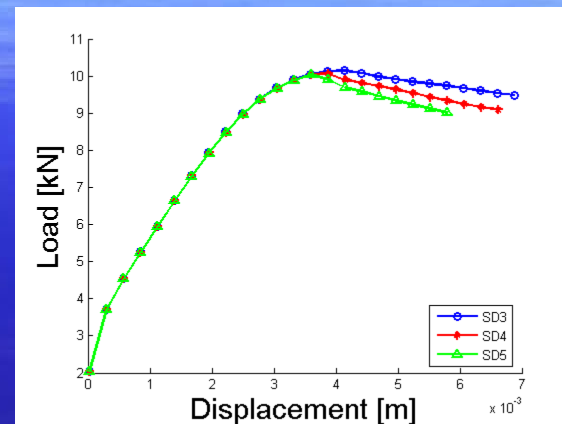
Medium



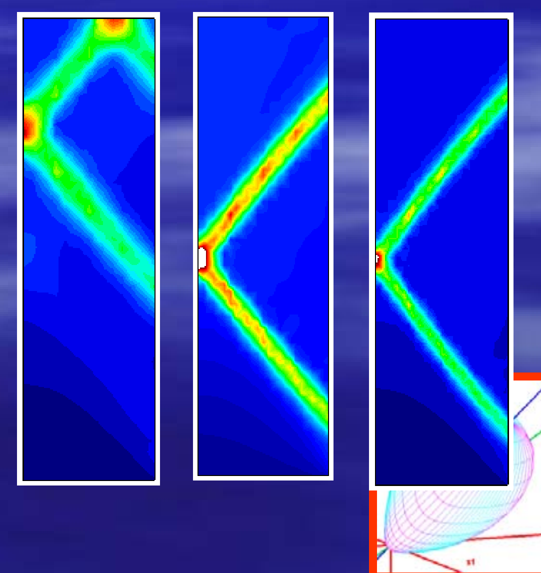
SD3 SD4 SD5



Fine

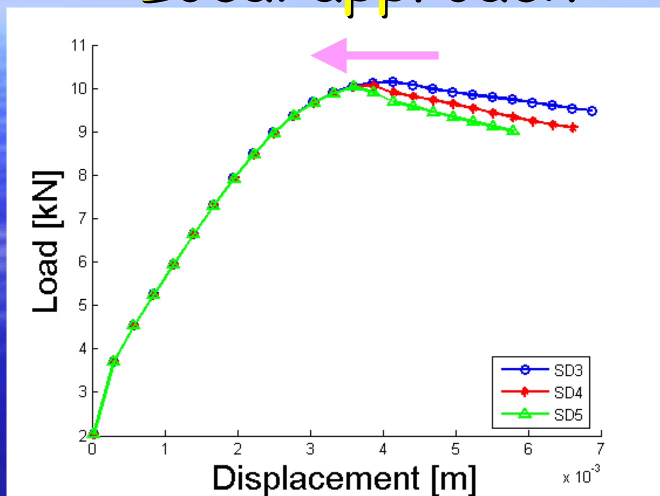


SD3 SD4 SD5

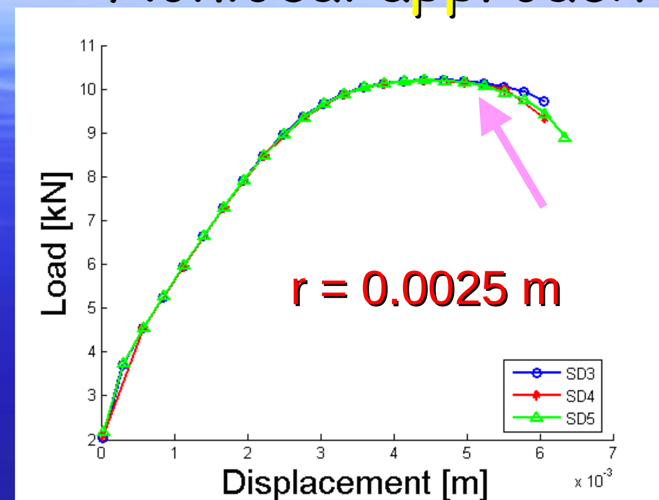


Mesh "fine"

