Toward inversion of broadband seismic body-wave waveform data for Earth structure

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Introduction

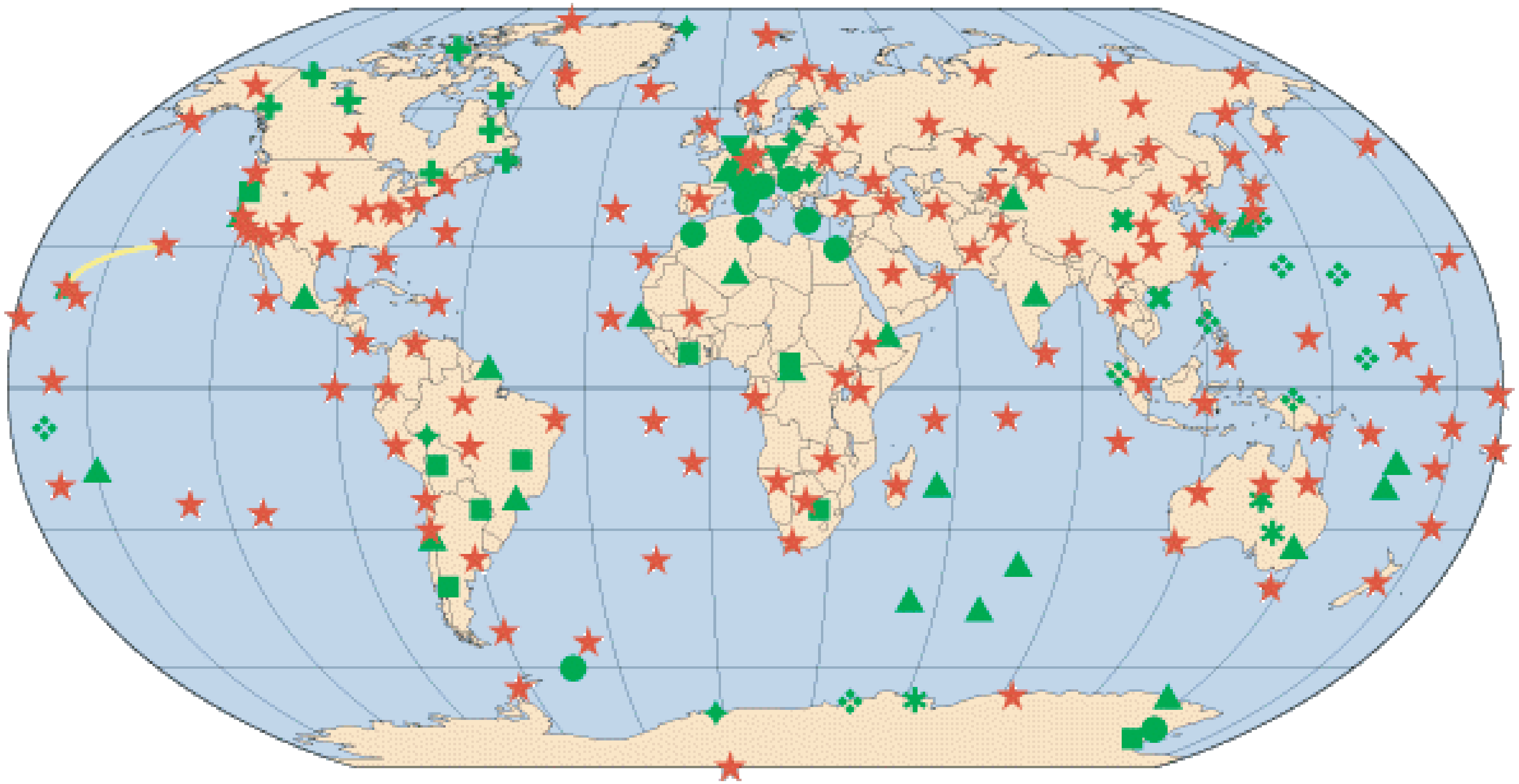
- Global and regional networks have significantly improved over the past 20 years
- But methods for analyzing these data have not made commensurate progress
- Waveform inversion is a promising method for obtained more reliable Earth models
- But many technical issues must be resolved

Waveform inversion

- Inverting seismic waveform data (rather than just travel times) for Earth structure, allows observed data to be fully utilized.
- This will provide information which is useful for studying geodynamics, mantle convection, and mineral physics on a global scale.
- Waveform inversion methods should also be useful for local and regional studies.

Global networks

GSN/FDSN

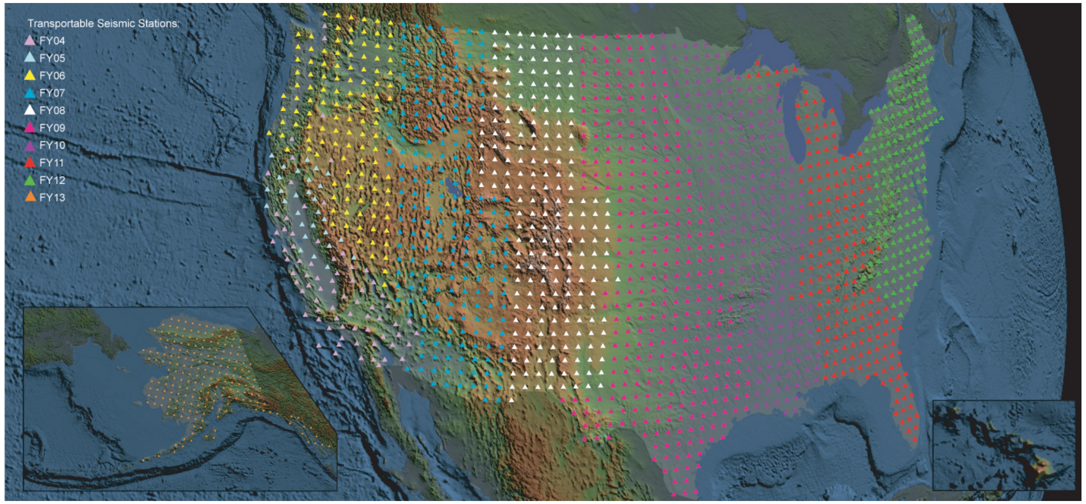


Distribution of receivers is inhomogeneous.

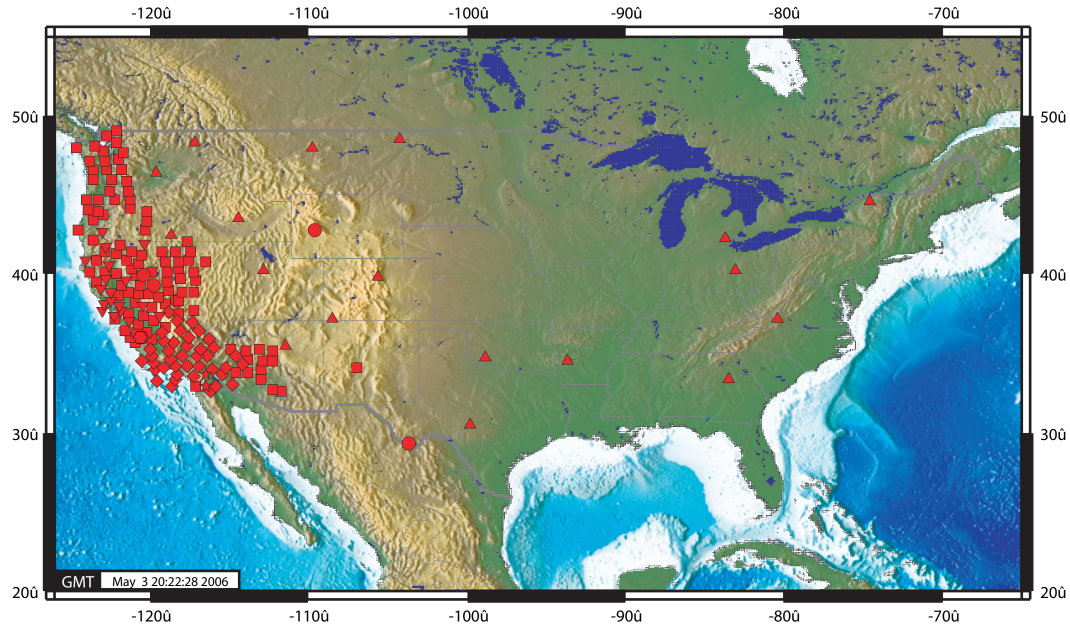
Array networks

New deployments of broadband array networks

(USArray, F-net, Hi-net, NECESSArray)



USArray



(Currently operating stations)

New data for inferring fine structure

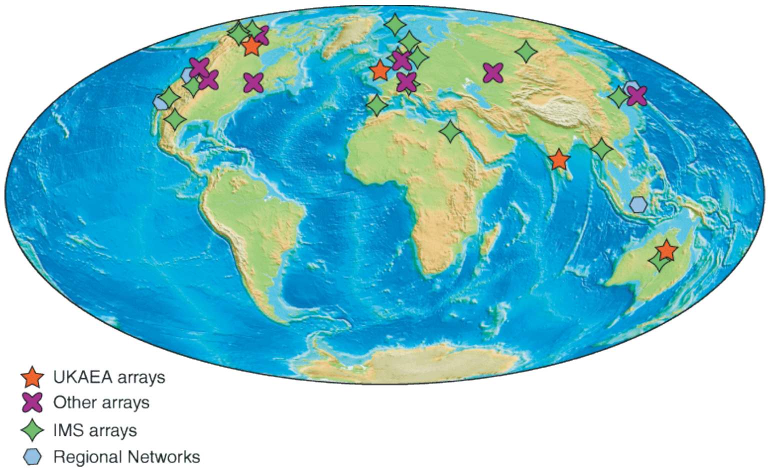
Heterogeneity of resolution of Earth structure

Distribution of both sources and receivers is biased.

