Politecnico di Milano
Department of Structural Engineering

SPECTRAL ELEMENTS FOR THE ASSESSMENT OF THE SEISMIC VULNERABILITY OF STRUCTURES

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Aussois

GEO-ELSE
(GEO-ELasticity by Spectral Elements)

- GEO-ELSE is a Spectral Elements code for the study of wave propagation phenomena in 2D or 3D complex domain

- Developers:
  - CRS4 (Center for Advanced, Research and Studies in Sardinia)
  - Politecnico di Milano, DIS (Department of Structural Engineering)

- Native parallel implementation

- Naturally oriented to large scale applications (> at least 10^6 grid points)

Why using spectral elements ?

- Suitable for modelling a variety of physical problems (acoustic and elastic wave propagation, thermo elasticity, fluid dynamics)

- Accuracy of high-order methods

$$\left\| \bar{u} - u_{h,N} \right\| \leq C\, h^{N} e^{-N}$$

- Suitable for implementation in parallel architectures

- Great advantages from last generation of hexahedral mesh creation program (e.g.: CUBIT, Sandia Lab.)

Formulation of the elastodynamic problem

Dynamic equilibrium in the weak form:

$$\frac{\partial^2}{\partial t^2}\int_\Omega \rho u_i v_i d\Omega + \int_\Omega \sigma_{ij}\varepsilon_{ij} d\Omega = \int_\Gamma t_i v_i + \int_\Omega f_i v_i$$

where u_i = unknown displacement function

v_i = generic admissible displacement function (test function)

t_i = prescribed tractions at the boundary Γ

f_i = prescribed body force distribution in Ω

Time advancing scheme

Finite difference 2nd order (LF2 – LF2)

$$\begin{cases} \dfrac{\partial^2 \boldsymbol{u}}{\partial t^2} = \dfrac{\boldsymbol{u}_{n+1} - 2\boldsymbol{u}_n + \boldsymbol{u}_{n-1}}{\Delta t^2} \\ \dfrac{\partial \boldsymbol{u}}{\partial t} = \dfrac{\boldsymbol{u}_{n+1} - \boldsymbol{u}_{n-1}}{2\Delta t} \end{cases}$$

Courant-Friedrichs-Levy (CFL) stability condition

$$\Delta t \le \eta \left(\frac{\Delta x}{c} \right)_{\min}$$

Spatial discretization

Spectral element method SEM (Faccioli et al., 1997)

Bibliography

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Forward modelling by the Fourier method
Geophysics 1982; 47: 1402-1412

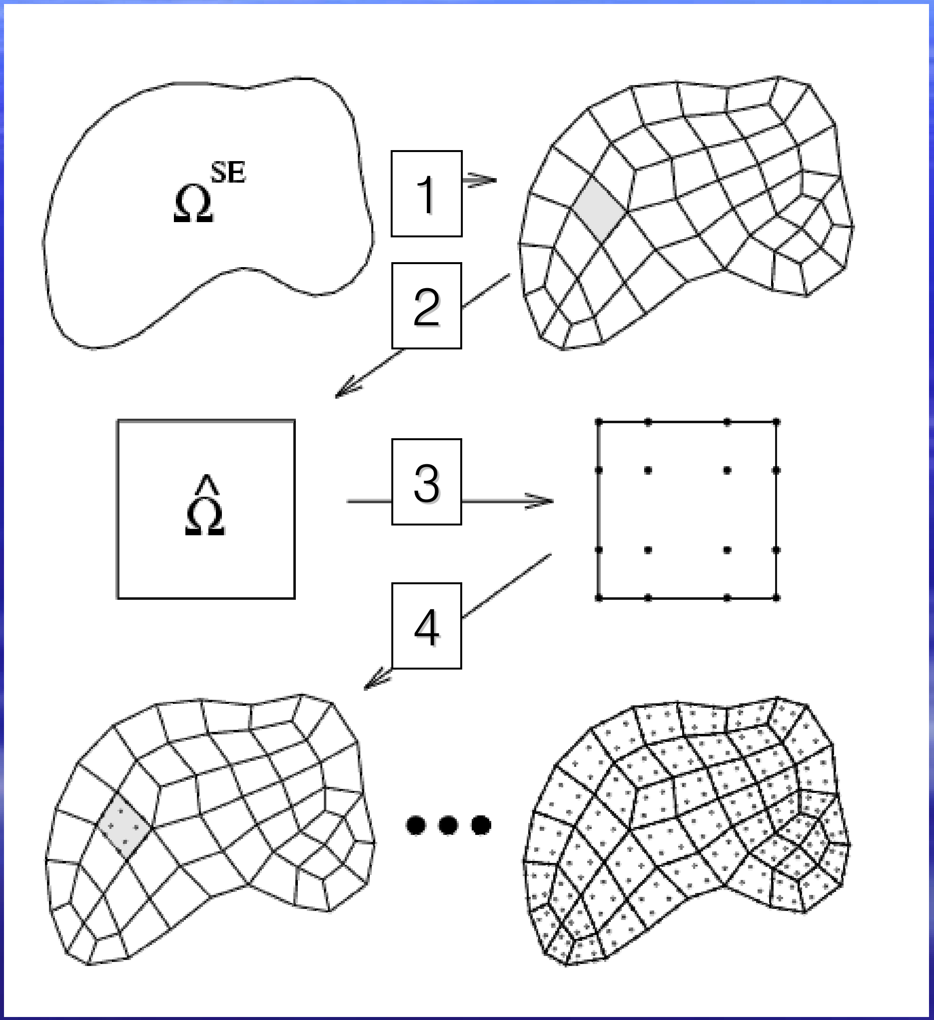
Kosloff D, Kessler D, Filho AQ, Tessmer E, Behle A, Strahilevitz R.
Solutions of the equations of dynamics elasticity by a Chebyshev spectral method
Geophysics 1990; 55: 748-754

Faccioli E, Maggio F, Paolucci R, Quarteroni A.
2D and 3D elastic wave propagation by a pseudo-spectral domain decomposition method
Journal of Seismology 1997; 1 237-251.

Komatitsch D, Vilotte J-P.
The spectral element method: an efficient tool to simulate the seismic response of 2D and 3D geological structures.
Bull. Seism. Soc. Am. 1998; 88: 368-392.

Spectral discretization of the spatial domain

- The domain is split into quadrilaterals (hexahedra)
- Each subdomain is mapped onto a reference element
- LGL nodes are introduced
- Spectral grid-points are mapped back onto the domain



The Legendre-Gauss-Lobatto quadrature formula

$$\int_{-1}^{1} f(x)dx \cong \sum_{k=0}^{N} \alpha_k f(x_k)$$

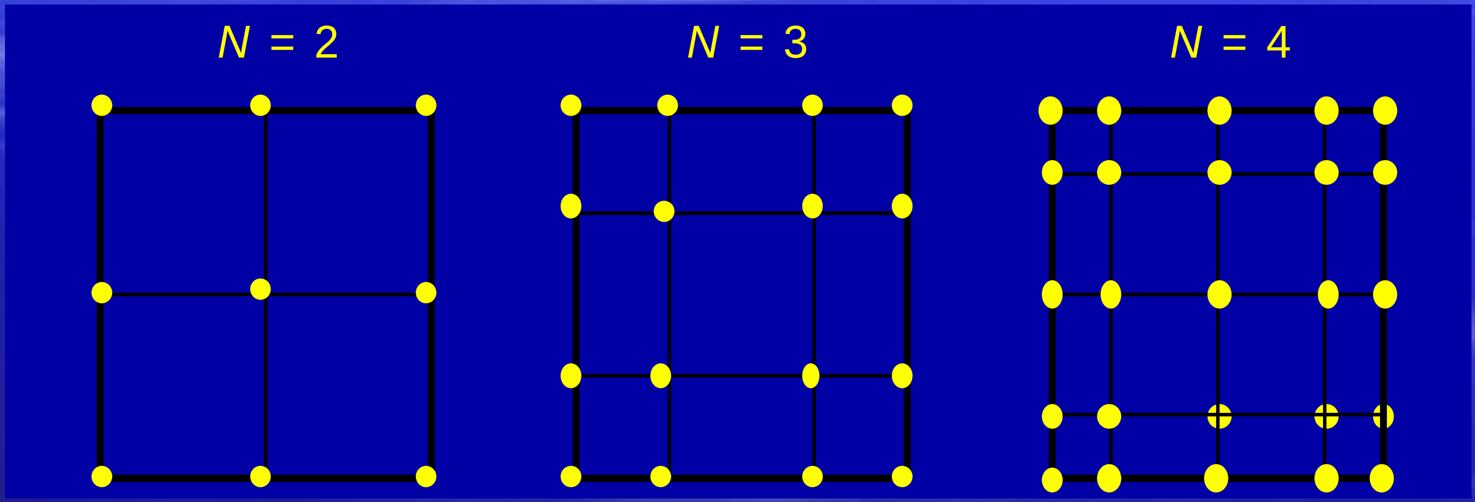
where

$$\alpha_k = \frac{2}{N(N+1)} \frac{1}{L_N^2(x_k)}$$

$L_N(x_k)$ being the Legendre orthogonal polynomial of degree N, calculated at the LGL node x_k

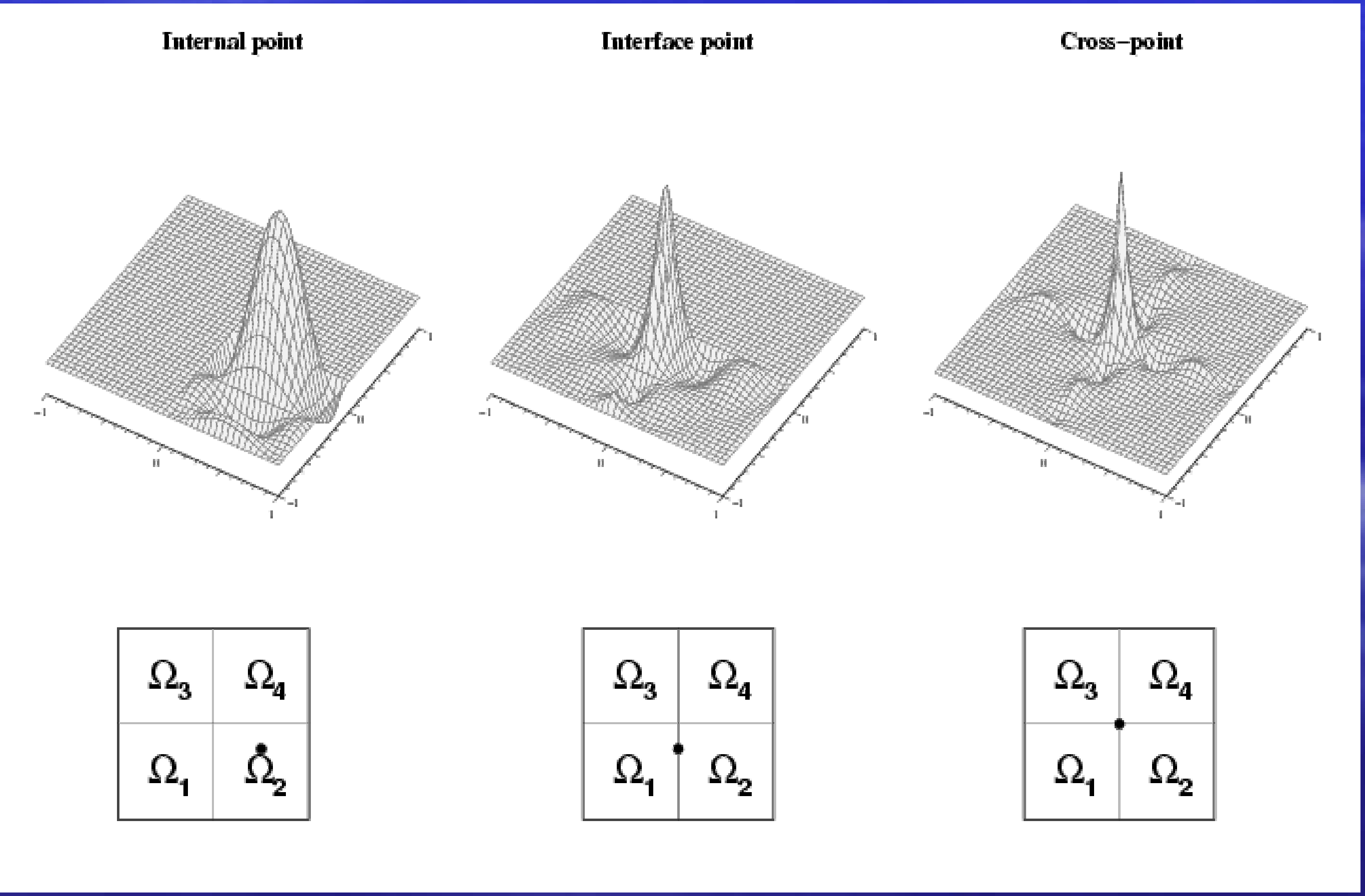
LGL nodes: $L'_N(x_k) = 0 \quad [x_0 = -1, \ldots, x_N = 1]$

2D spectral elements and LGL nodes for different values of the polynomial degree



Selection of the test functions

A suitable choice is the Lagrange polynomial of degree N, which is equal to one at the j^{th} LGL node and vanishes at all other nodes