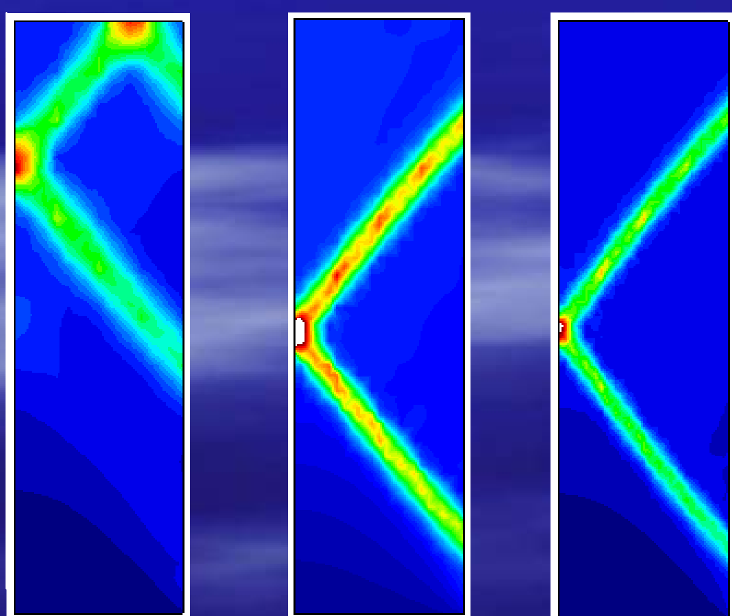
Local approach

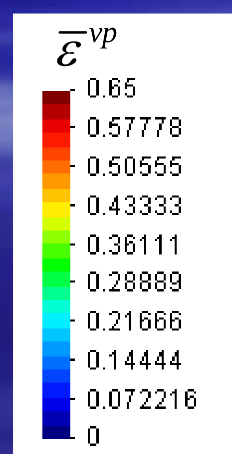


Nonlocal approach

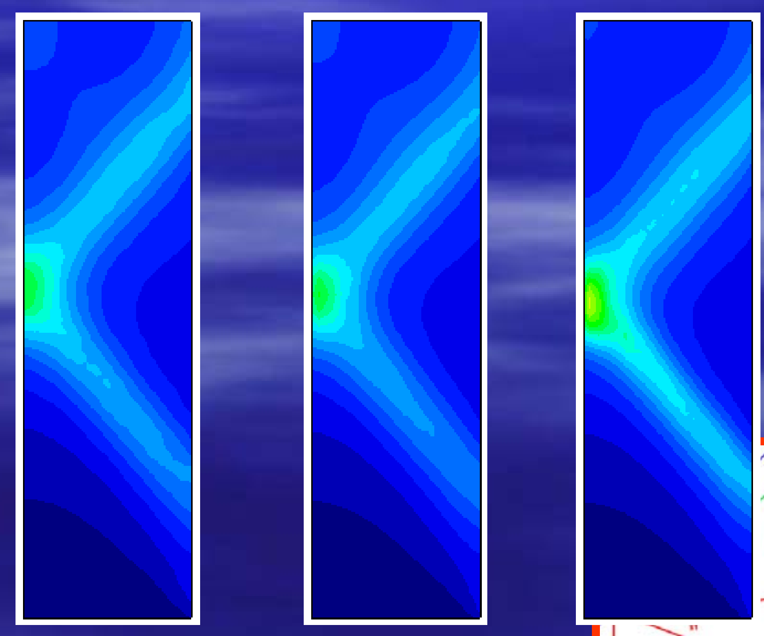


SD3 SD4 SD5





SD3 SD4 SD5

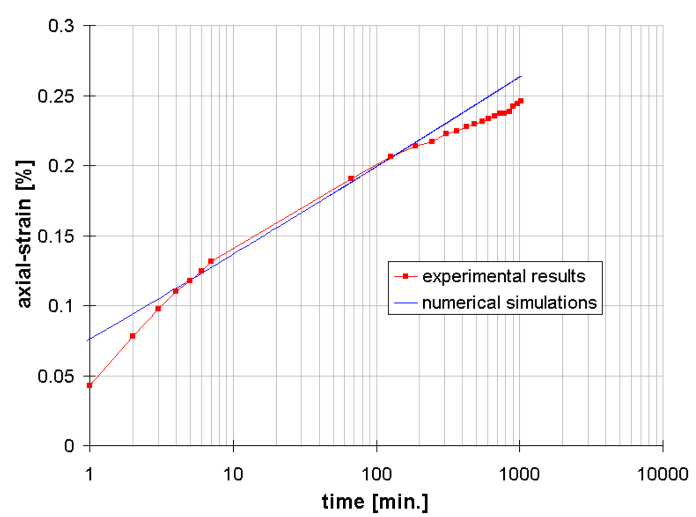


Definition of the viscous nucleus