(1/3 of surface is land and 2/3 is sea.)

→ Resolution in the Earth is inhomogeneous.

It is necessary to develop new methods
to quantitatively and systematically
infer Earth structure,
considering inhomogeneous resolution.

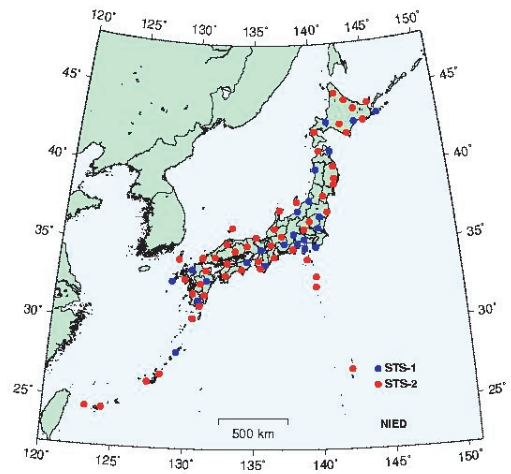


Global array network

Data such as

- broad band data
- array data
- borehole data

becoming increasingly available.



(F-net, NIED)

It is necessary to develop new methods
in order to fully take advantage of these data.

Previous research using waveforms (1)

Global (Spherical harmonics basis)

- Waveform inversion for either isotropic or TI models for the upper mantle (e.g. Su & Dziewonski 1997, Li & Romanowicz 1996, Hara & Geller 2000)
- Waveform inversion for TI (e.g. Panning & Romanowicz 2004)

→Since these studies use spherical harmonics as the basis functions, there can be artifacts in areas of poor coverage.

Local (1-D structure for a particular region)

- Waveform modeling for Earth structure such as
  transition zone (e.g. Tajima & Grand 1996)
  D" layer (e.g. Lay & Helmberger 1983)
- Partitioned waveform inversion (e.g. Nolet 1990)

→Quantitative inversion and error estimation are difficult

Previous research using waveforms (2)

Synthetic seismograms in previous studies
were computed using various approximations:

- Ray approximation (Generalized ray theory)
- Earth flattening (Reflectivity)
- Great circle approximation
- Perturbation approximation

In our research we compute synthetics for 1-D media without such approximations. Synthetics for 3-D media can be computed accurately using higher order terms of the Born approximation.

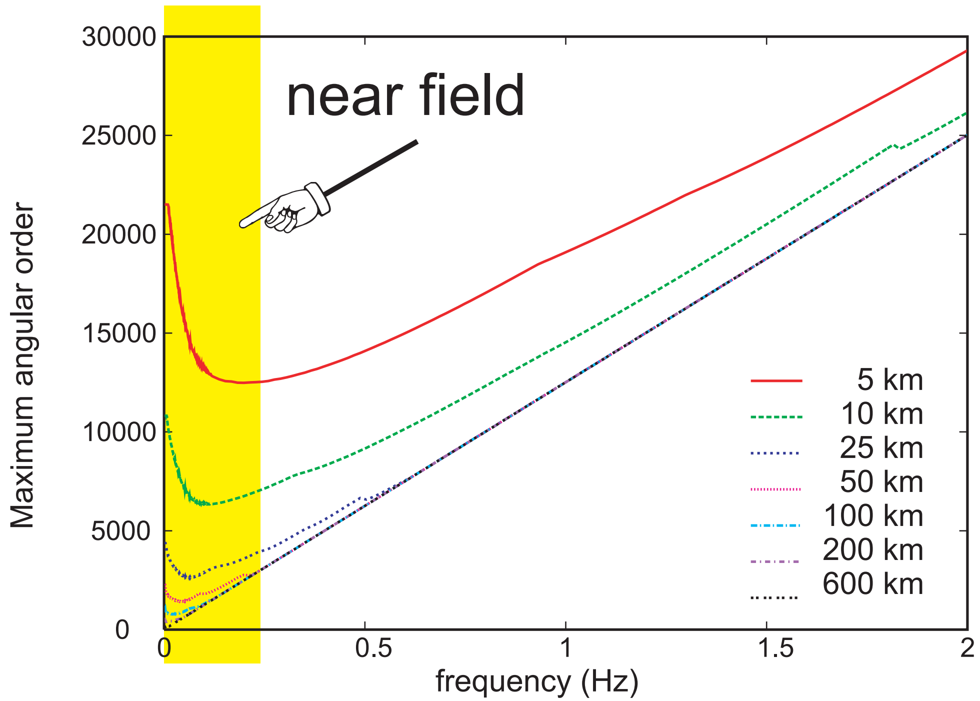
DSM full-wave synthetics for spherically symmetric media

Previous work	**This study**
isotropic (usually)	**transverse isotropy (TI)**
deep focus events only	deep and **shallow events**
long period only	**broad band (up to 2 Hz)**

→Development and testing of software
for accurate and efficient computation of full-wave synthetics

Difficulty of computation for shallow events (1)

There is a linear relation between frequency and maximum angular order for deep events, but...

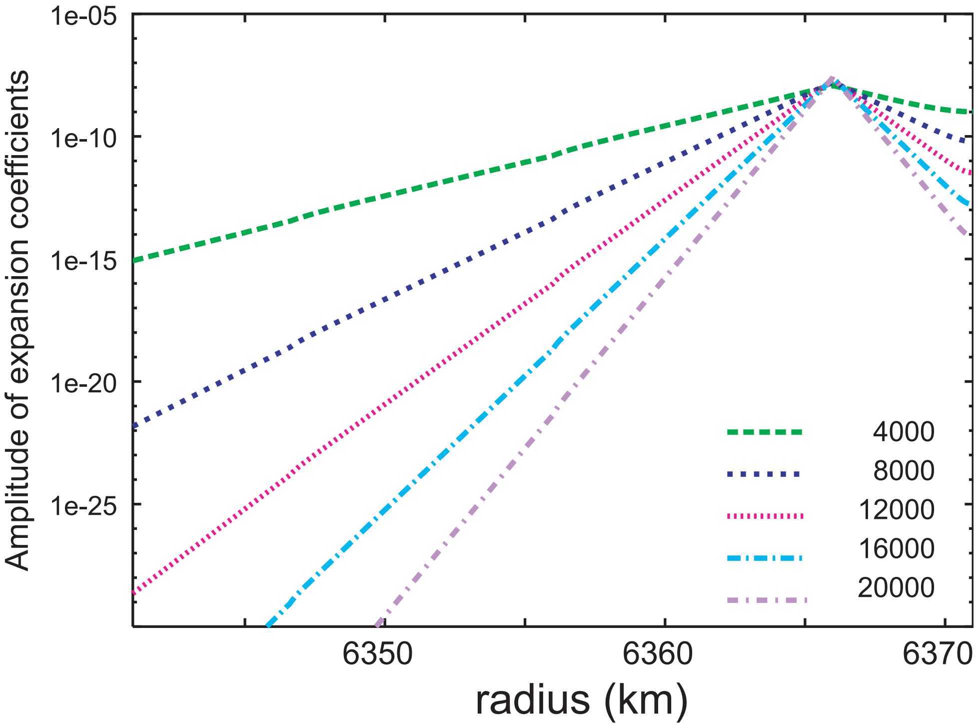


As the source depth becomes shallower,
a linear relation is still observed at high frequencies
with a higher maximum angular order
but the required maximum angular order also increases greatly at low frequencies.

Difficulty of computation for shallow events (2)

Decay of the amplitude of the depth-dependent part of the wavefield

100s



The source depth is 5 km.

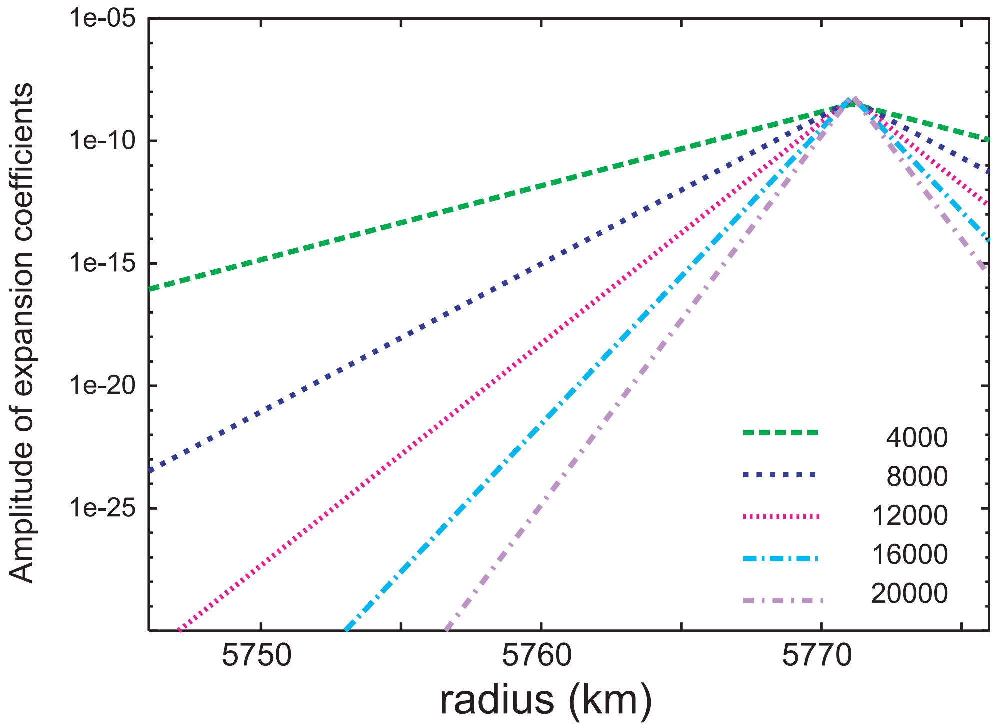
High angular order components of the wavefield with significant amplitudes exist in the vicinity of the source depth.