FEM/SEM comparison

Convergence of the method

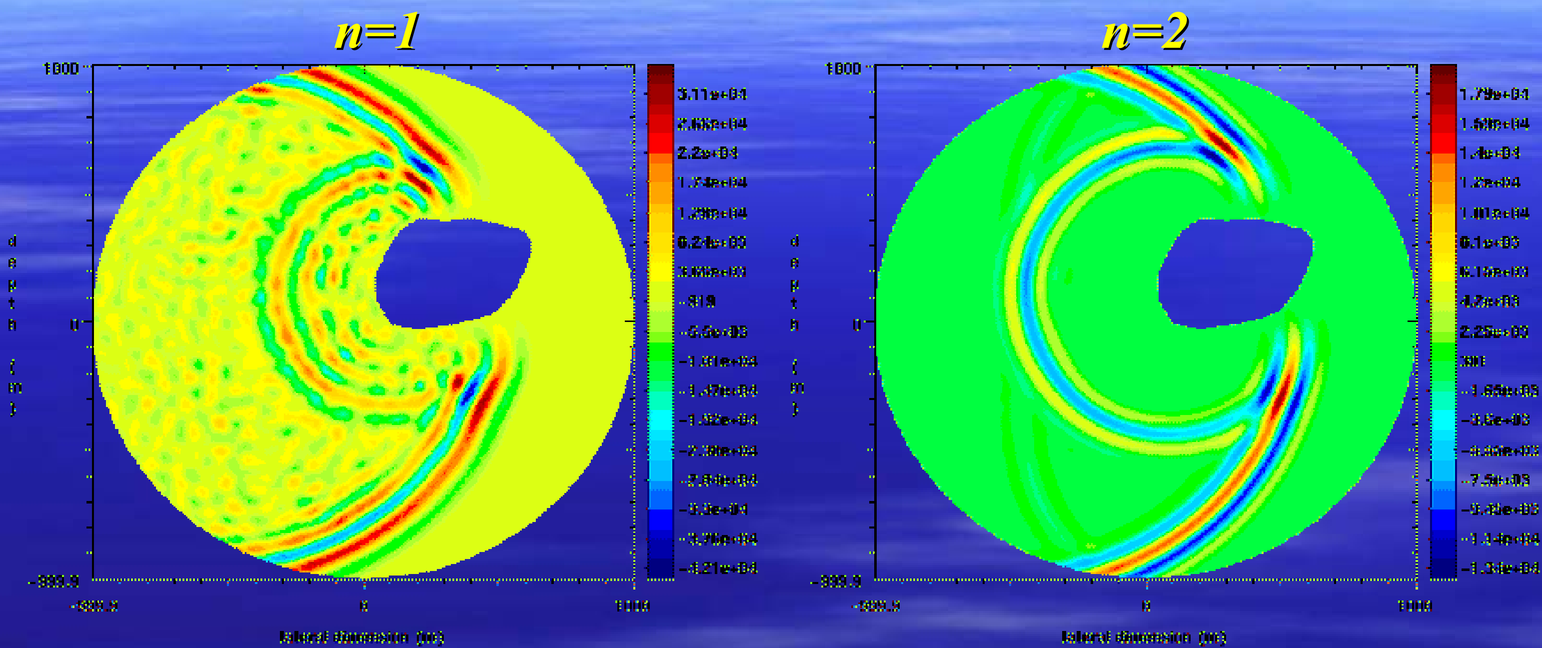
FEM → $\left\| \bar{u} - u_{h,N} \right\| \leq C\,h^{N}$

SEM → $\left\| \bar{u} - u_{h,N} \right\| \leq C\,h^{N} e^{-N}$

The SEM converges more rapidly then FEM or, equivalently, it is more accurate for a given number of nodal points (spectral accuracy)

Why using spectral elements ?

acoustic problem #1

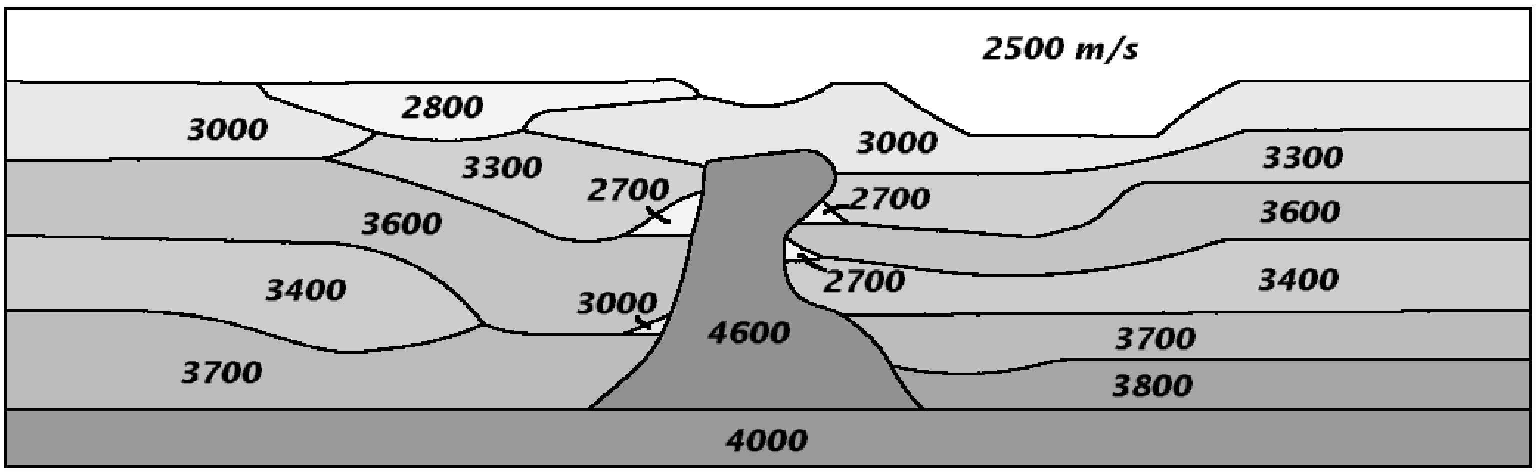


Acoustic wave propagation through an irregular domain. Simulation with spectral degree 1 *(left)* exhibits numerical dispersion due to poor accuracy.

Simulation with spectral degree 2 *(right)* provides better results. Change of spectral degree is done at **run time**.

Why using spectral elements ?
acoustic problem #2

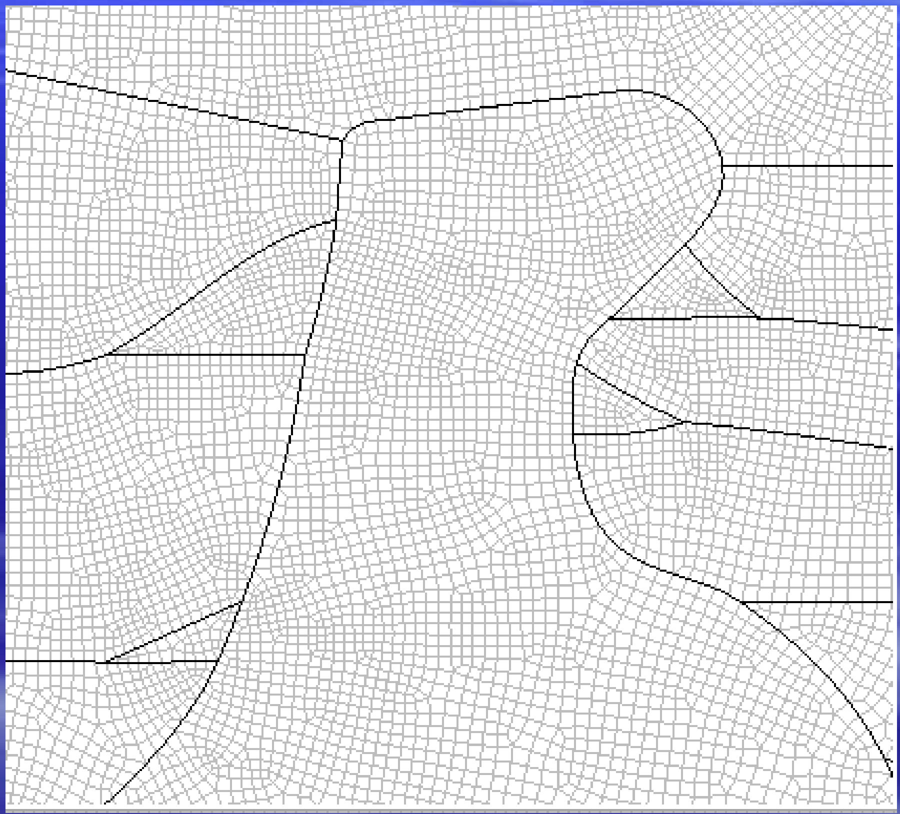
Domain geometry



Courtesy of Dr. F. Maggio

Why using spectral elements ?
acoustic problem #2

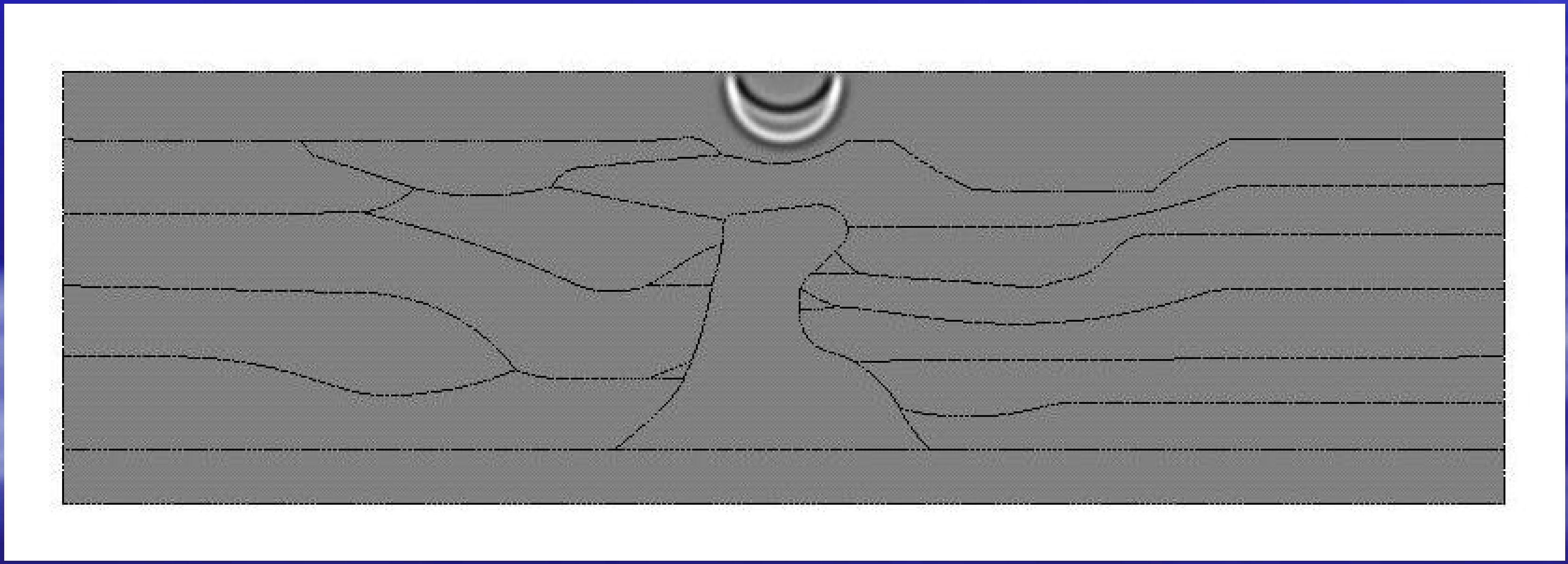
- Fully unstructured mesh (~ 39000 quads)
- Spectral degree *n*=4
- ~ 618000 spectral grid-points



Courtesy of Dr. F. Maggio

Why using spectral elements ?
acoustic problem #2

- Snapshots are taken every 0.25 s, up to 2.00 s
- Pressure values are normalized to [-1,+1]
  (from black to white).



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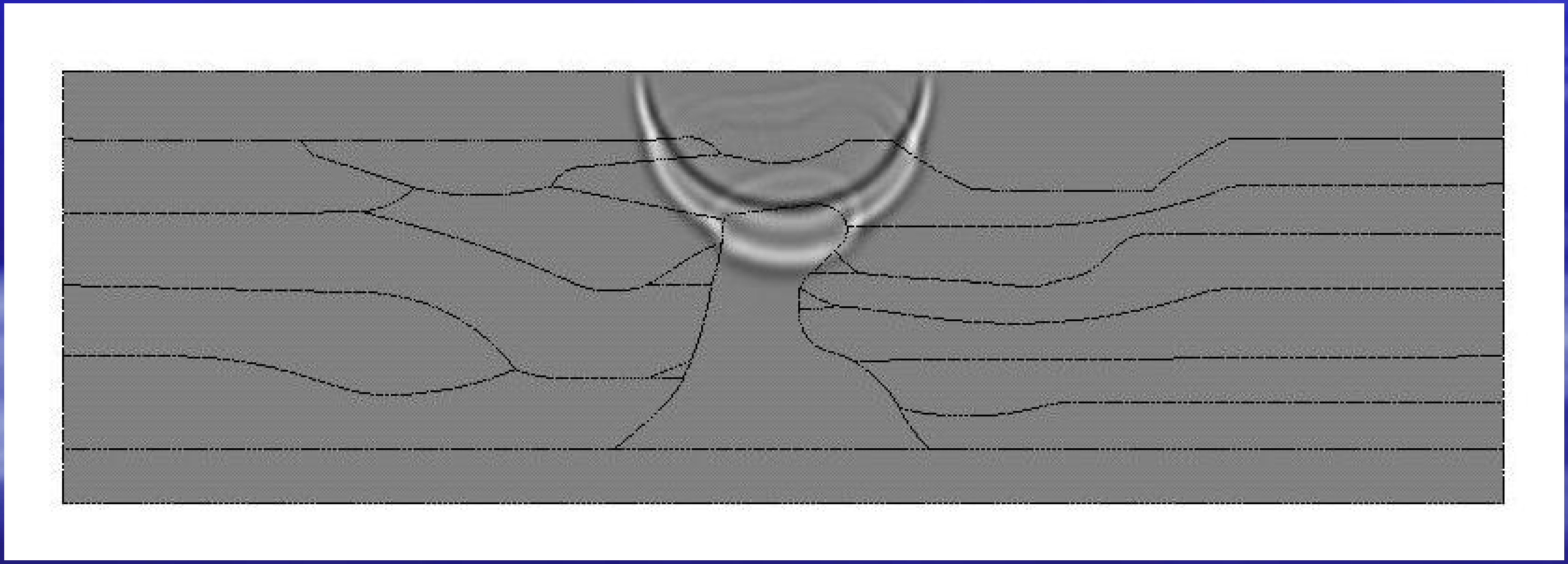