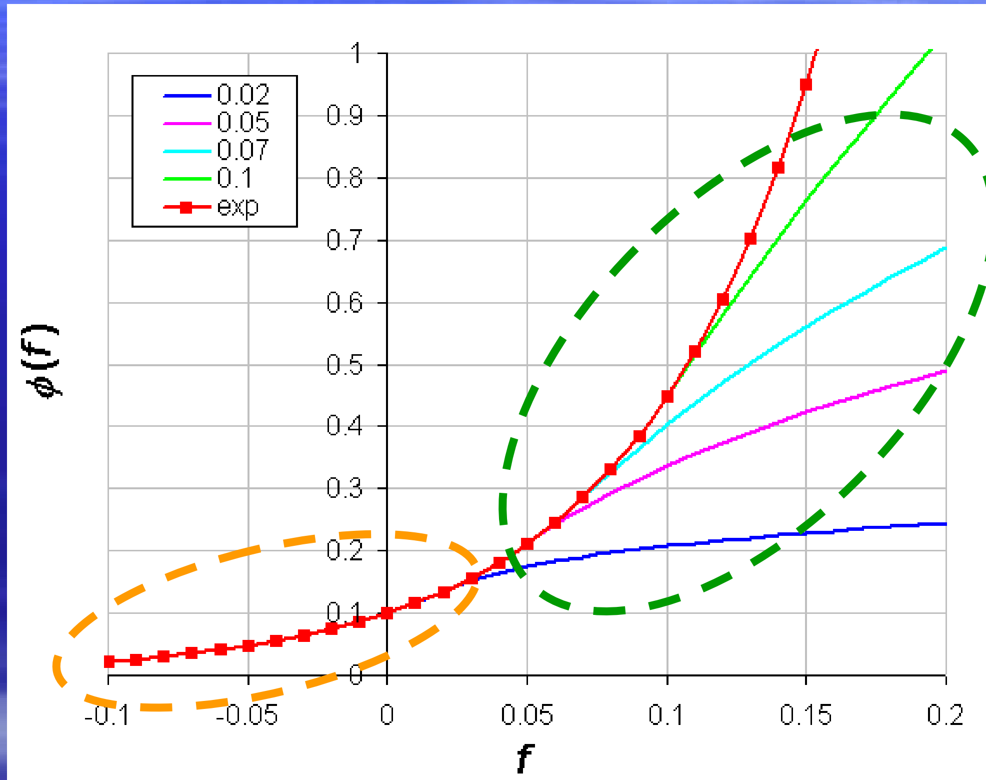
$$\dot{\varepsilon}^{vp}_{ij} = \gamma \phi(f) \frac{\partial g}{\partial \sigma'_{ij}}$$

Fast loading tests

Impact tests

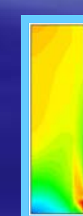


Creep test experimental data and numerical simulation [di Prisco et al. 1996]