Is this behavior seen only for shallow events?

Difficulty of computation for shallow events (3)

Decay of the amplitude of the depth-dependent part of the wavefield

100s



The source depth is 600 km.

The same basic behavior is observed for both shallow and deep events, but for deep sources the amplitude decays to a negligible value at the Earth's surface.

Grid spacing

Relative Error $\approx \left| \frac{k_z^2 \Delta z^2}{12} \right|$

(Geller & Takeuchi, 1995)

$$\frac{\omega^2}{c^2} = k_x^2 + k_z^2 = \left(\frac{l + \frac{1}{2}}{r} \right)^2 + k_z^2$$

k	wave number	ω	angular frequency
Δz	grid spacing	c	phase velocity
l	angular order	r	radius

The vertical grid spacing required to produce a given relative error can be quantitatively evaluated using the above formula

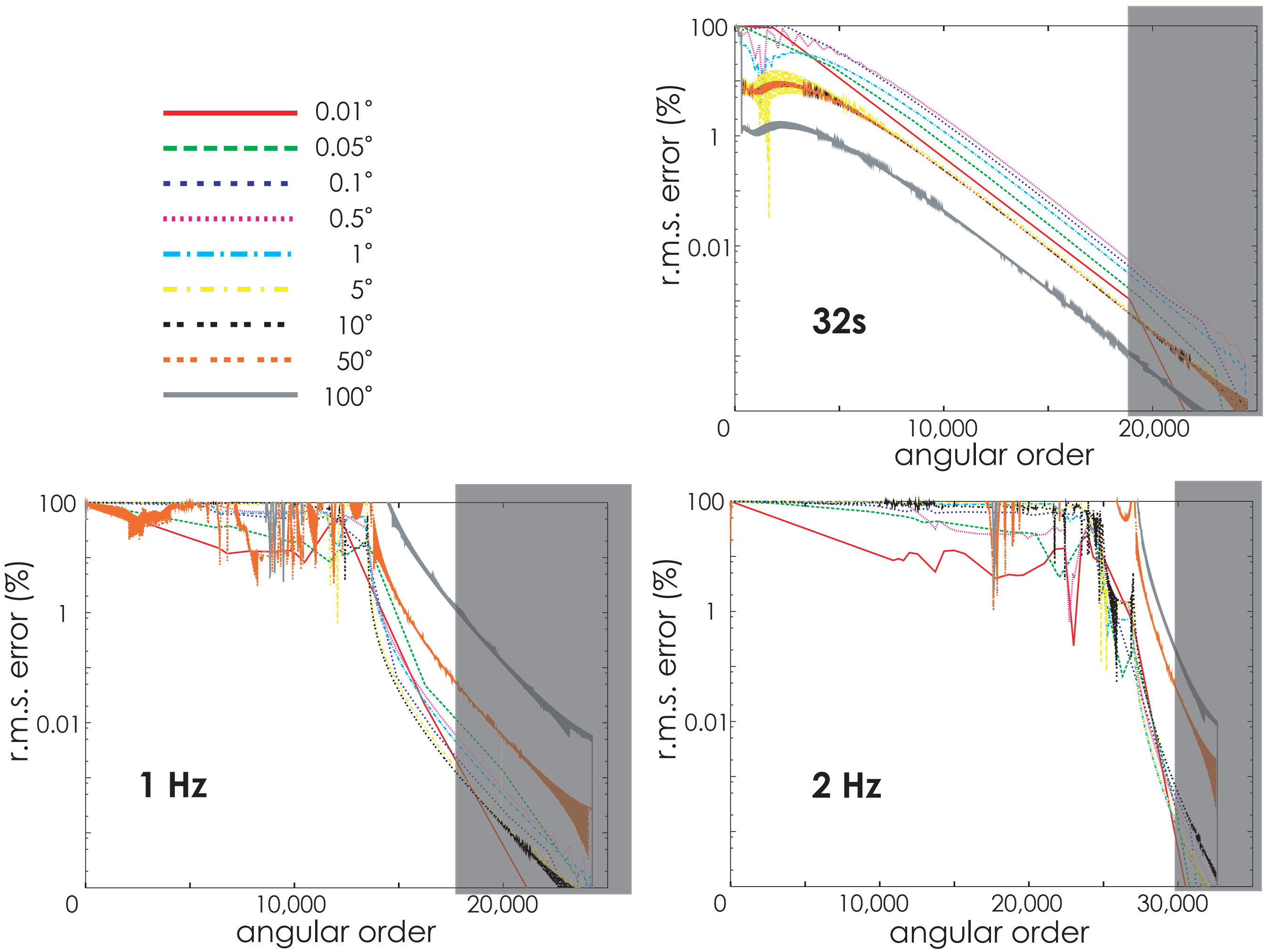
Evanescent waves ($k_z^2 < 0$) control the grid spacing for shallow events.

This method can quantitatively control numerical error.

Computational efficiency is also significant...

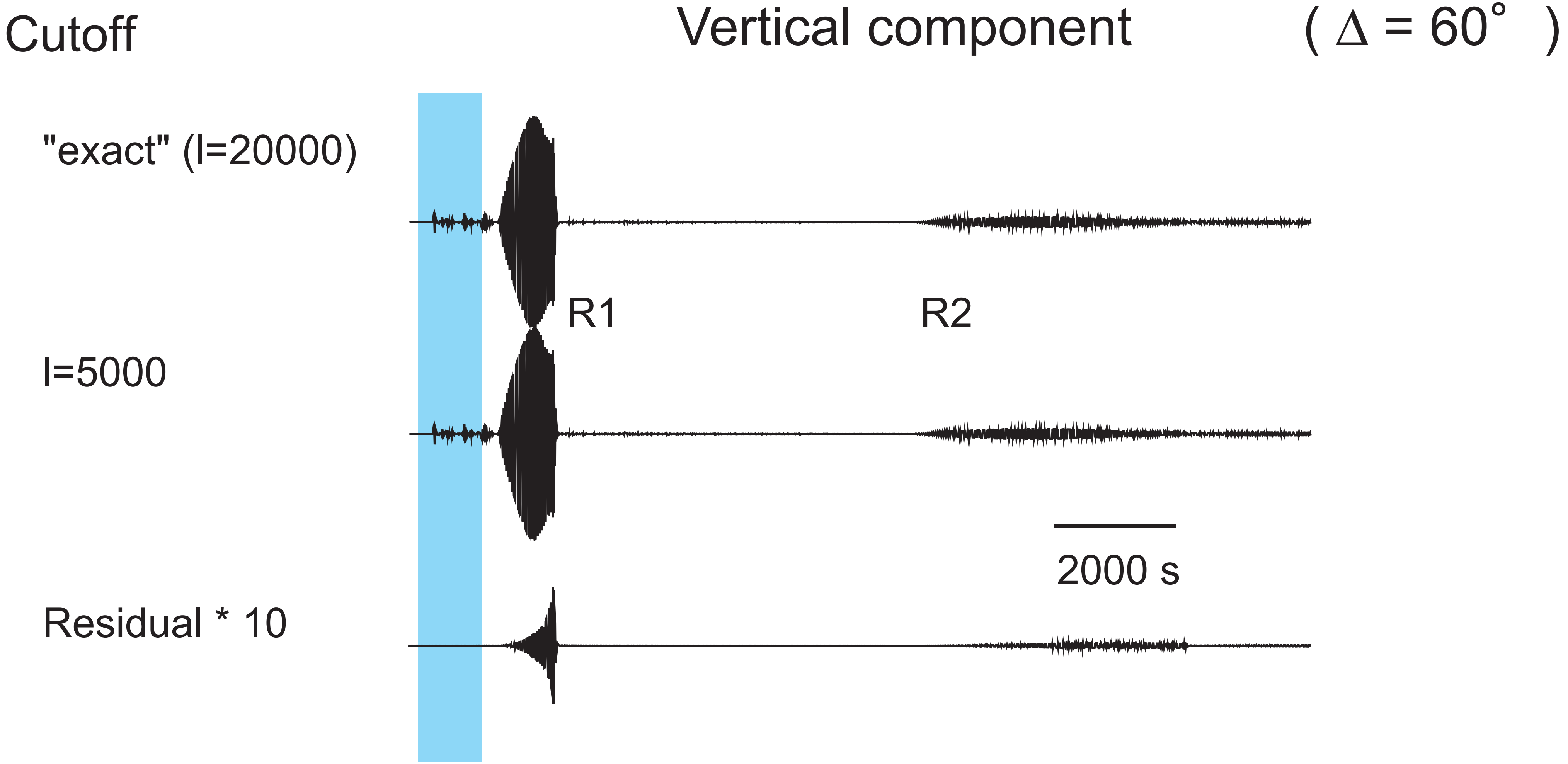
Automatic angular order cutoff (1)

Convergence of solutions



Our program stops computing for a given frequency when the amplitude of the expansion coefficient decays beneath a user-specified fraction of the maximum amplitude for that frequency.