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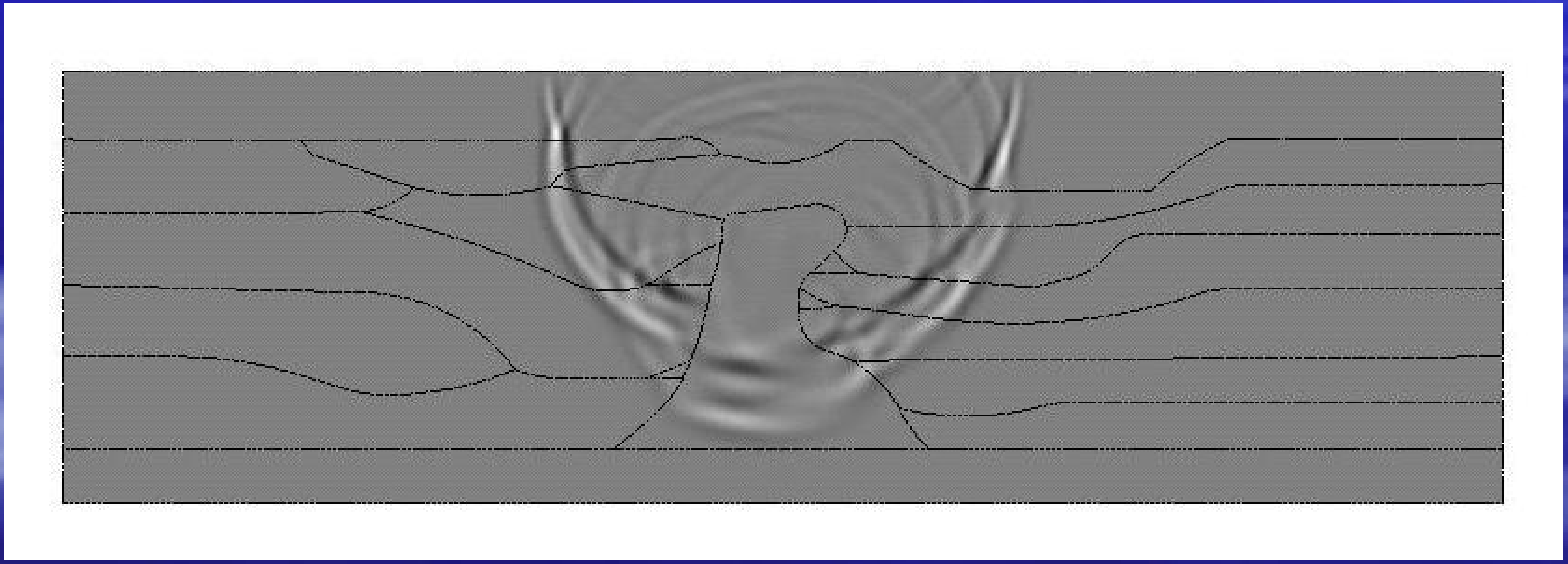

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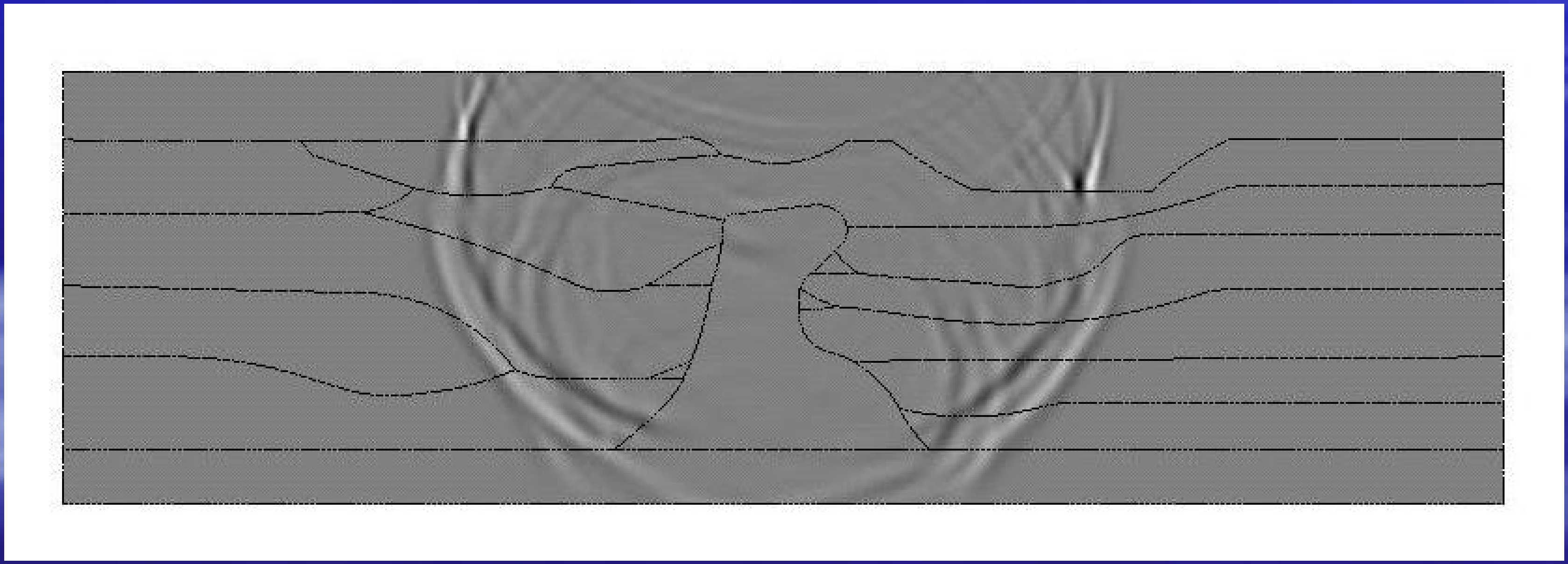

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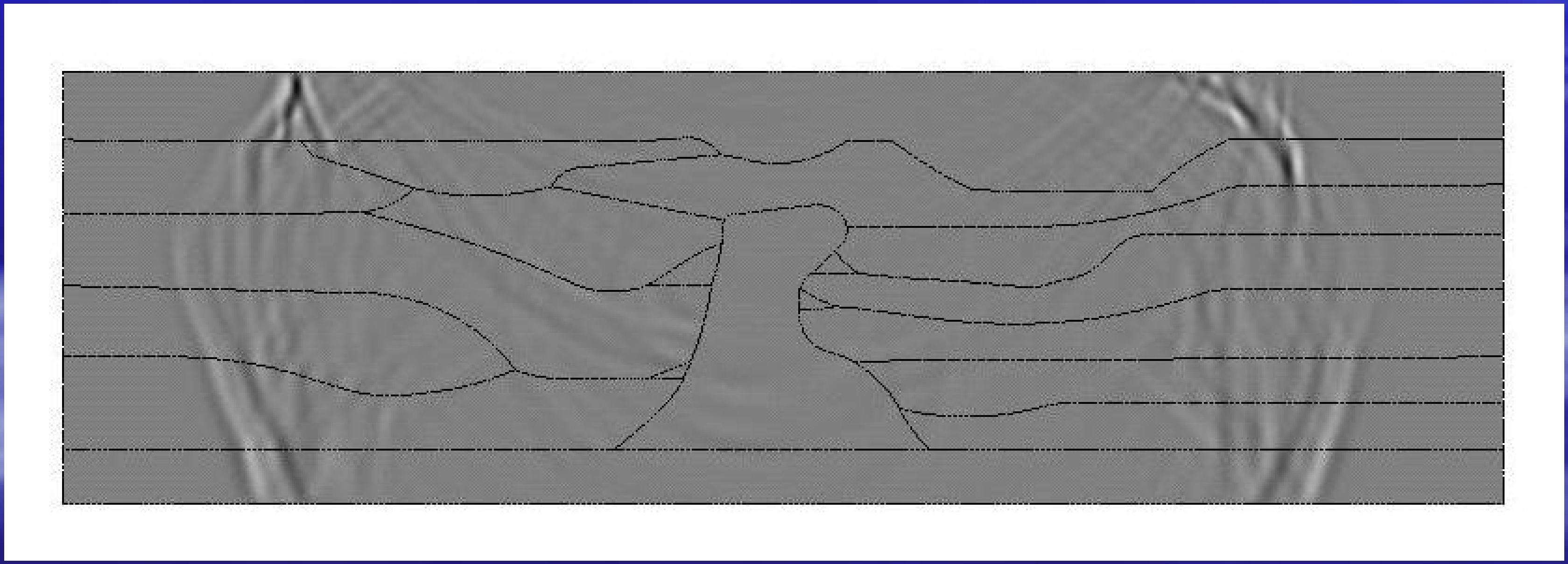

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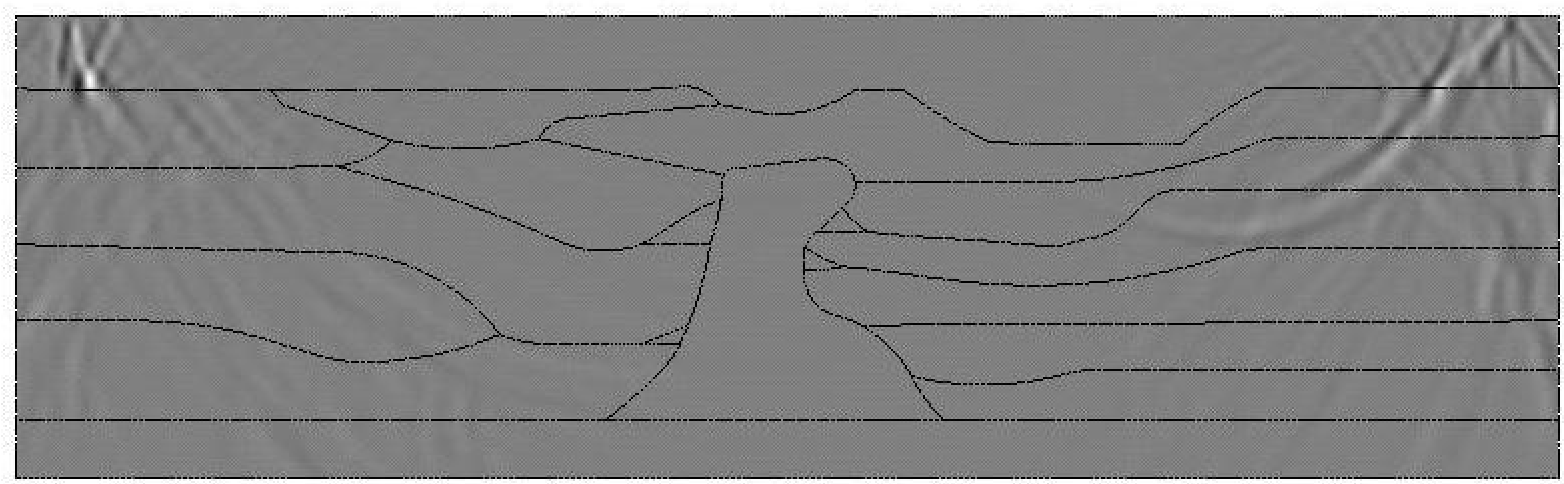

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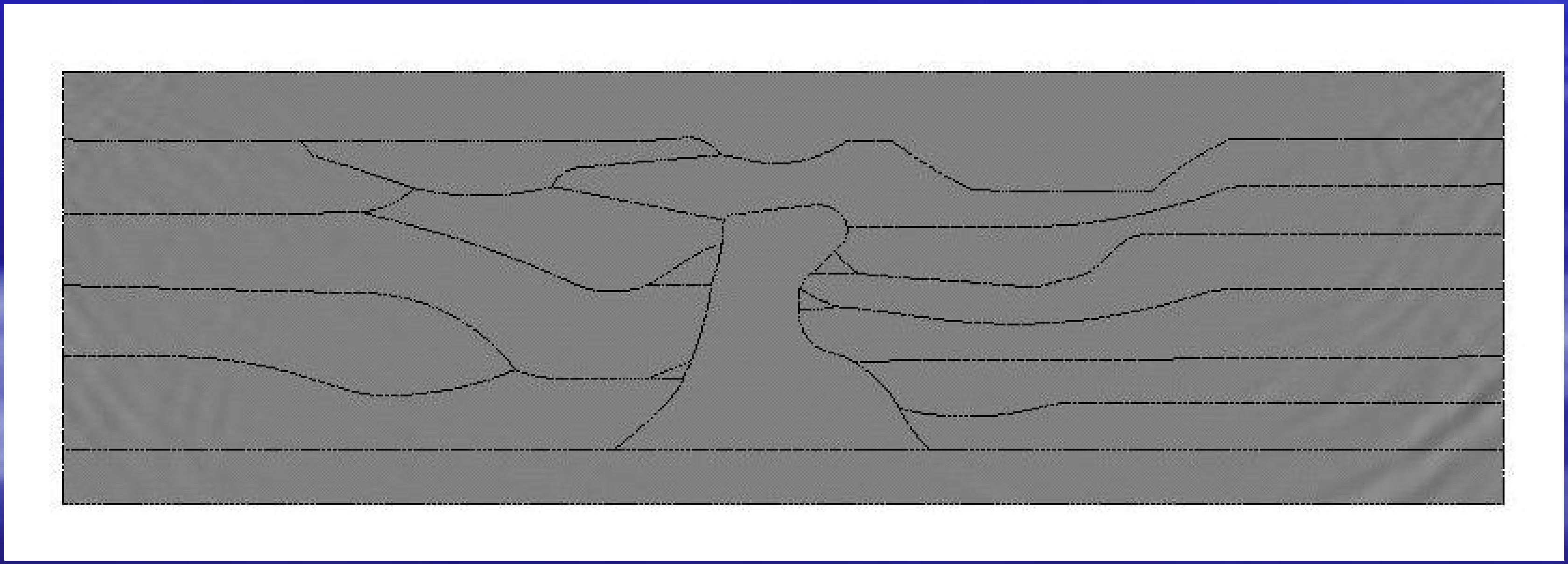