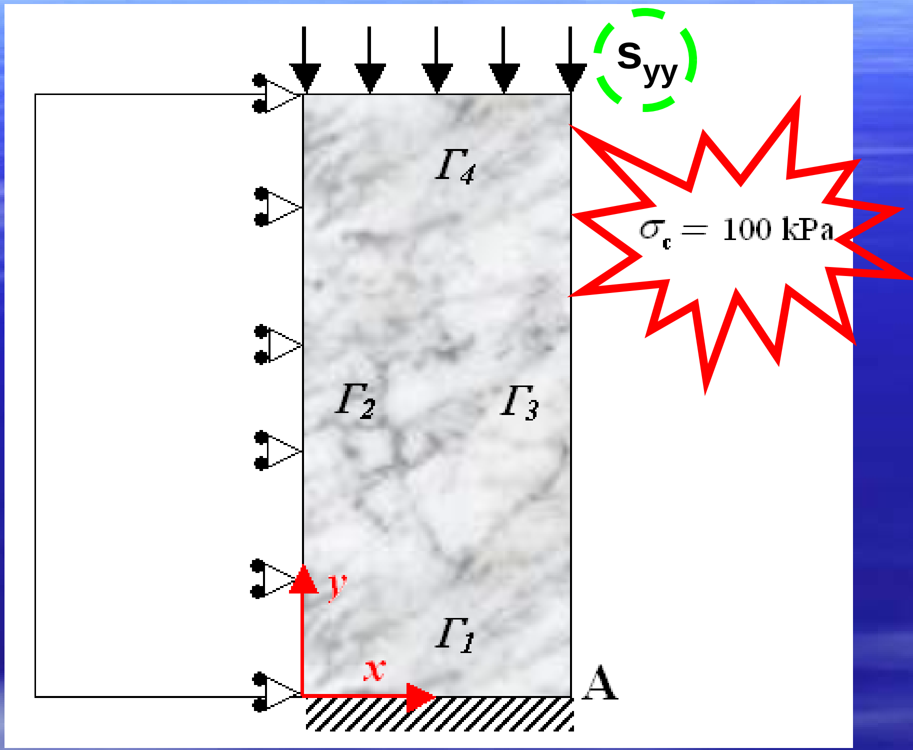


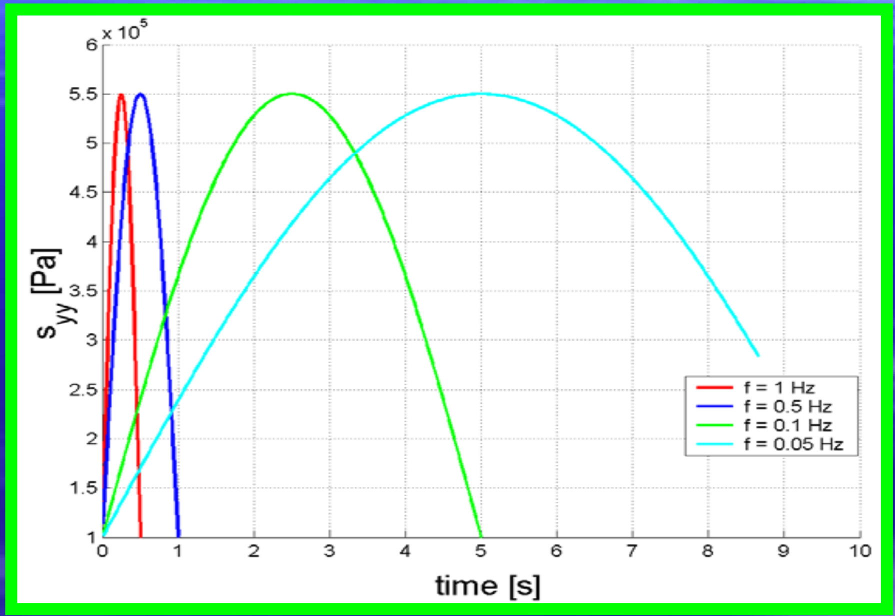
$$\Phi_1 = \gamma_{visco} \exp(\alpha f) \quad if \quad f \leq f_0$$