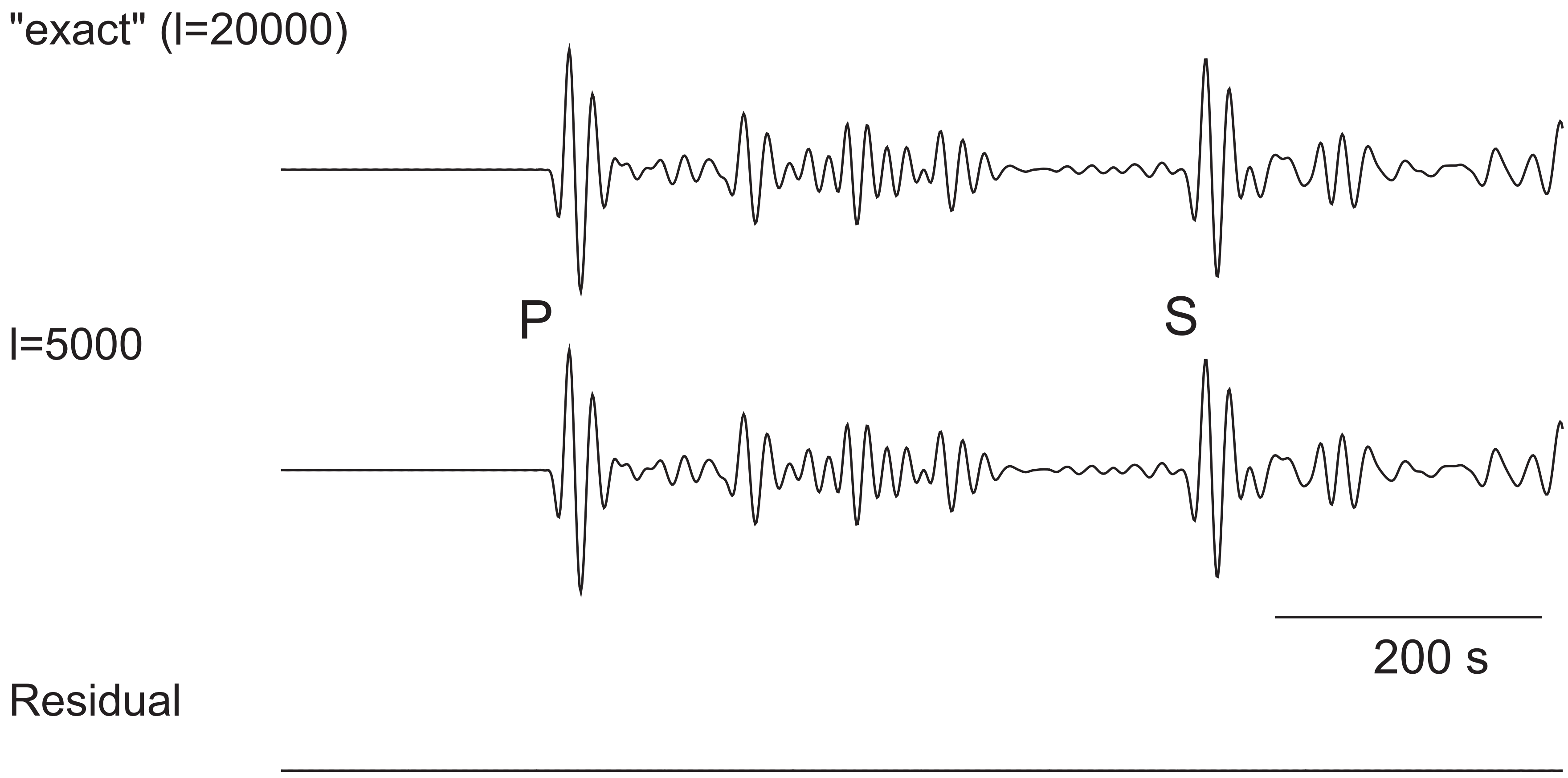
Automatic angular order cutoff (2)



Predominance of error for surface waves with slow phase velocity caused by truncation at too low an angular order.

Automatic angular order cutoff algorithm (3)

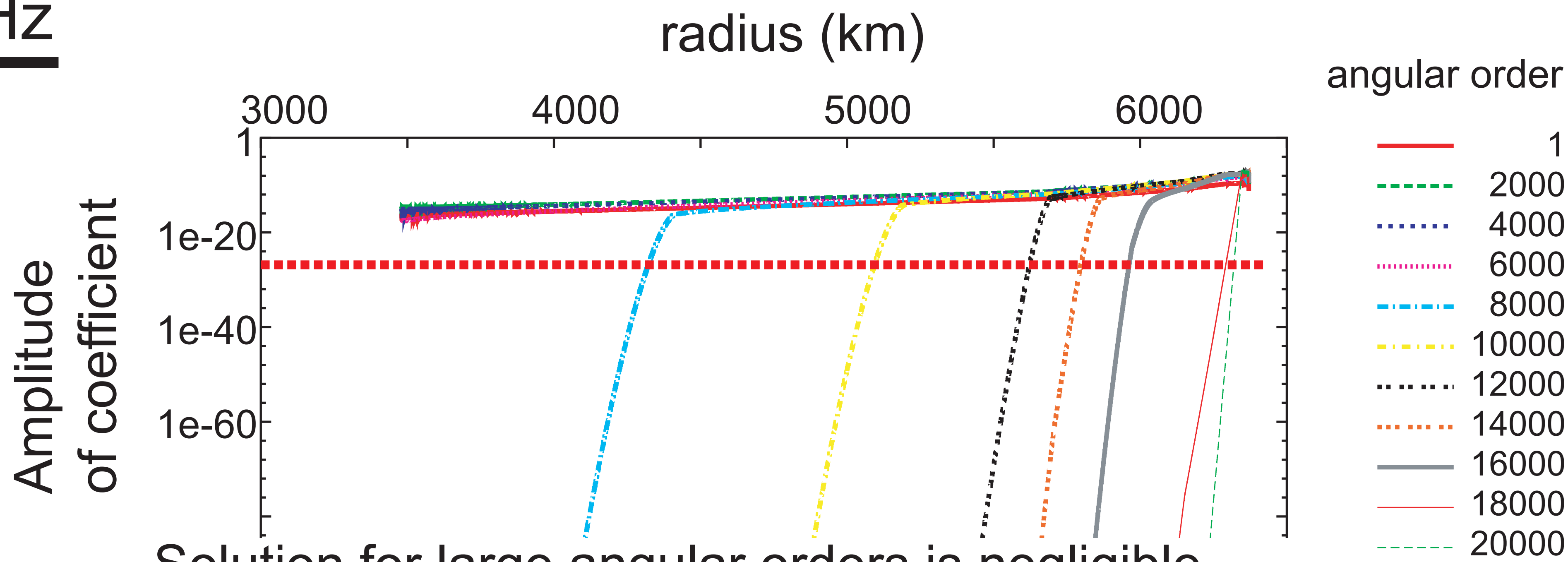
Vertical component ($\Delta = 60^\circ$)