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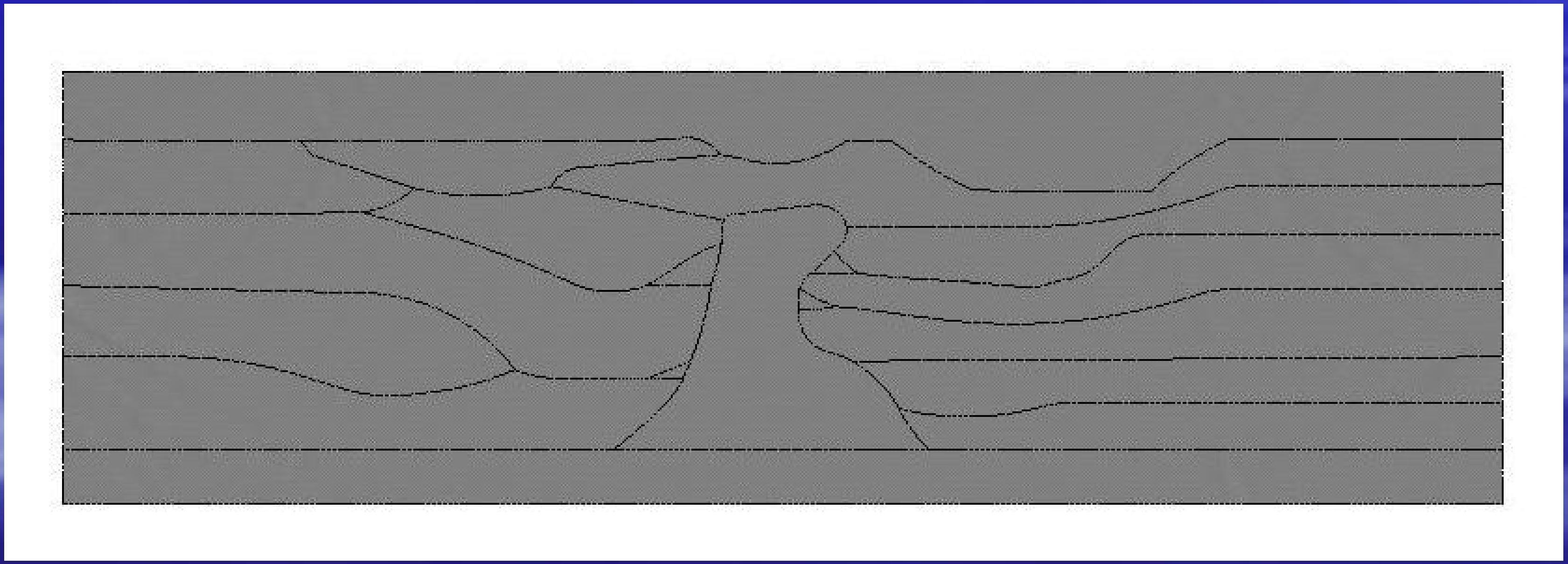


Courtesy of Dr. F. Maggio

FEM/SEM comparison

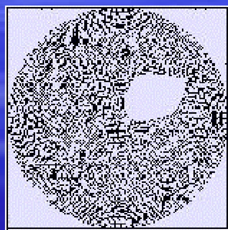
Mass matrix

FEM → Non diagonal and symmetric

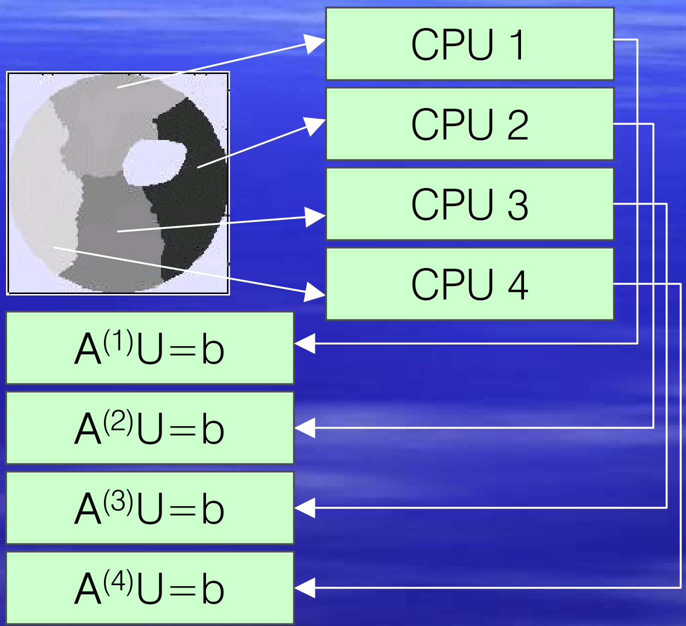
SEM → Diagonal

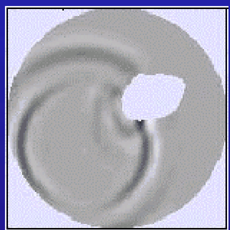
SEM coupled with an explicit time approximation scheme is very effective in term of CPU time saving

Parallel Strategy adopted in GEO-ELSE



Computational domain partitioning (METIS, University of Minnesota, USA)





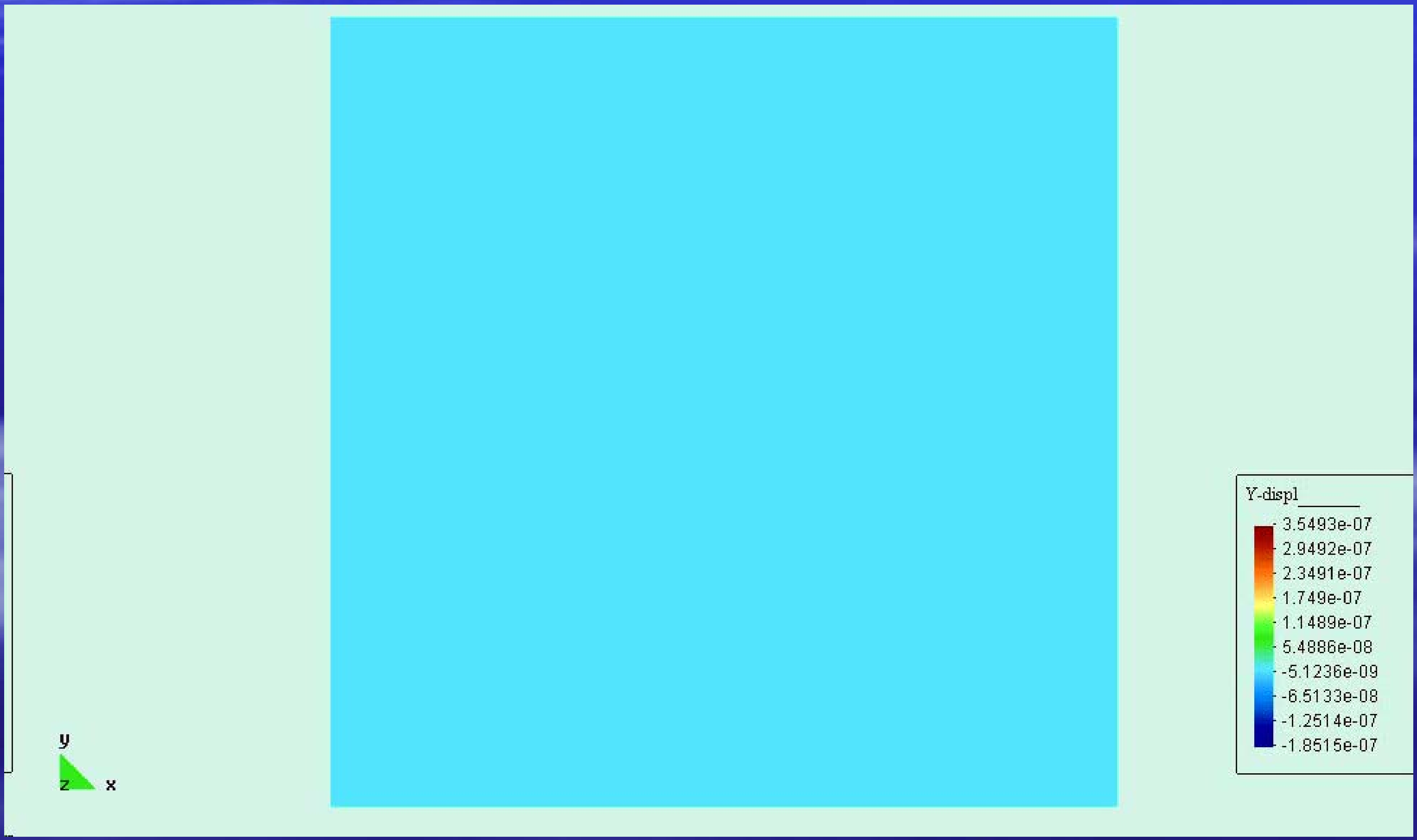
Algebraic system solution (AZTEC, Sandia National Laboratories, USA)

Parallel Efficiency = $T_{seq} / (N_{CPU} \cdot T_{par}) \approx 90\%$

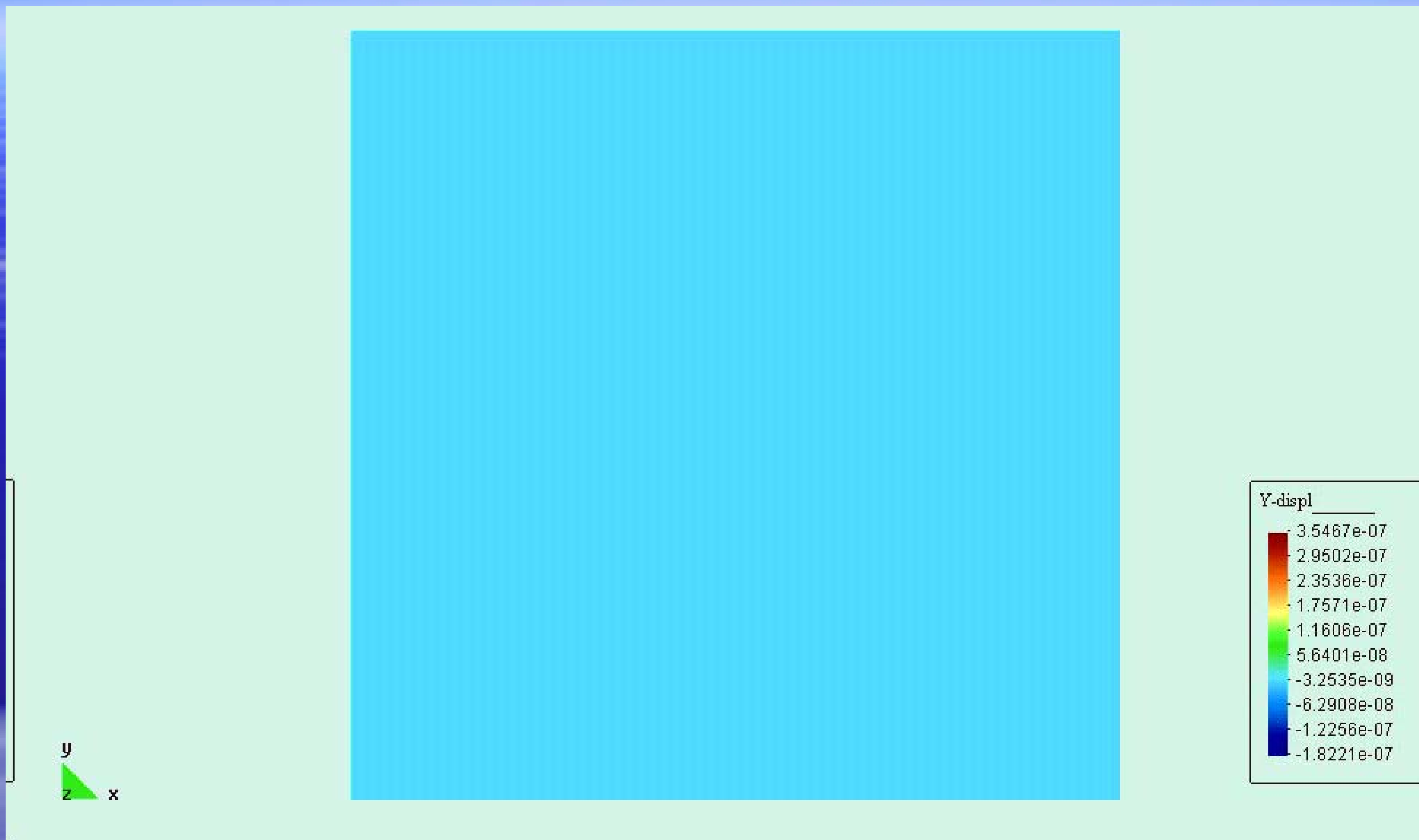
Courtesy of Dr. F. Maggio

Absorbing boundaries

Stacey 1st order (1988, "improved" paraxial conditions)



Viscoelastic damping



Modified wave equation

$$\frac{\partial^2 u}{\partial t^2} \Rightarrow \frac{\partial^2 u}{\partial t^2} + 2\gamma \frac{\partial u}{\partial t} + \gamma^2 u$$