$$\Phi_2 = \beta \sqrt[\delta]{\log(\zeta f)} \quad if \quad f \rangle f_0$$

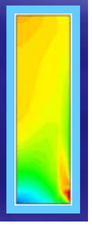


Shock load controlled biaxial compression test

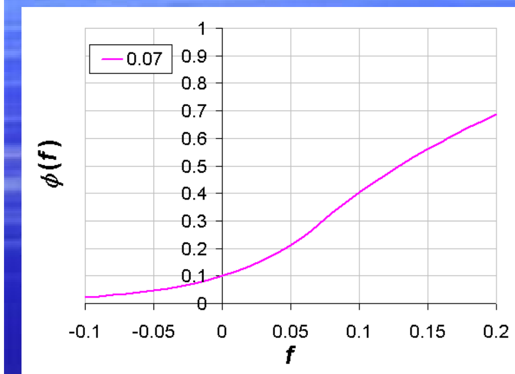


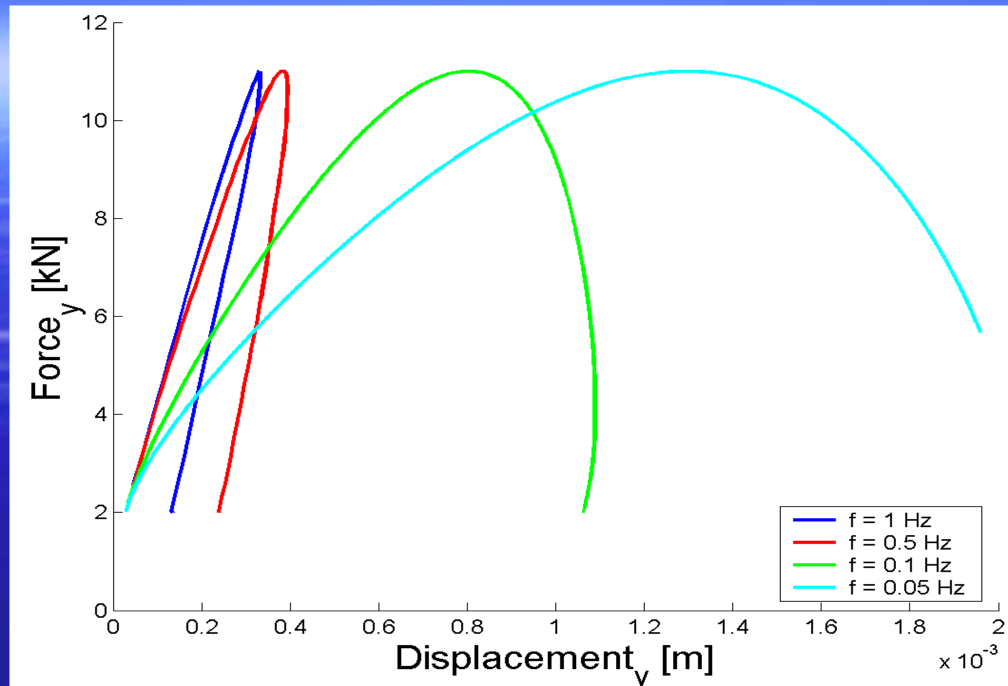


Dense sand homogeneous specimen



Shock load controlled biaxial compression test