Small error in body waves

Depth cutoff

2 Hz

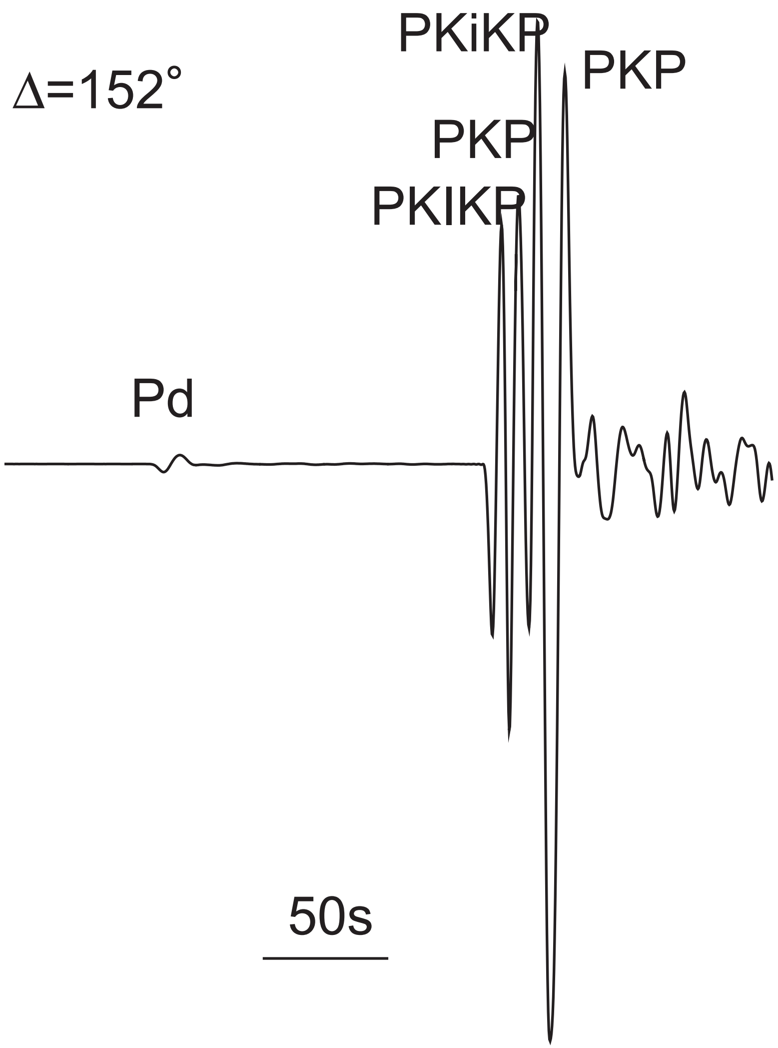


Solution for large angular orders is negligible in deep interior

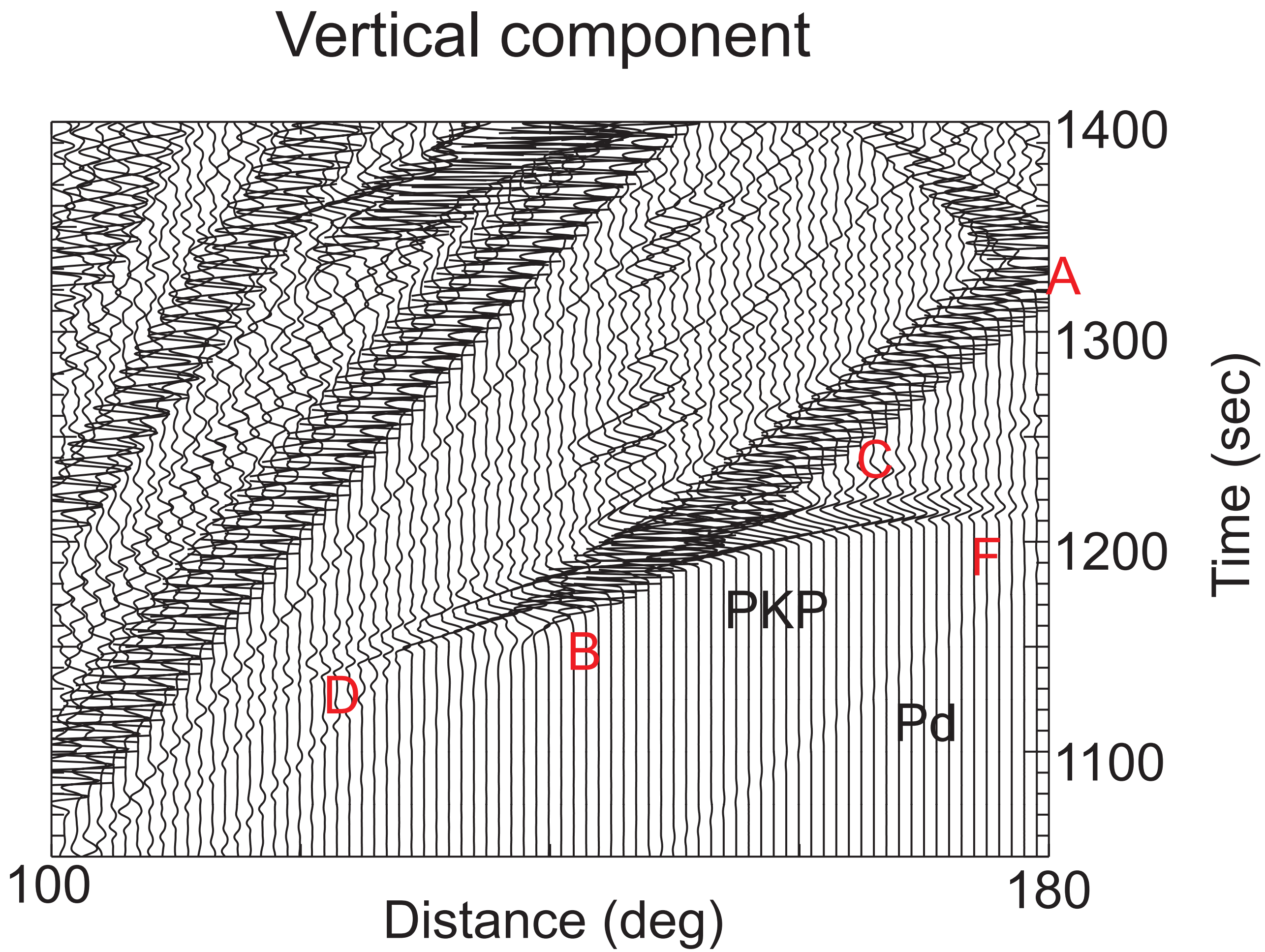
Reduction of CPU time by truncation of computational grids

To obtain 7 significant digits we use a depth cutoff of 10^{-10} of maximum amplitude

Core phases

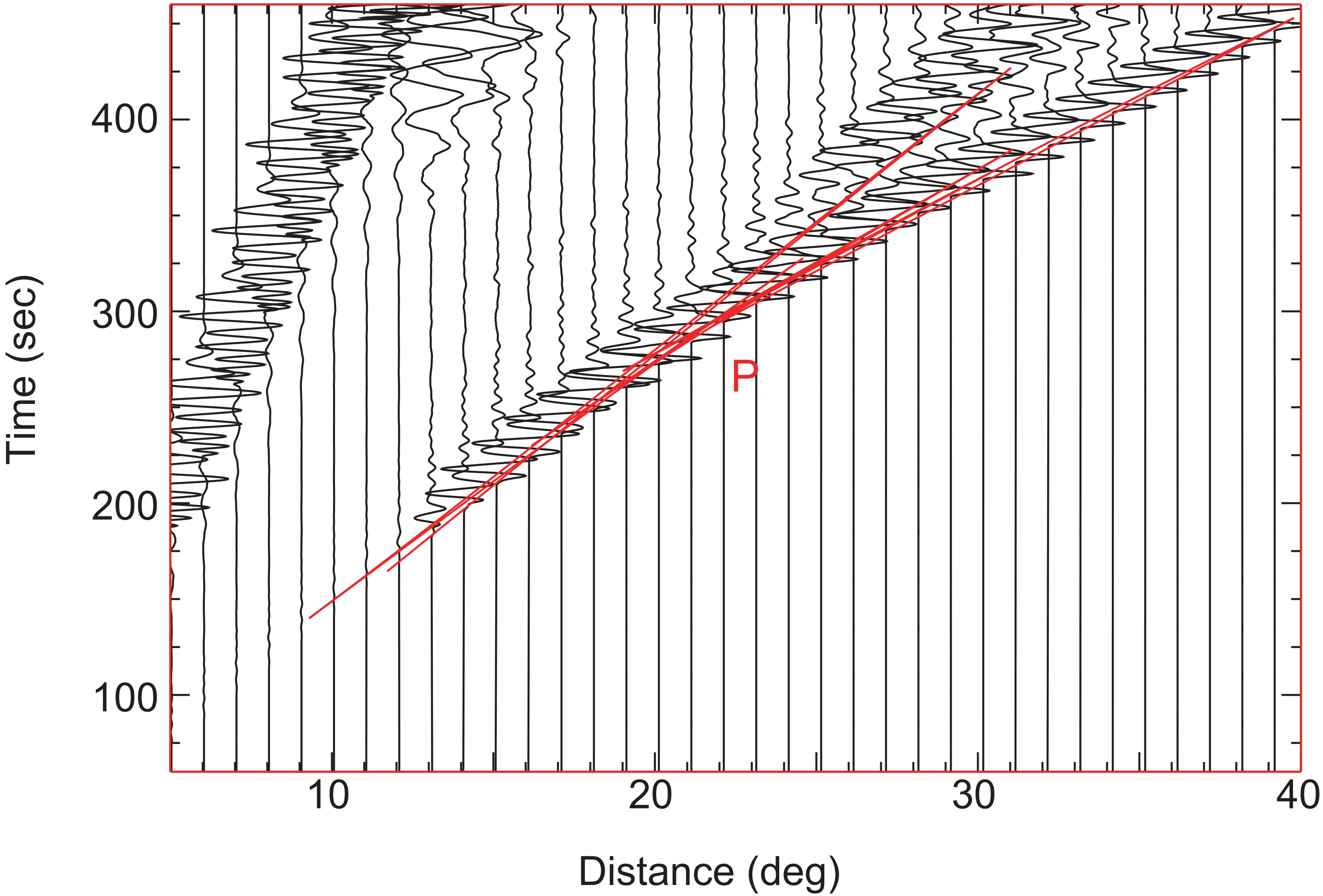


Δ=152°
PKiKP
PKP
PKP
PKIKP
Pd
50s



Vertical component
1400
1300
1200
1100
Time (sec)
A
C
F
B
D
PKP
Pd
100
180
Distance (deg)