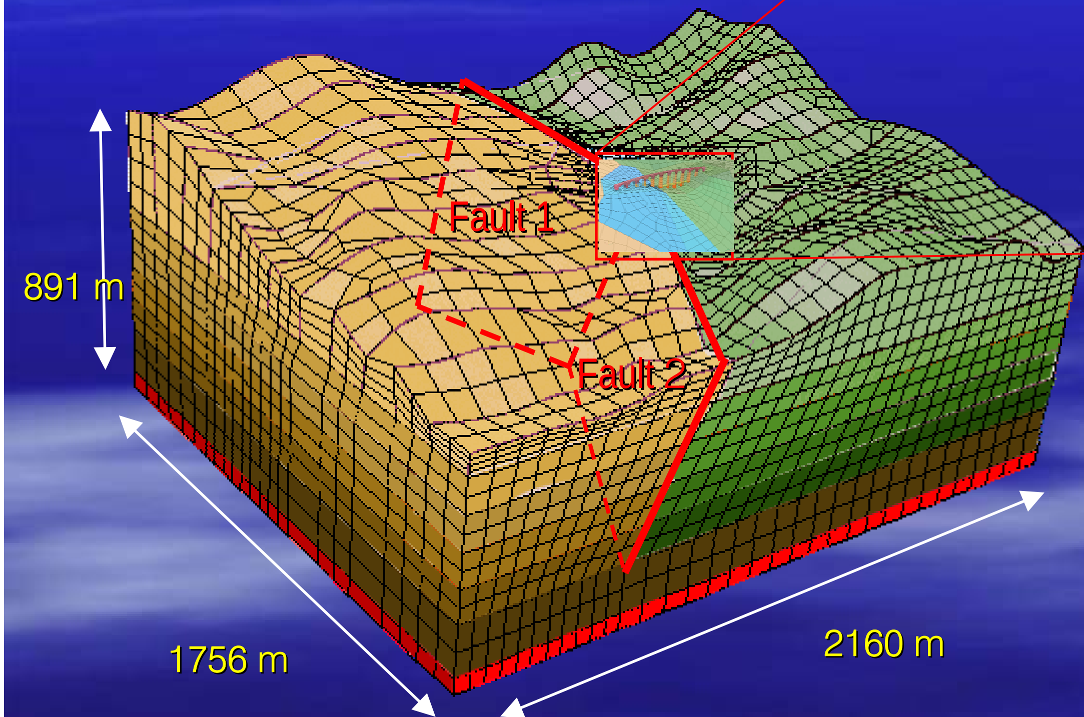
Q factor, function of the frequency

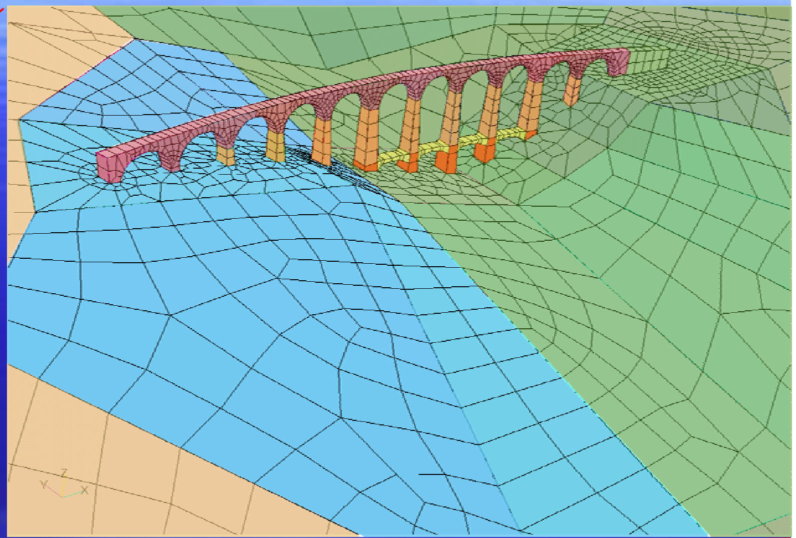
$$Q(f) = \frac{\pi}{\gamma} f$$

3D Soil-Structure – Acquasanta viaduct

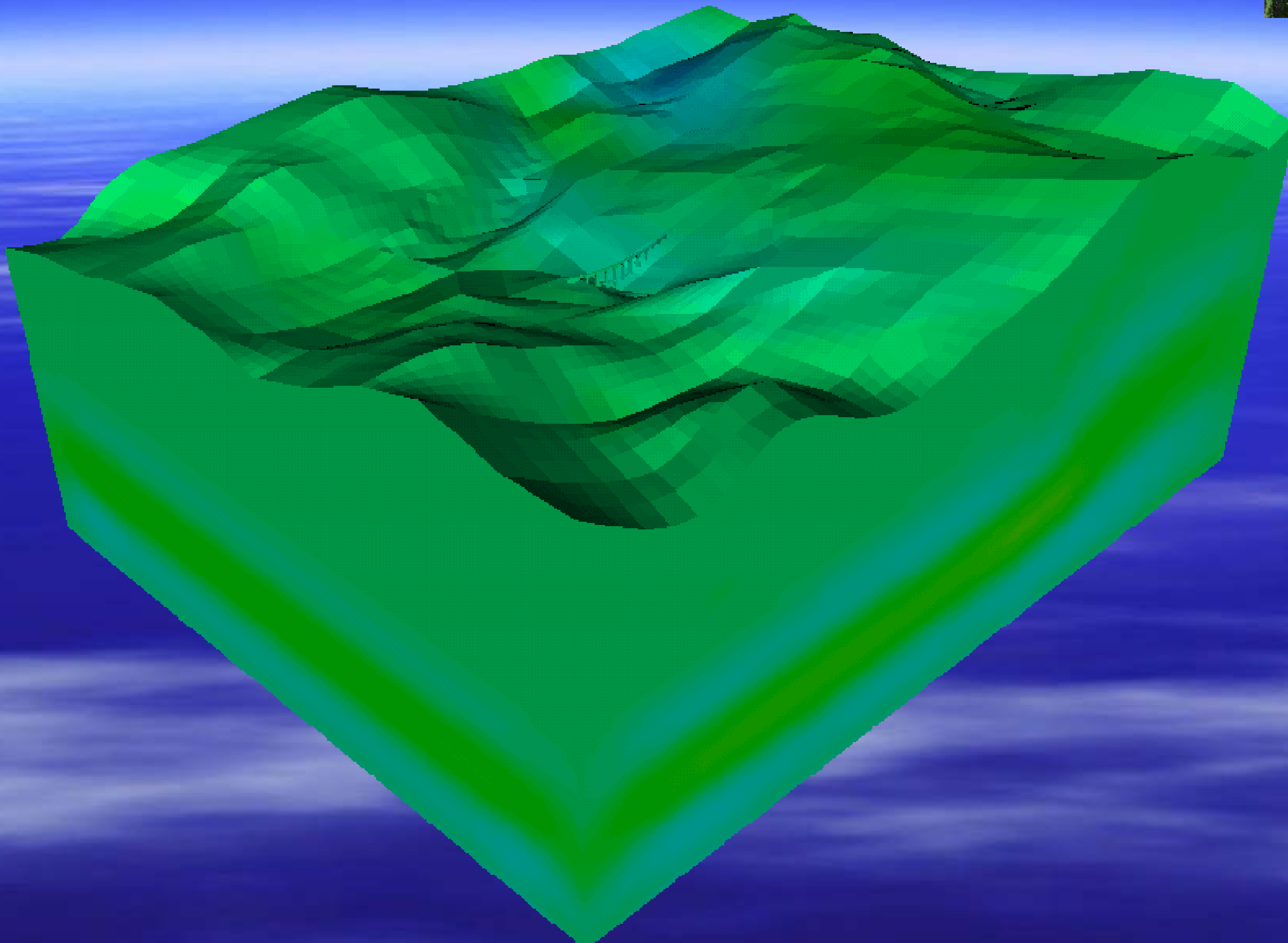
3D Soil-Structure – Acquasanta viaduct

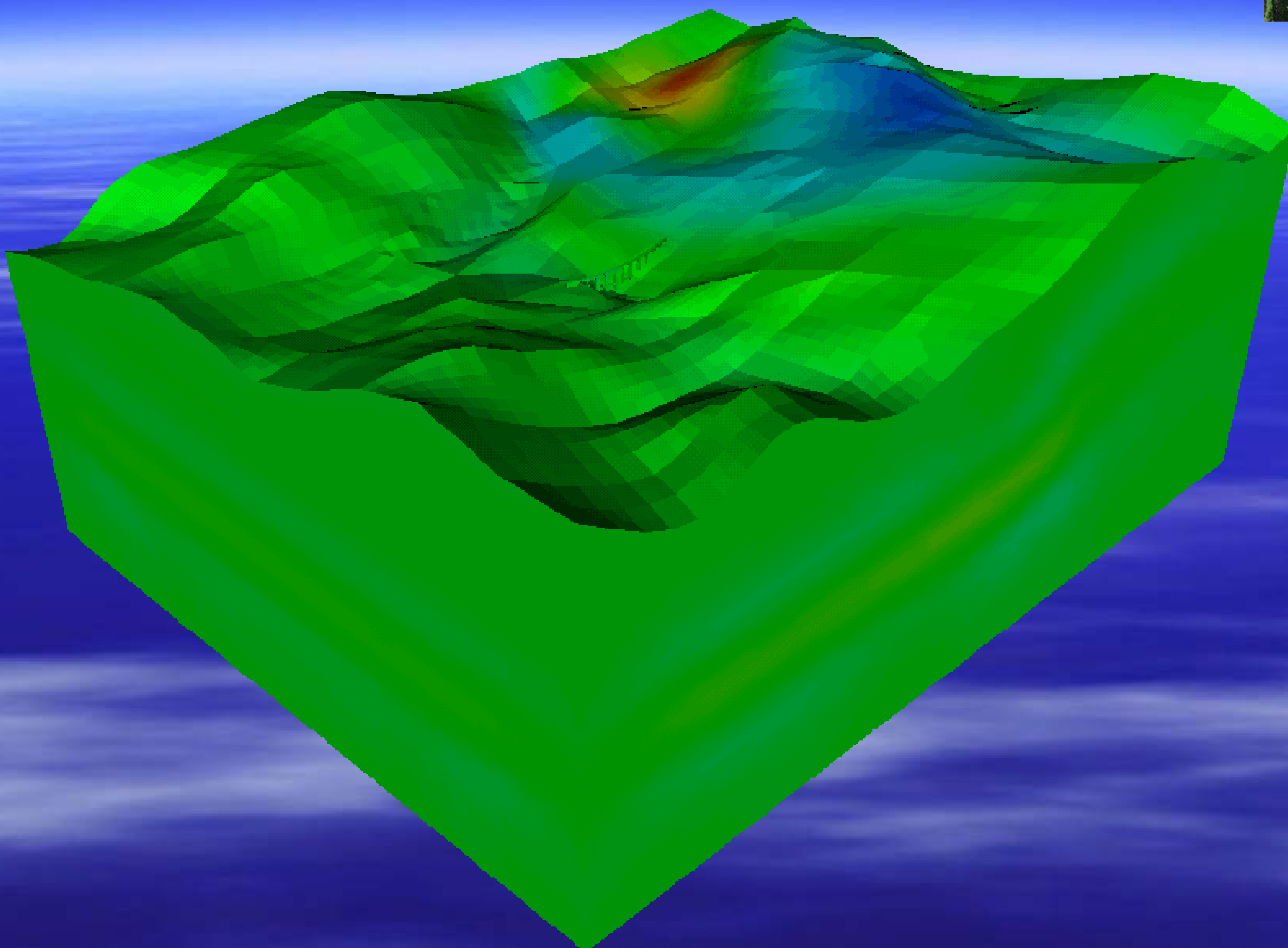
Complex 3D layered structures
with two main faults