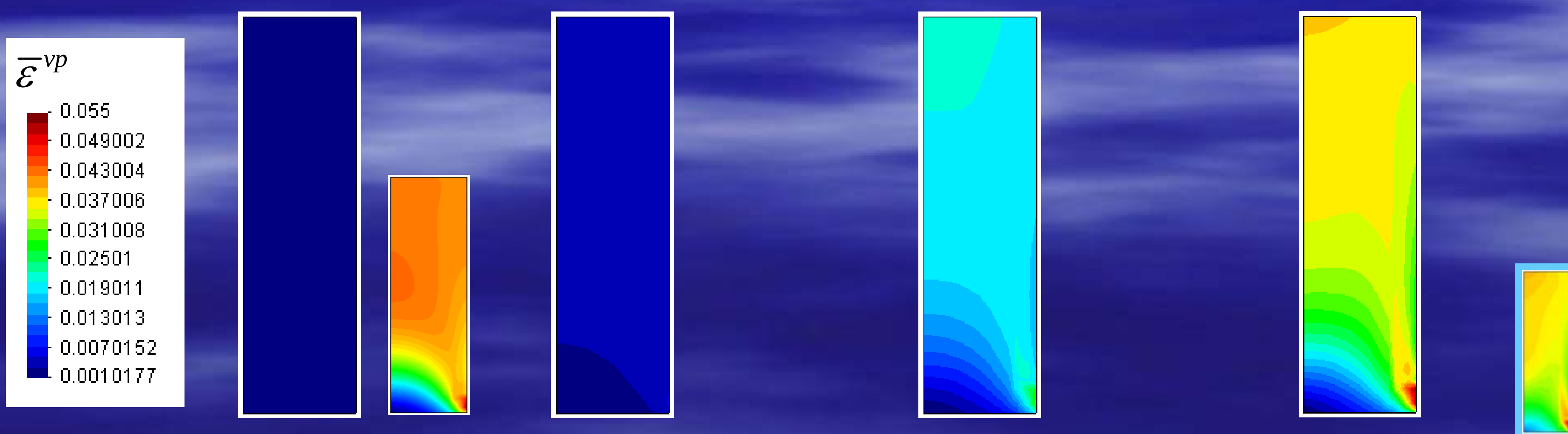


fr = 1 Hz

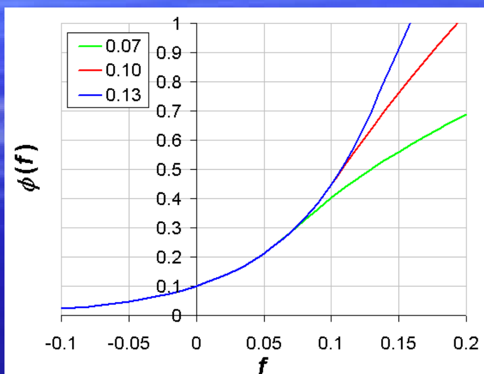
fr = 0.5 Hz

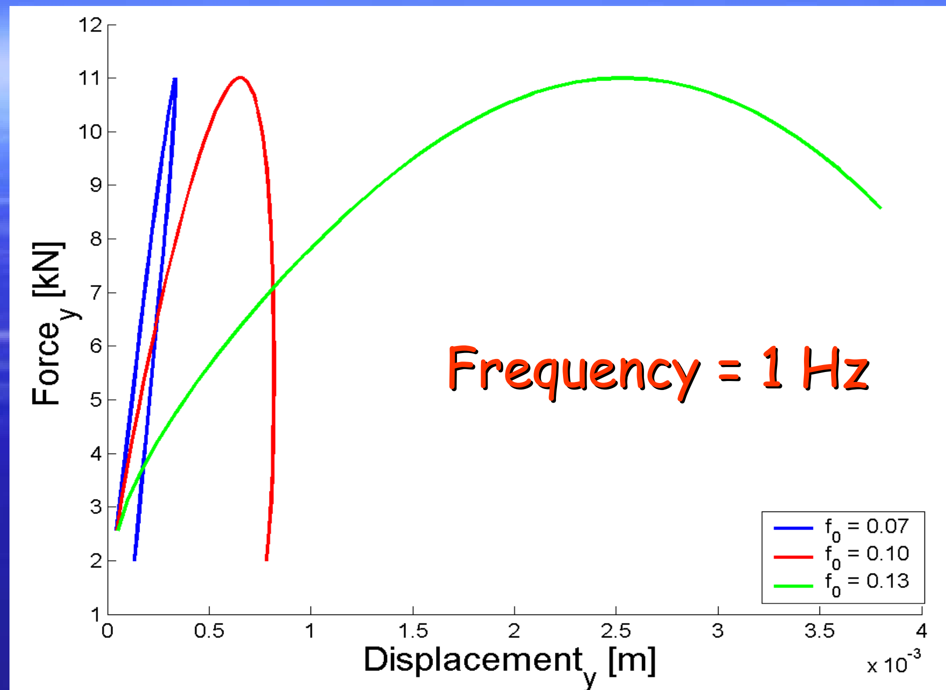
fr = 0.1 Hz

fr = 0.05 Hz

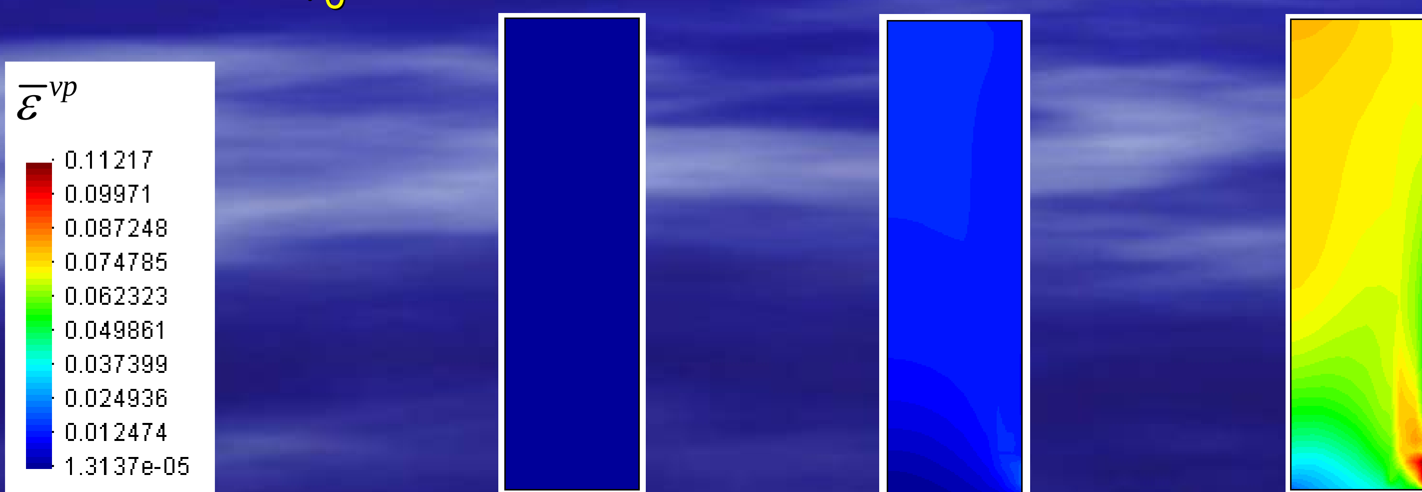


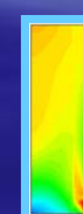
Shock load controlled biaxial compression test