Triplication in P waves sampling the transition zone



Waveform inversion

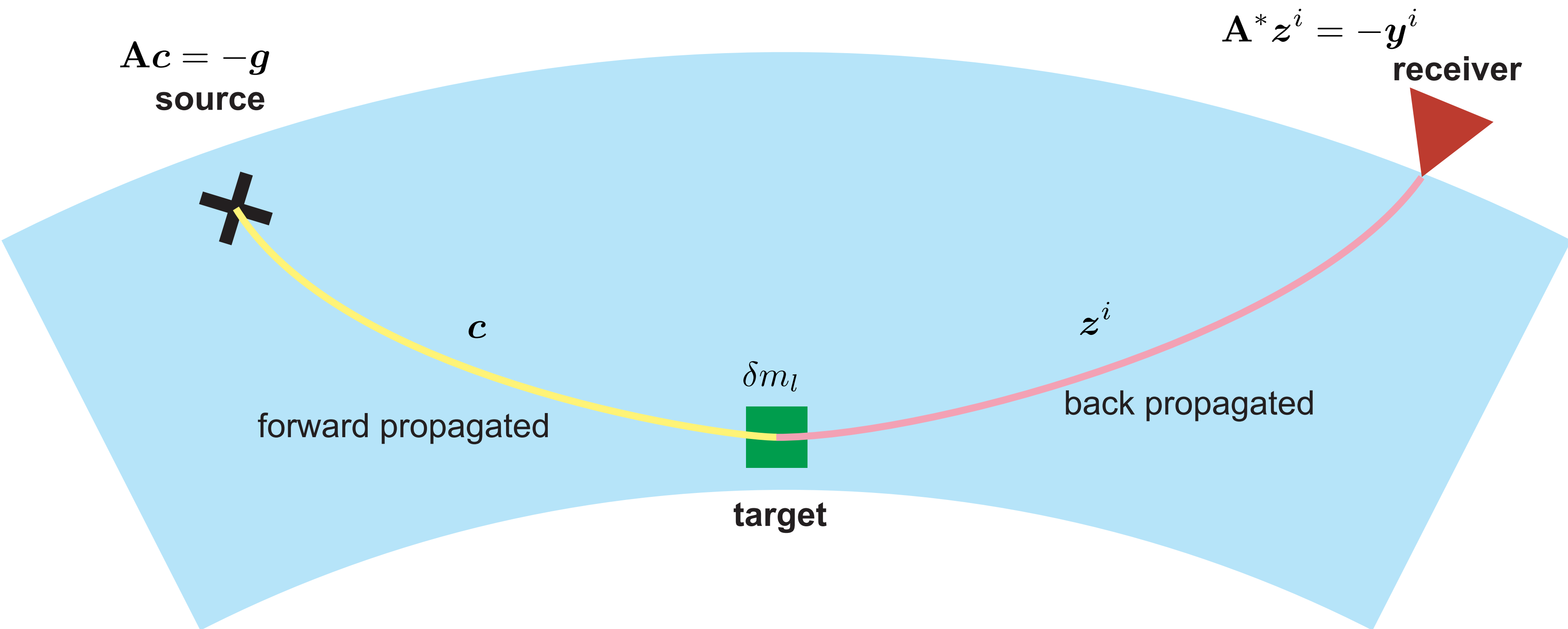
1. Efficient computation of partial derivatives

- Efficient algorithm for computation of partial derivatives (Geller and Hara 1993).
  → Feasibility of computation of partial derivatives for "pixel models".

2. Partial derivatives for point and shell perturbations

- Delta functions are used for the dependence of the model parameters rather than spherical harmonics.
  → Waveform inversion for local structure can be conducted without artifacts due to regions of poor coverage.

Cross Correlation of forward and back propagated wavefields (1)



$\mathbf{A}\ (= \omega^2\mathbf{T} - \mathbf{H})$ discretized structure matrix

m_l *l*-th model parameter

Forward propagated wavefield

$\boldsymbol{c}$ forward propagated wavefield

$\boldsymbol{g}$ excitation (moment tensor at source)

Back propagated wavefield

$\boldsymbol{z}$ back propagated wavefield

$\boldsymbol{y}^i$ *i*-th component point force at receiver

Cross Correlation of forward and back propagated wavefields (2)

Partial derivatives of the *i*-th component of displacement for a perturbation to the *l*-th model parameter at the p-th receiver for the k-th earthquake

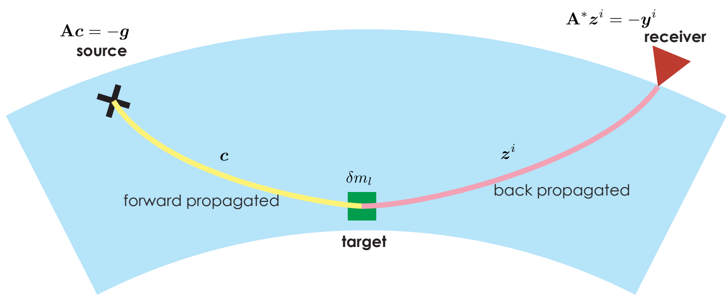
$$\left\{ \frac{\partial u_i^{(k)}[\boldsymbol{r}^{(p)}]}{\partial m_l} \right\}^* = [\boldsymbol{c}^{(k)}]^* [\omega^2 \mathbf{T}^{(l)} - \mathbf{H}^{(l)}]^* \boldsymbol{z}_i^{(p)}$$

$$= \int_V (\omega^*)^2 [u_j^{(k)}]^* [\rho^{(l)}]^* \eta_{ji}^{(p)} dV - \int_V [u_{j,q}^{(k)}]^* [C_{jqrs}^{(l)}]^* \eta_{ri,s}^{(p)} dV$$

u_i	*i*-th component displacement
$\boldsymbol{c}$	forward propagated wavefield

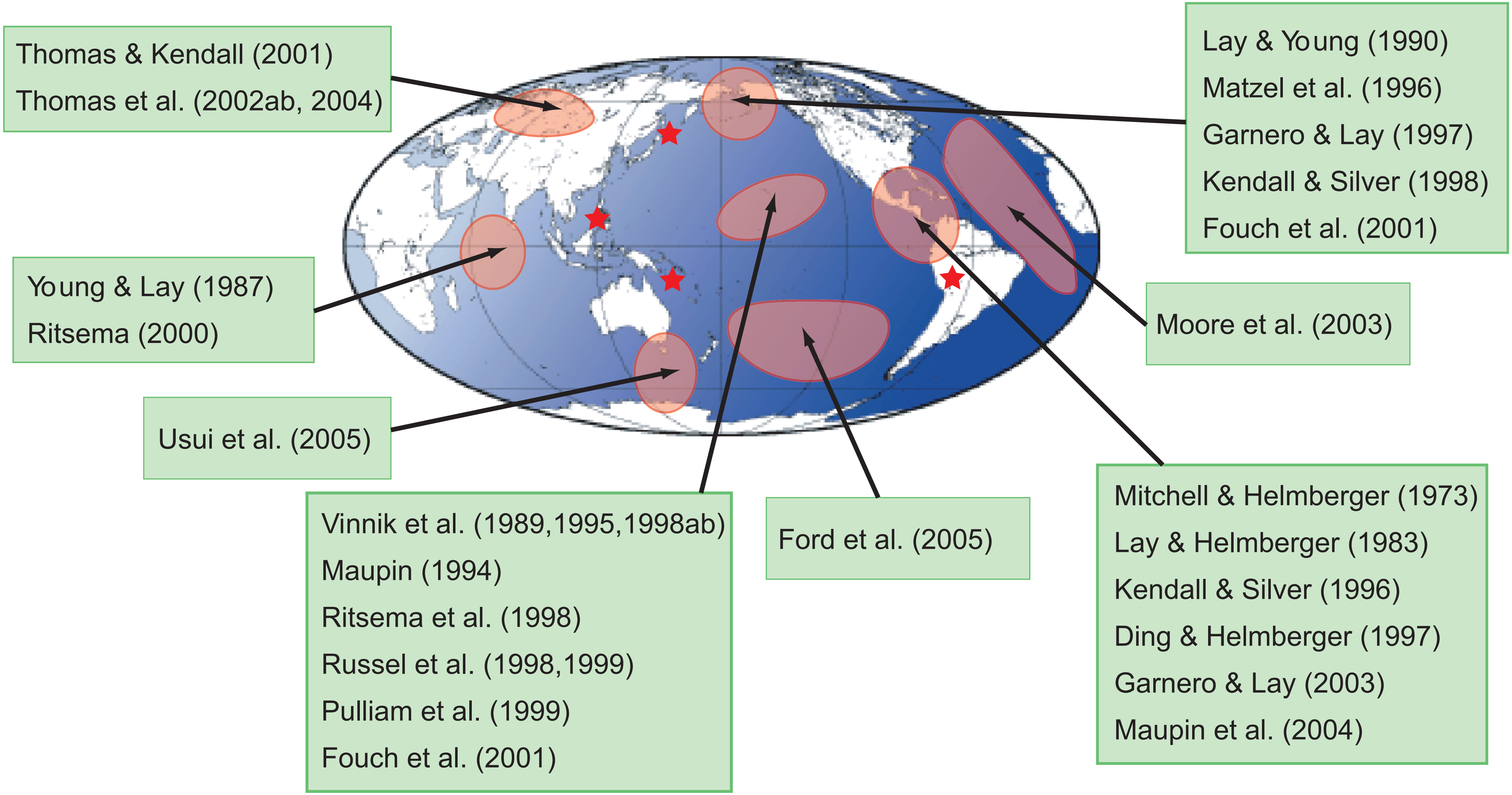
η	back propagated displacement
$\boldsymbol{z}$	back propagated wavefield