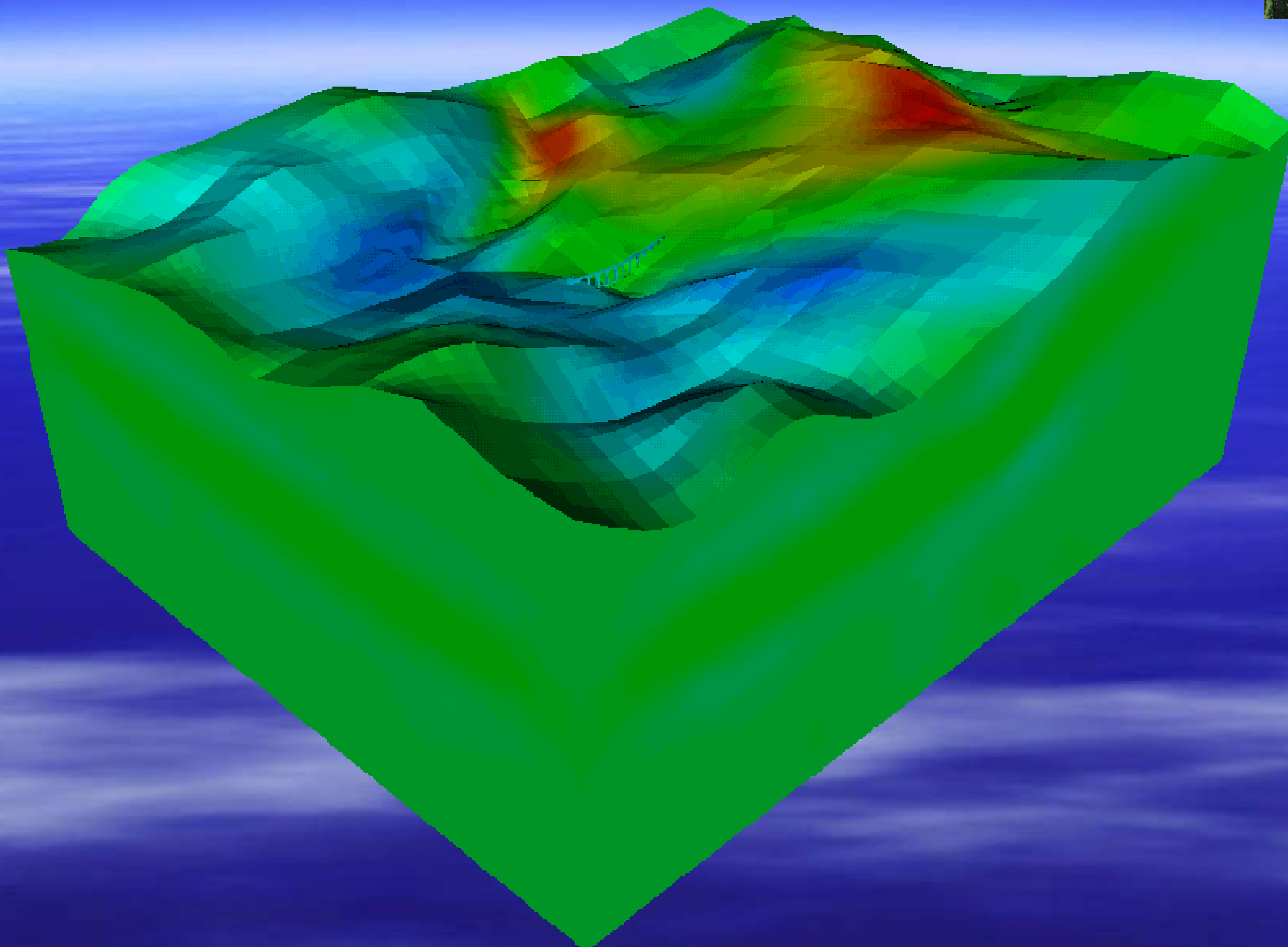


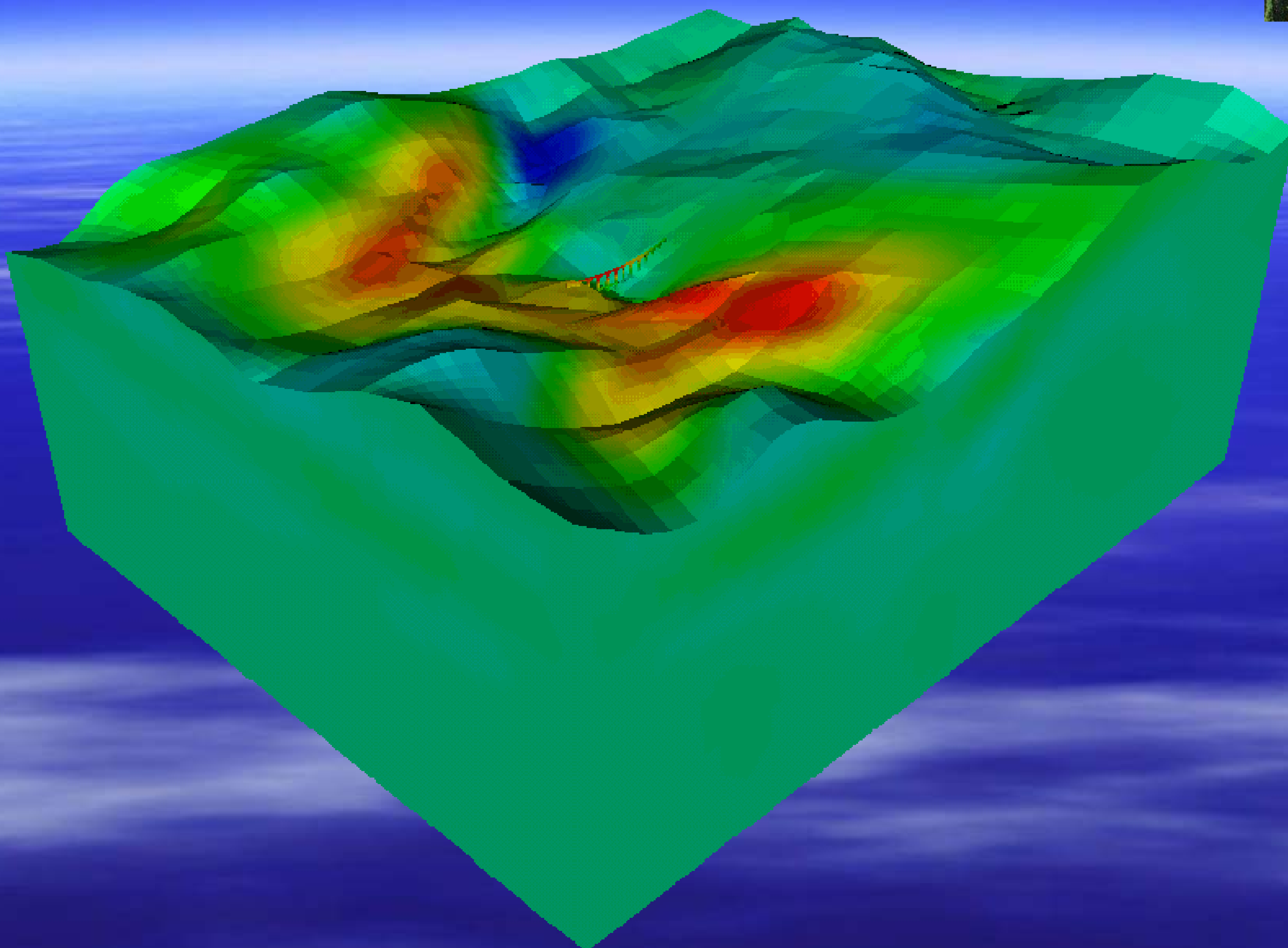


Alluvial Deposits
(max depth 30 m) with a
masonry railway bridge
crossing the valley

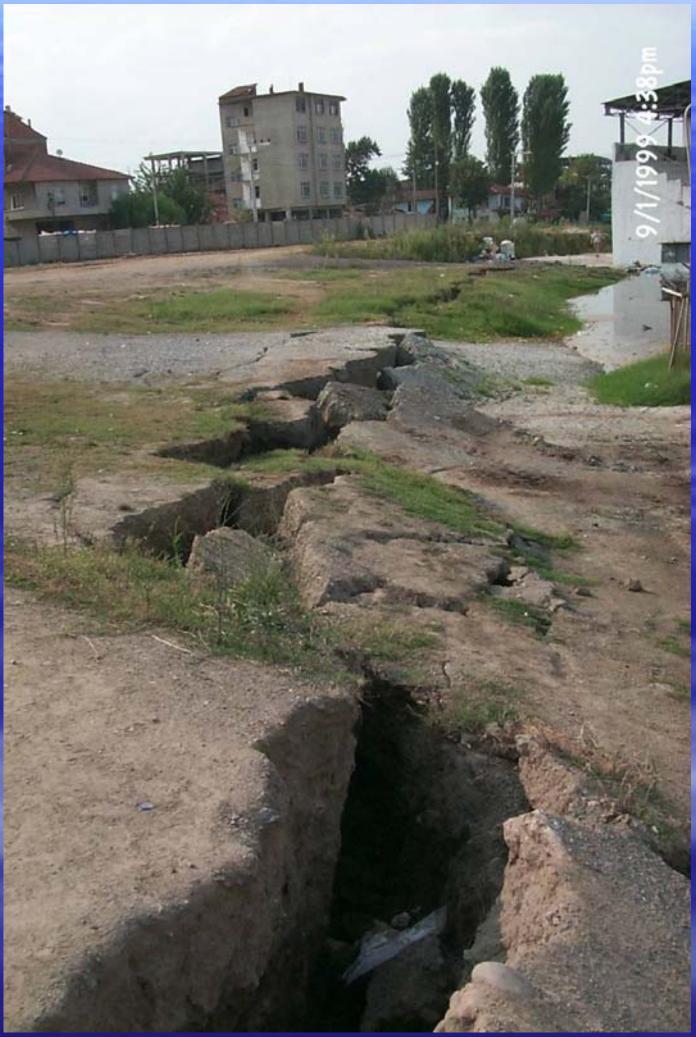








Turkey earthquake 1999



GEO-ELSE Viscoplastic

⇒ Elastoviscoplastic constitutive model

⇒ Total strain tensor can be written as:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^{el} + \dot{\varepsilon}_{ij}^{vp}$$

and constitutive behaviour is given by: $\frac{\partial \sigma}{\partial t} = D^e : \left(\frac{\partial \varepsilon}{\partial t} - \frac{\partial \varepsilon^{vp}}{\partial t} \right)$

⇒ The flow rule can be written as below

$$\frac{\partial \varepsilon^{vp}}{\partial t} = \gamma m \langle \phi(f) \rangle$$ (Perzyna, 1963)

where *f* is the yield function, $\phi(f)$ is the viscous nucleus, γ is a parameter which describes the system evolution rate, *m*, is the gradient to the plastic potential $m = \partial g / \partial \sigma_{ij}$, and characterizes the direction of viscoplastic strain.

GEO-ELSE Viscoplastic

⇒ Viscous nucleus definition:

$$\phi(f) = \left(\frac{f - f_0}{f_0} \right)^N$$

where f_0 is the value characterizing the stress level below which no flow occurs, N is constitutive parameter

⇒ Von Mises o Drucker-Prager yield surface

⇒ The size of the yield surface varies according to a suitable softnening law (linear dependence on the equivalent deviatoric viscoplastic strain $\overline{\varepsilon}^{vp}$)

Implementation of the viscoplastic model in GEO-ELSE

For simplicity absence of absorbent conditions and not viscous material

$$[M]\ddot{\boldsymbol{u}}(t)+[K]\boldsymbol{u}(t)=\boldsymbol{F}_{ext}(t)$$

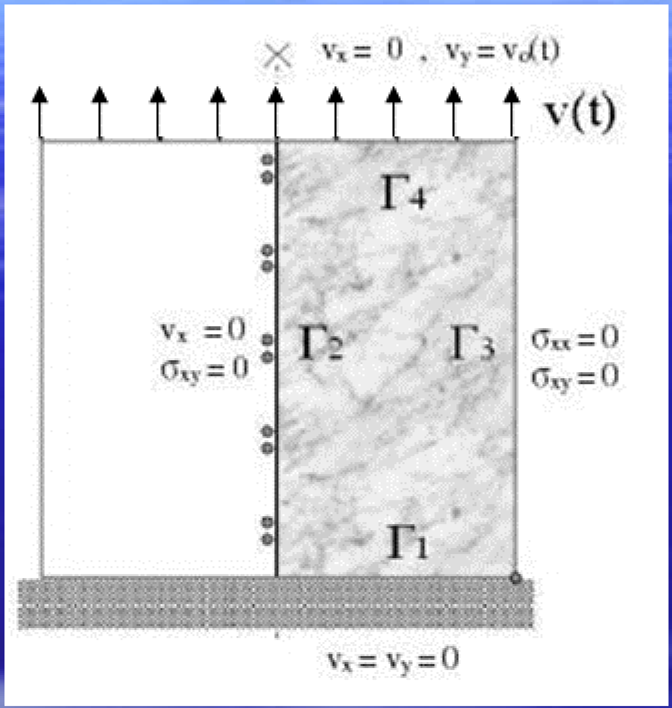
with $[K]\boldsymbol{u}(t)=F_{\text{int}}=\int_{\Omega}\sigma d\Omega$

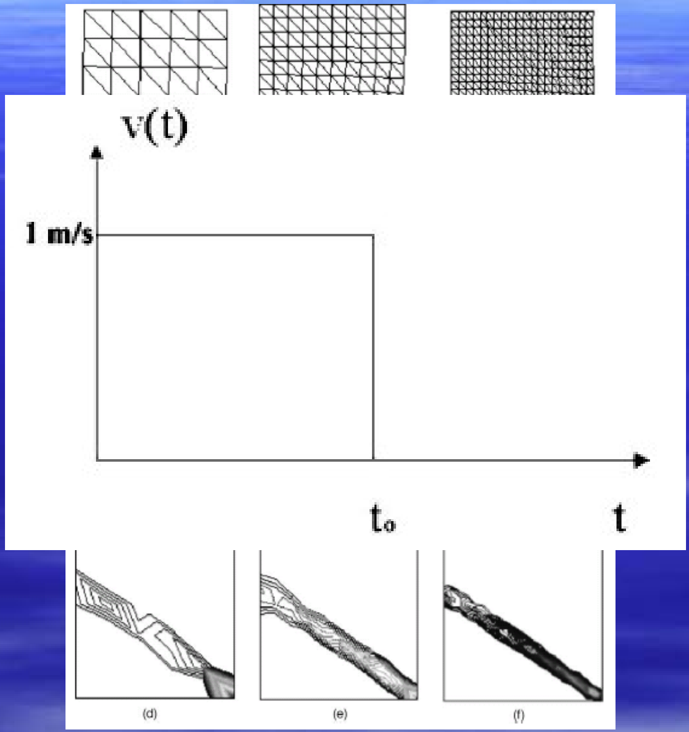
$$\boldsymbol{\sigma}_n=D^e:\left(\boldsymbol{\varepsilon}_n-\boldsymbol{\varepsilon}_n^{vp}\right)$$

The viscoplastic model introduces an additional right hand side term

$$\boldsymbol{u}_{n+1}=\left(\boldsymbol{F}_{ext}-\boldsymbol{F}_{int}\right)[M]^{-1}\Delta t^2+2\boldsymbol{u}_n-\boldsymbol{u}_{n-1}$$

1-Simulation of the shear band creation





$$\bar{\varepsilon}^{vp} = \sqrt{\frac{2}{3}} \left\{ \left(\varepsilon_{ij}^{vp}\right)\left(\varepsilon_{ij}^{vp}\right) \right\}^{1/2}$$

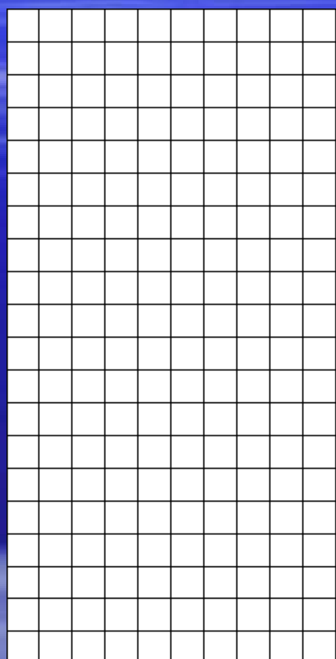
M. Mabssout, M. Pastor

"A Taylor-Galerkin algorithm for shock wave propagation and strain localization failure of viscoplastic continua"

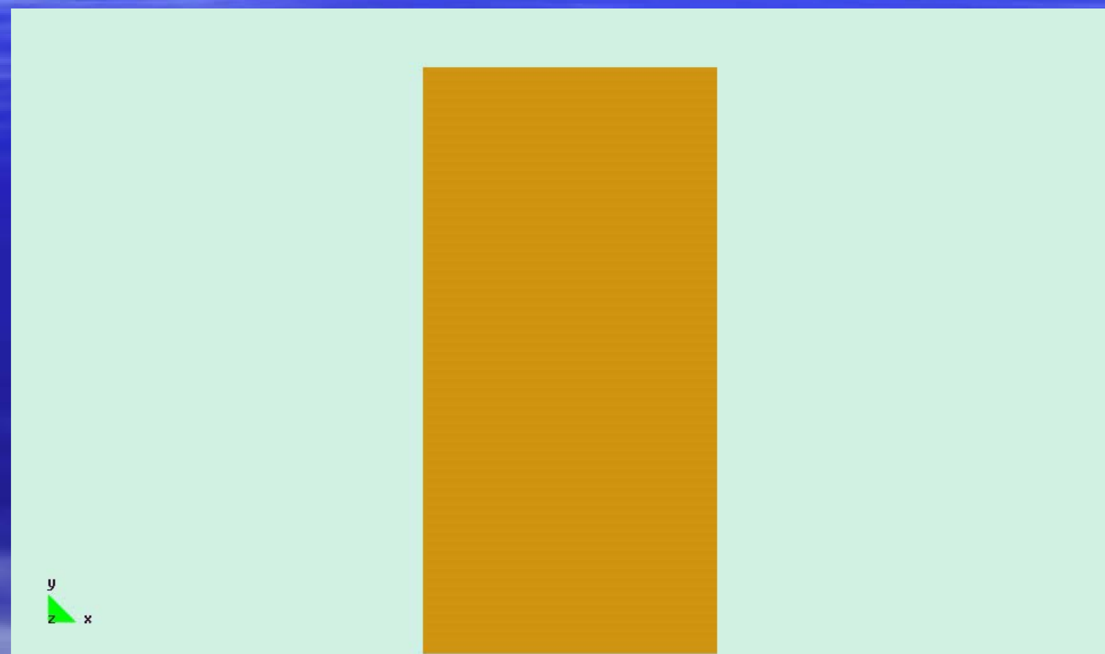
Comput. Methods Appl. Mech. Engrg., 2003