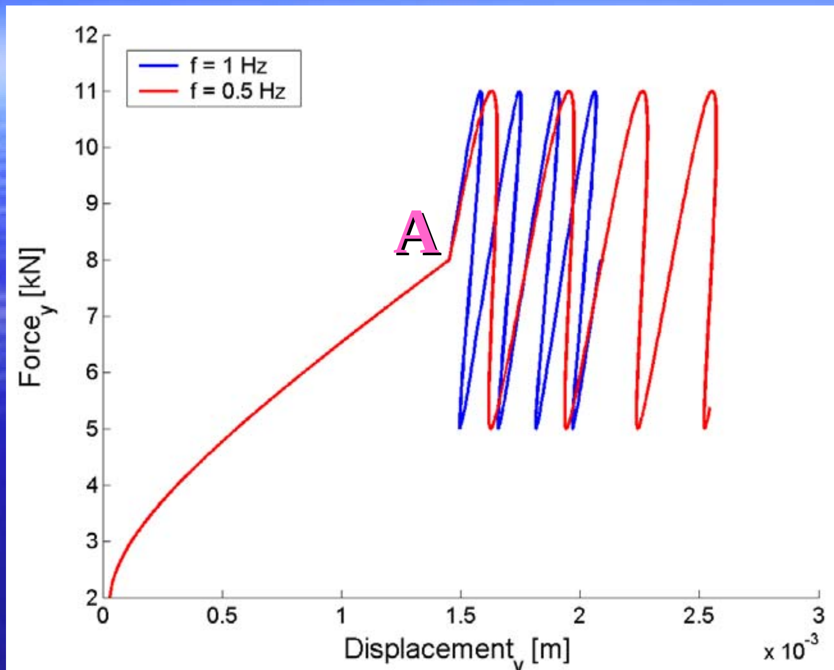


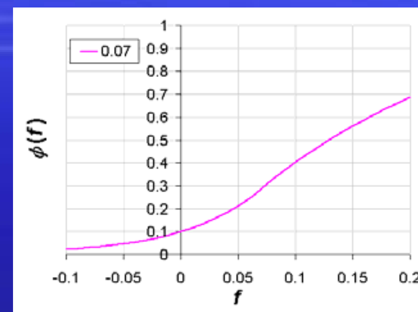
f_0 = 0.07 0.10 0.13

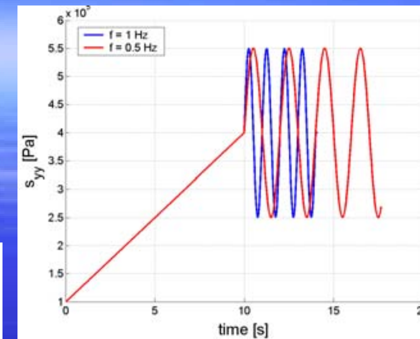


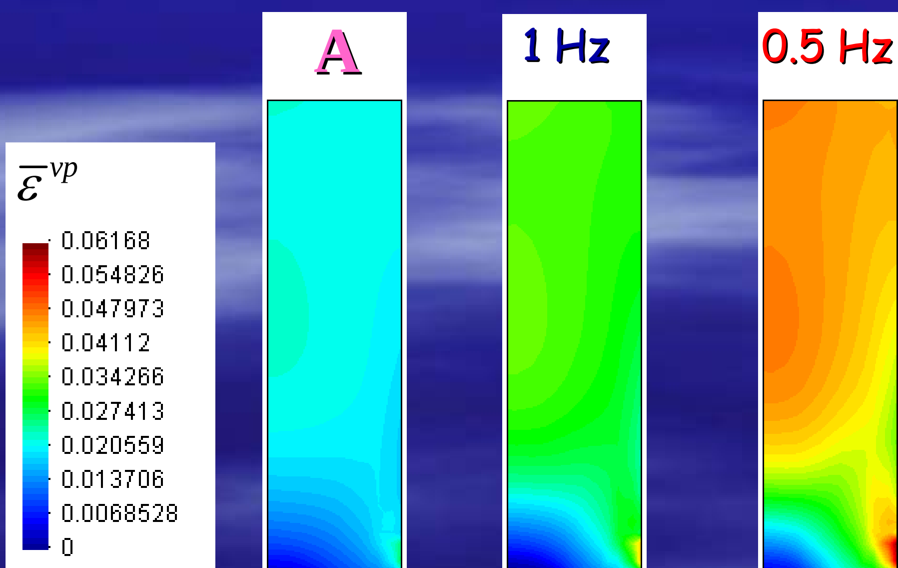


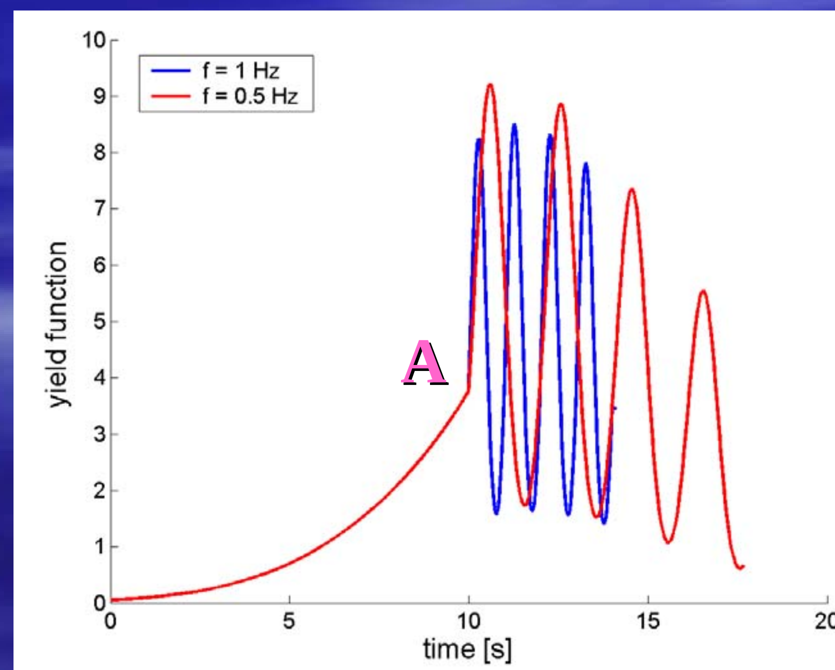
Shock load controlled biaxial compression test