$\mathbf{T}^{(l)}$	mass matrix for the *l*-th model parameter
$\mathbf{H}^{(l)}$	stiffness matrix for the *l*-th model parameter
m_l	*l*-th model parameter

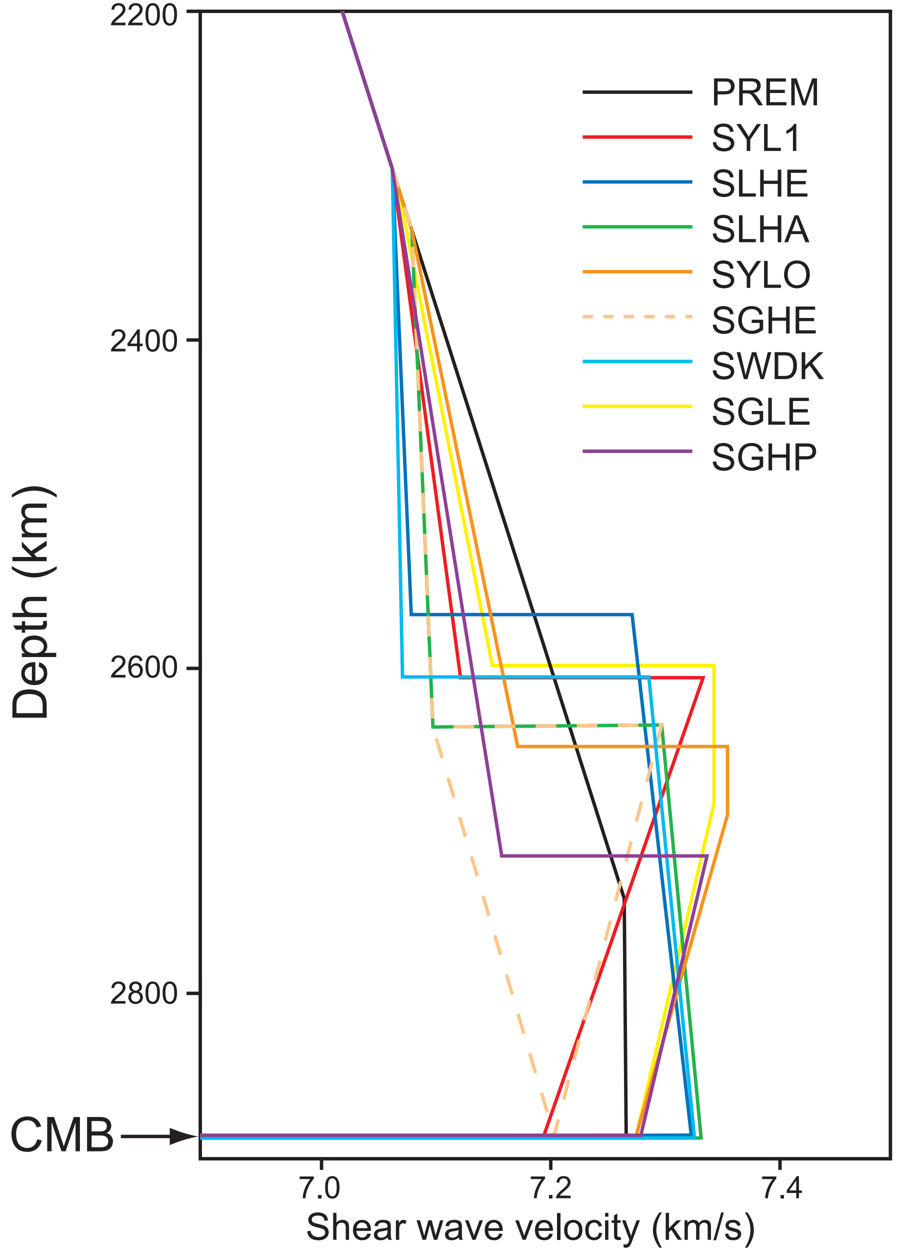


(Geller & Hara, 1993)

Regions of D" layer studied



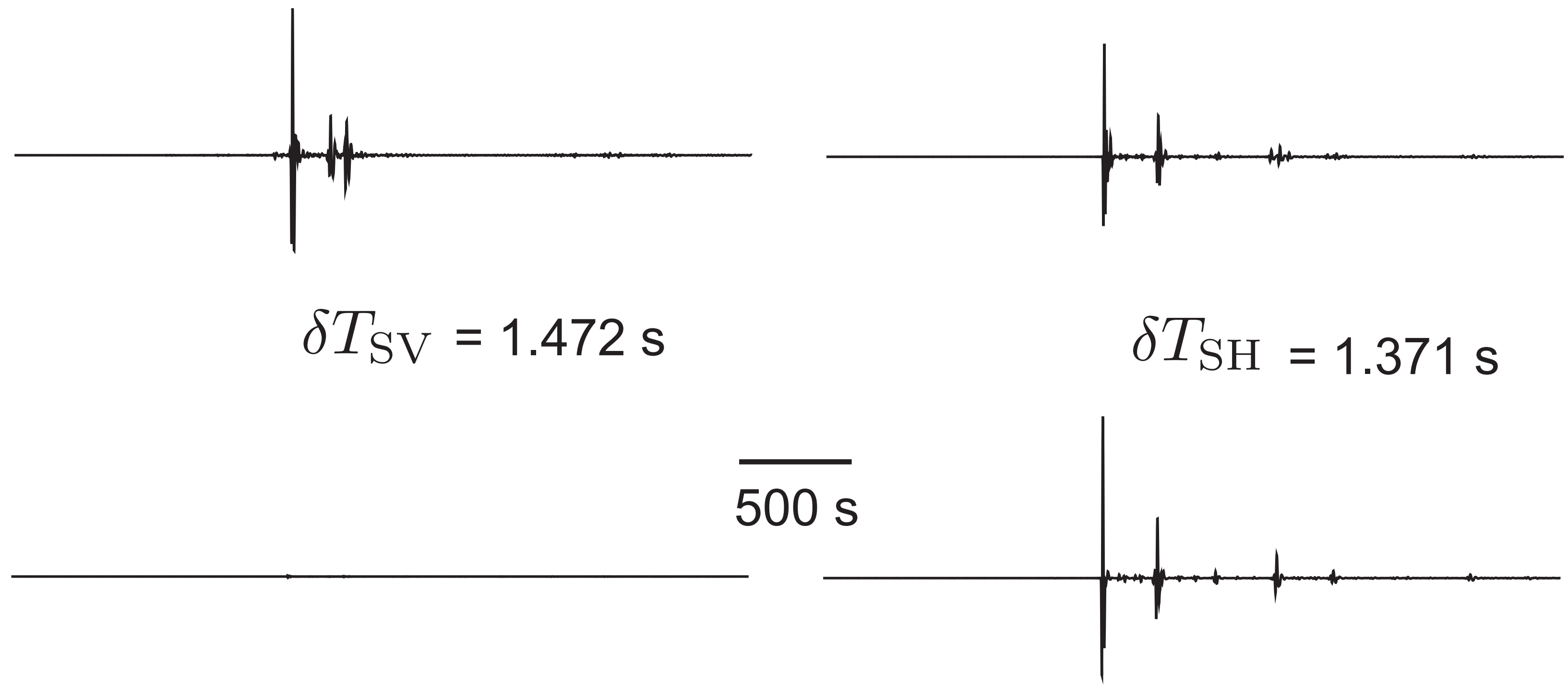
Previous studies of 1-D D" structure



(modified after Wysession et al 1998)