1-Simulation of the shear band creation

Mesh



Spectral degree = 4, Dt = 10% Dt_{CFL}



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1-Simulation of the shear band creation

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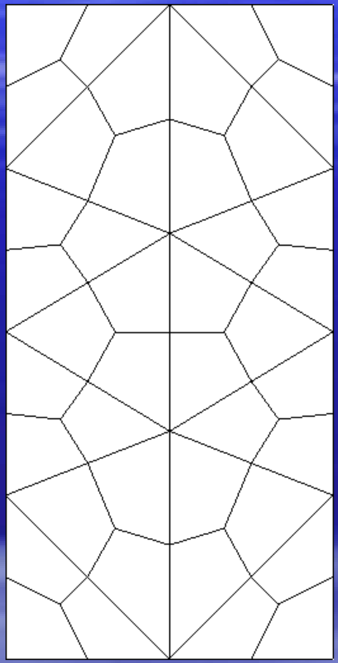
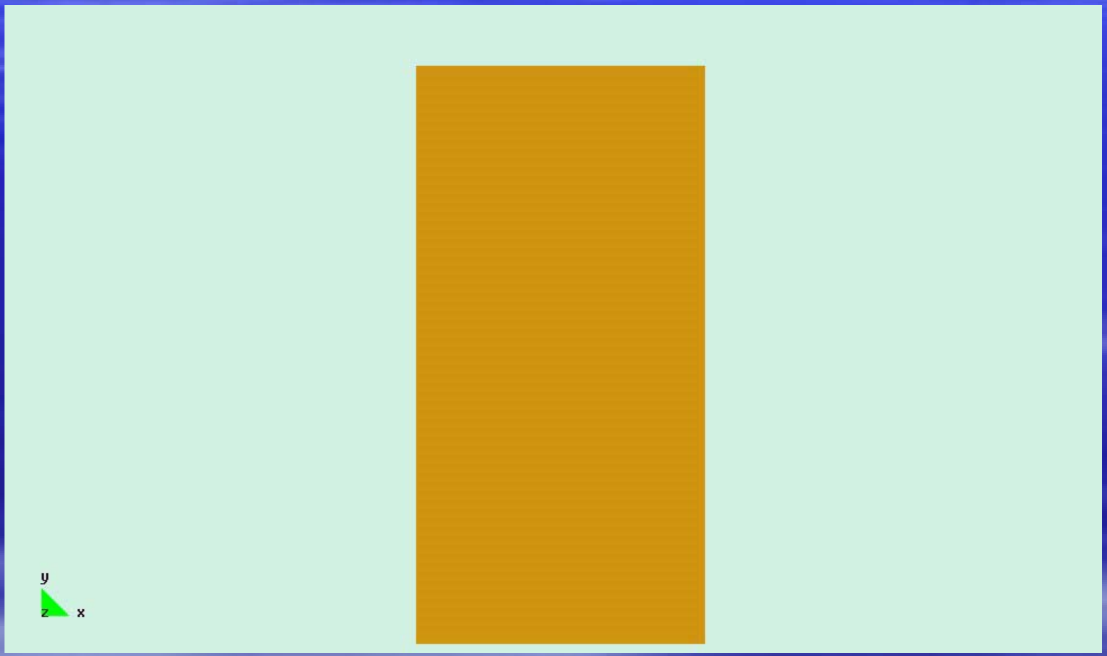
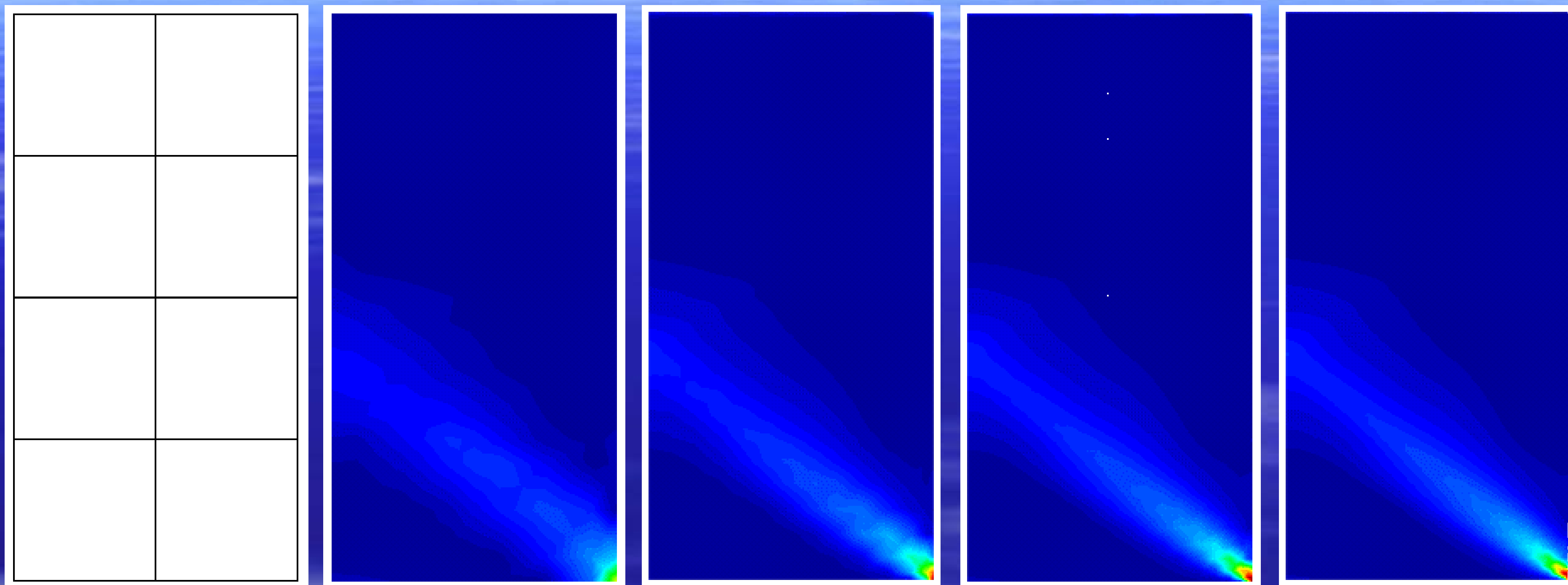


Spectral degree = 10, Dt = 10% Dt$_{CFL}$



$$\bar{\varepsilon}^{vp} = \sqrt{\frac{2}{3}} \left\{ \left(\varepsilon_{ij}^{vp} \right) \left(\varepsilon_{ij}^{vp} \right) \right\}^{1/2}$$