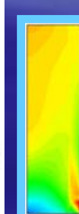




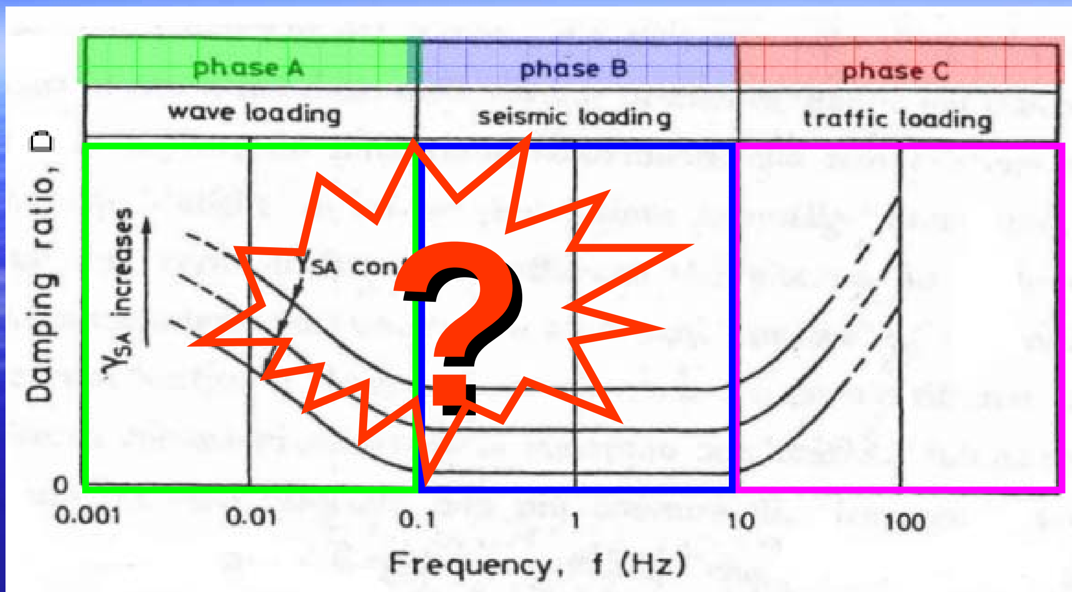


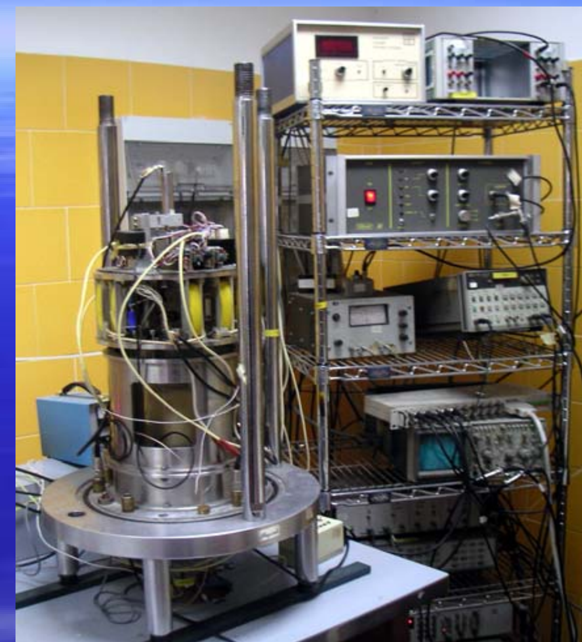


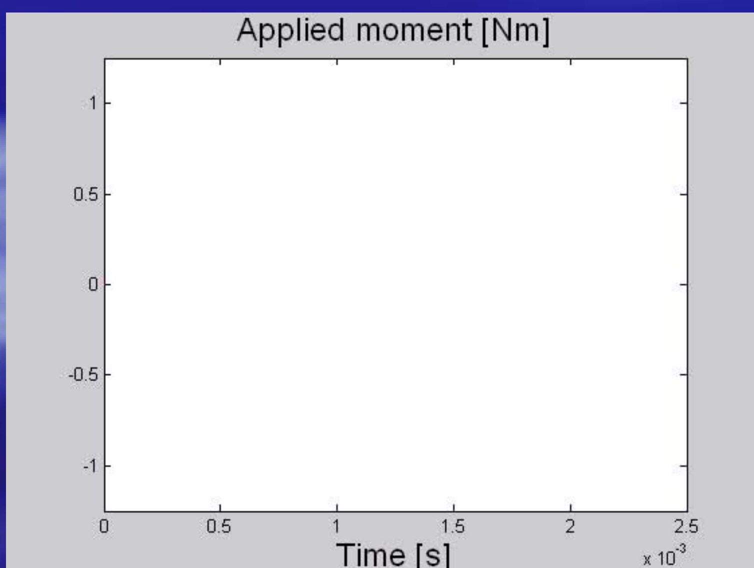


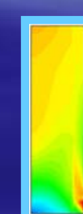


What viscous nucleus ?









THE END