1-Simulation of the shear band creation



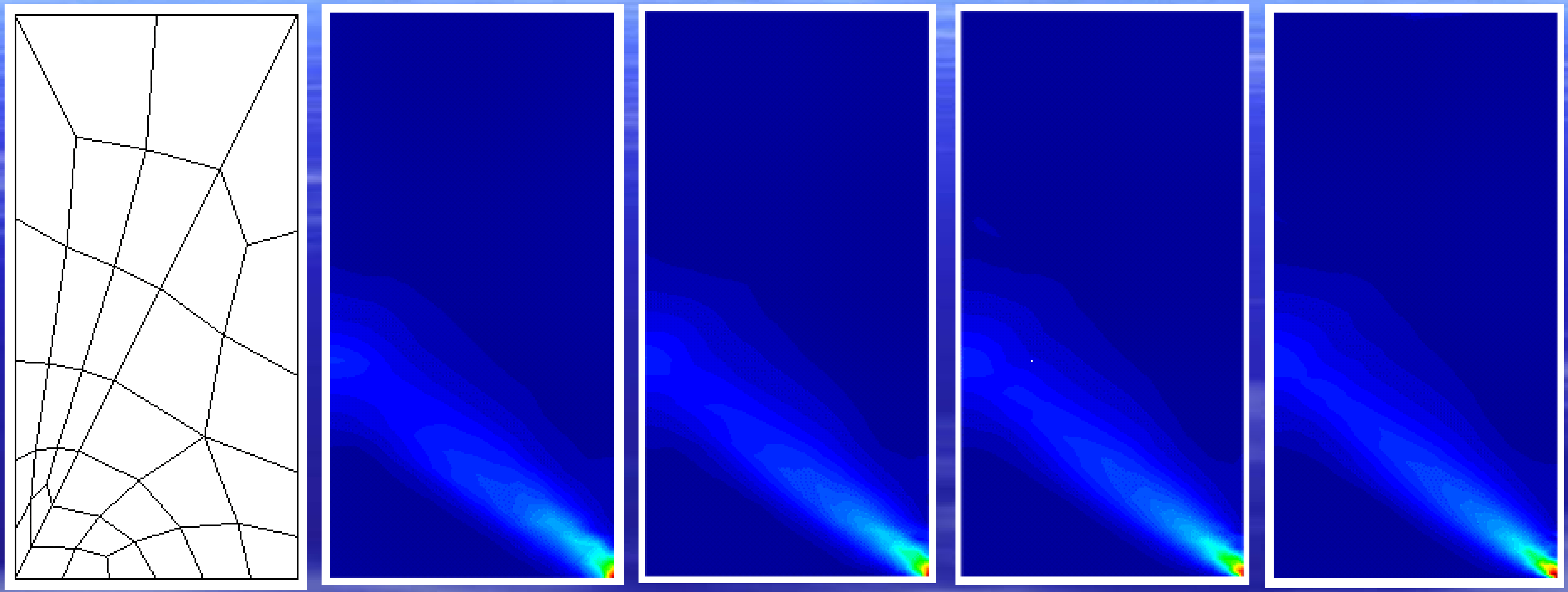
SD = 4, Nodes = 153

SD = 8, Nodes = 561

SD = 12, Nodes = 1225

SD = 16, Nodes = 2145

1-Simulation of the shear band creation



SD = 3, Nodes = 334

SD = 4, Nodes = 581

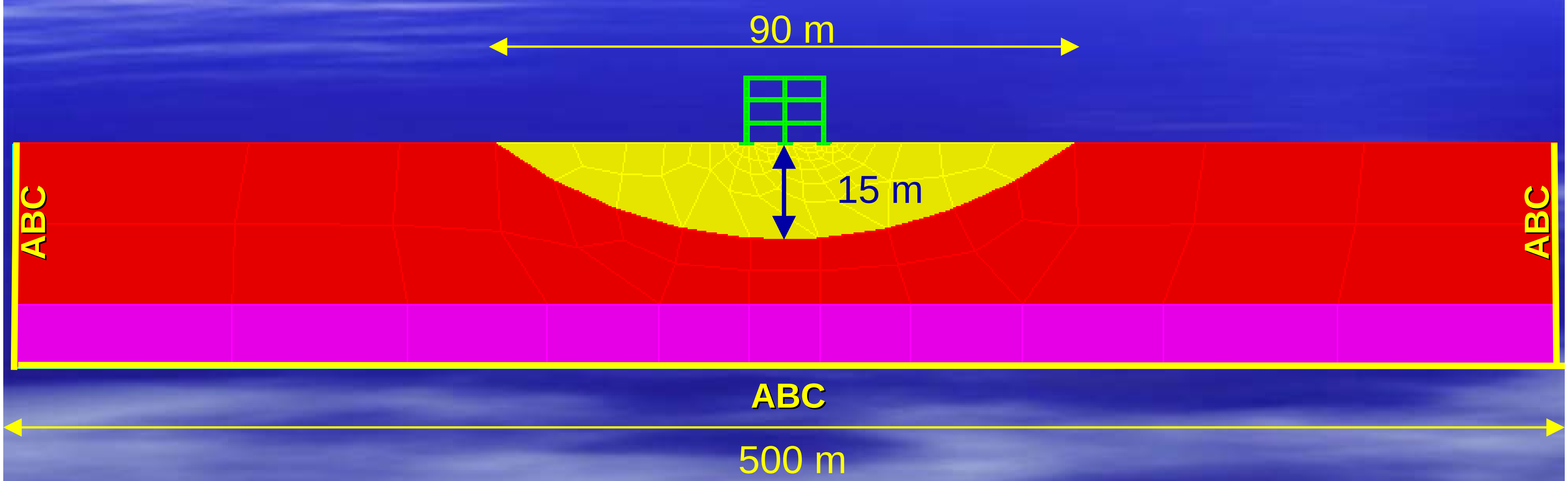
SD = 5, Nodes = 896

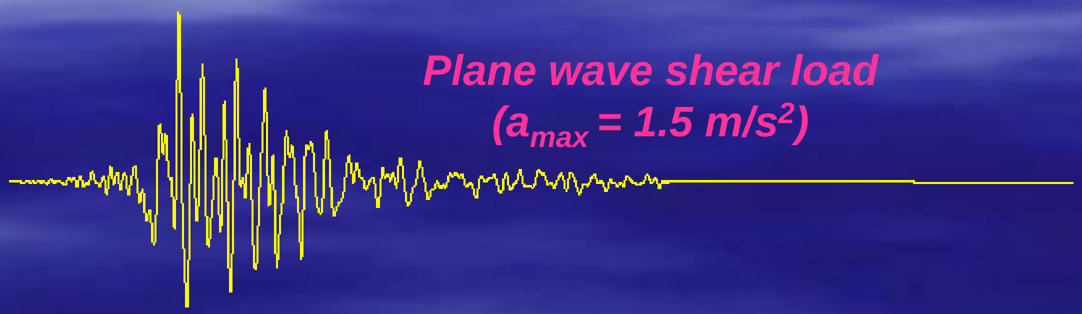
SD = 6, Nodes = 1279

2-Soil structure - hazard analsys

Alluvial valley:
V_P = 300 m/s
V_S = 150 m/s
ρ = 1800 kg/m3

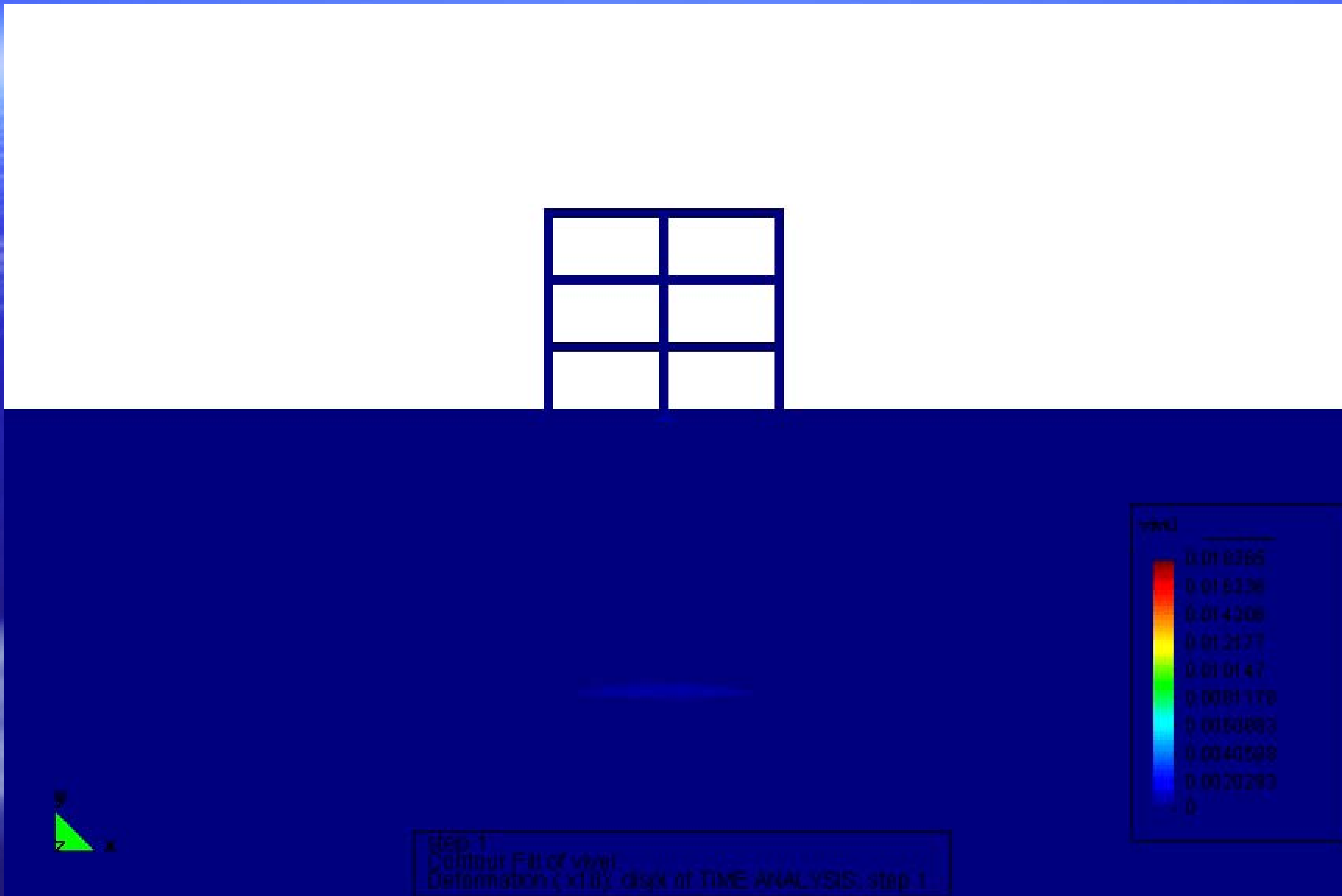
Bedrock:
V_P = 1000 m/s
V_S = 600 m/s
ρ = 2500 kg/m3



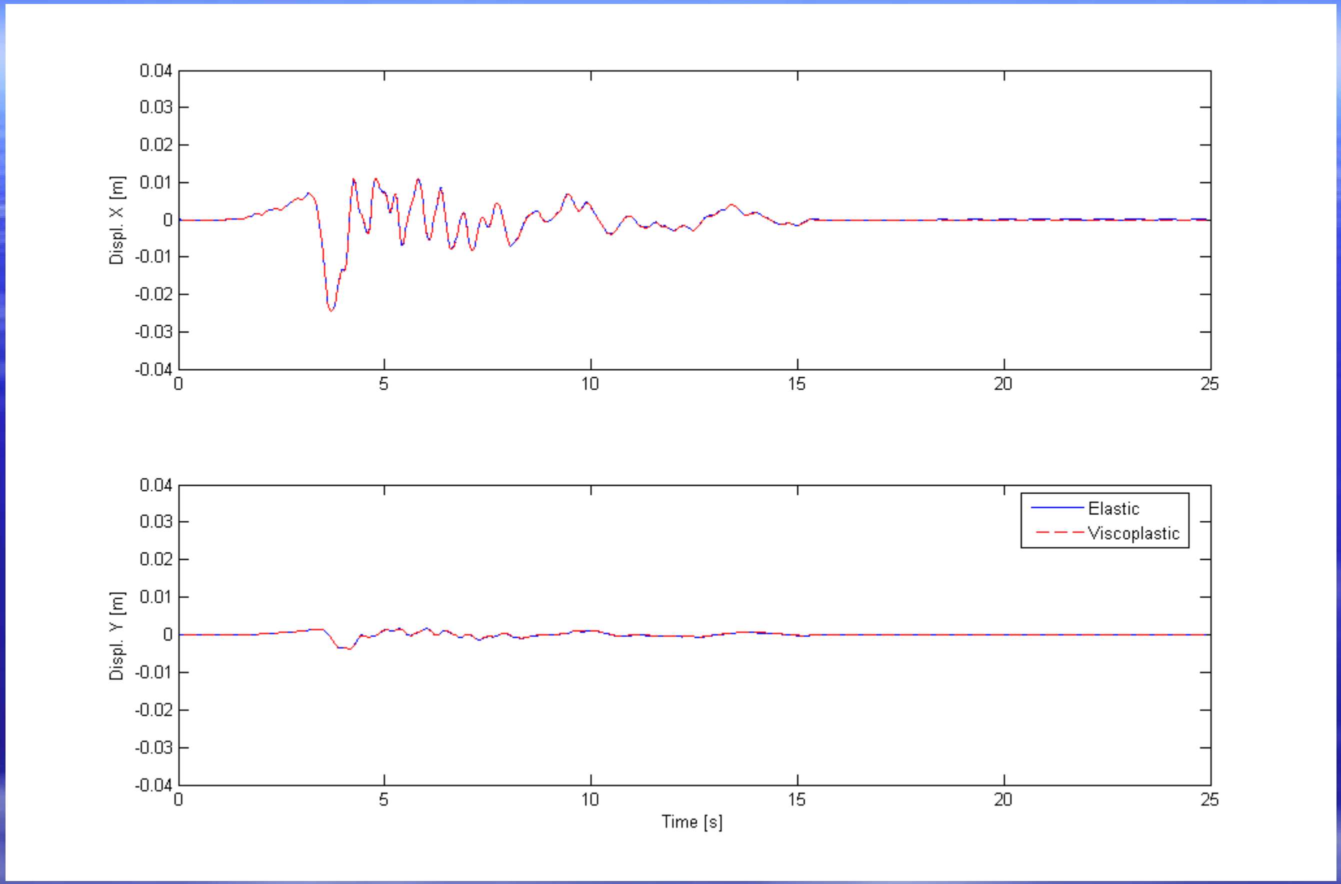


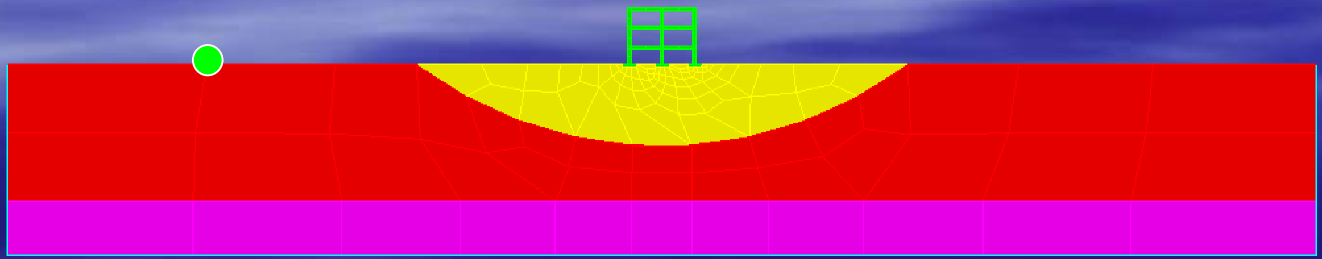
Plane wave shear load
(a_{max} = 1.5 m/s^2)

2-Soil structure – hazard analsys



2-Soil structure – time histories





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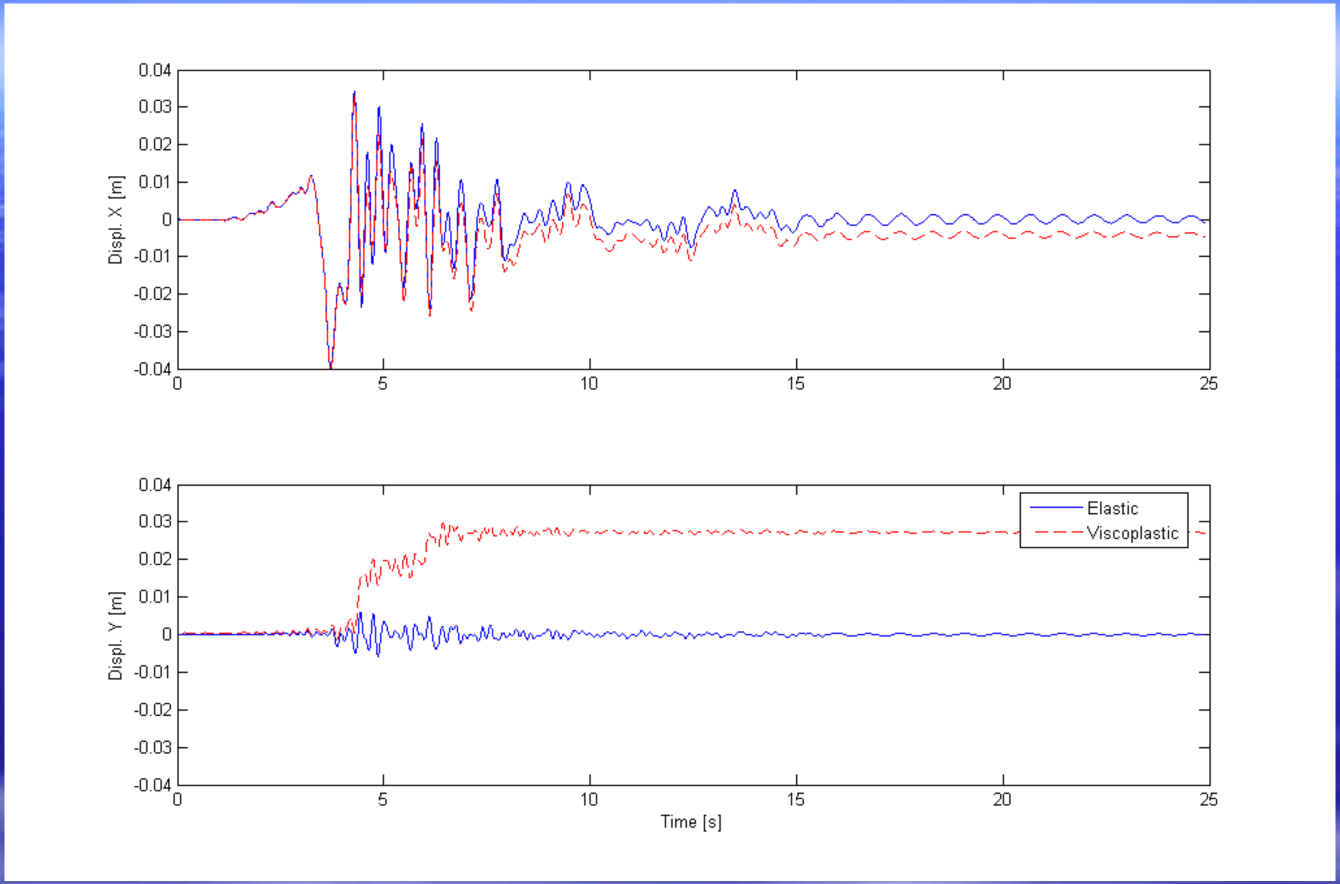

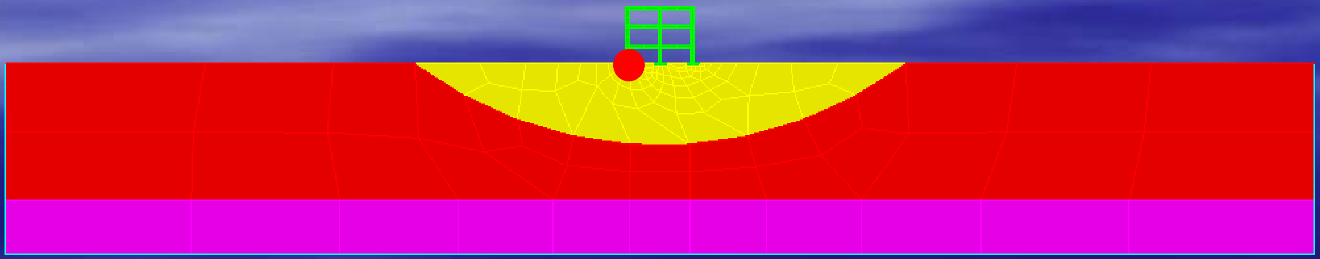

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