Limitation of ray theory

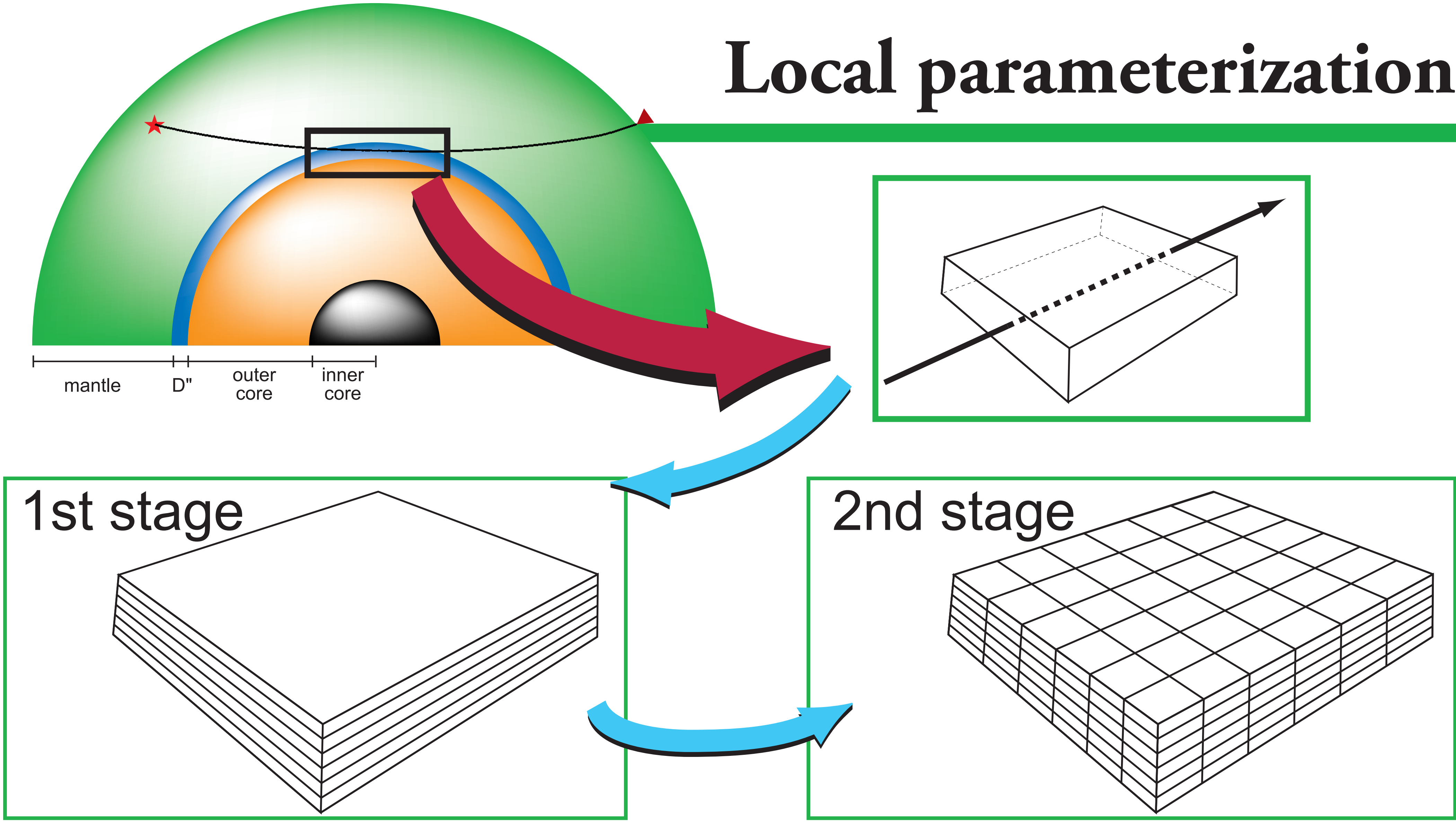
$\frac{\partial u}{\partial L}$ (SV) $\frac{\partial u}{\partial N}$ (SH)



200 km above CMB: δT_{SV} = 1.472 s, δT_{SH} = 1.371 s

above CMB: δT_{SV} = 1.072 s, δT_{SH} = 2.358 s

Local parameterization



Parameterization for (local) shell.

Parameterization for pixel.

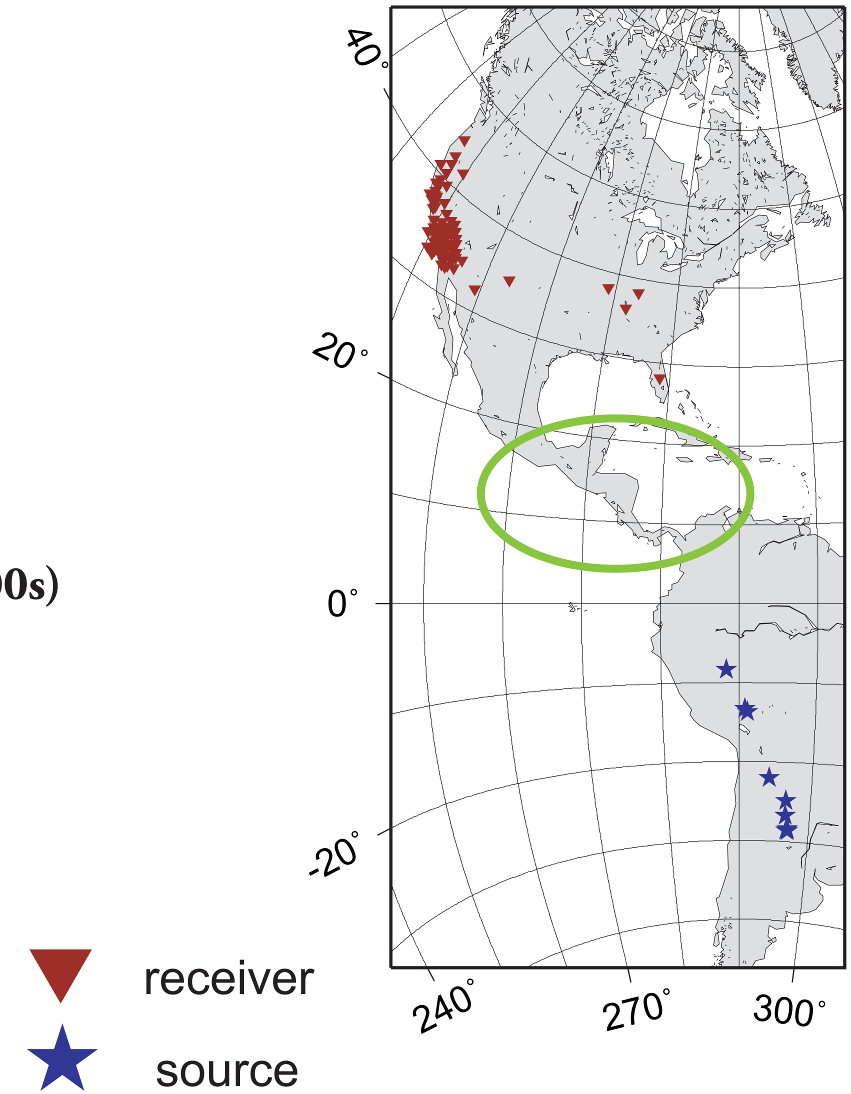
local 1-D inversion

local 3-D inversion

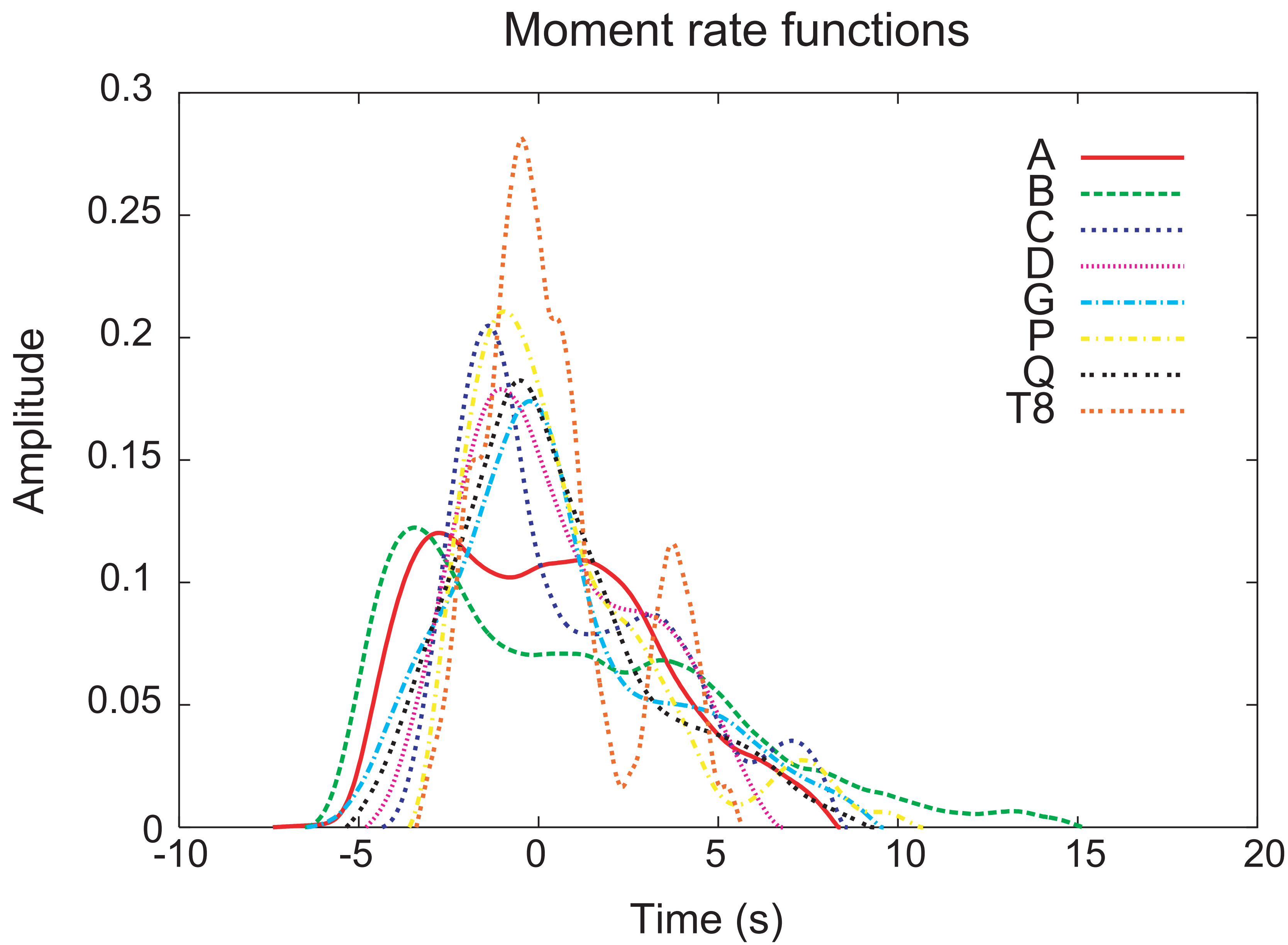
Waveform inversion - version 0

We conduct waveform inversion for local structure using body waves.

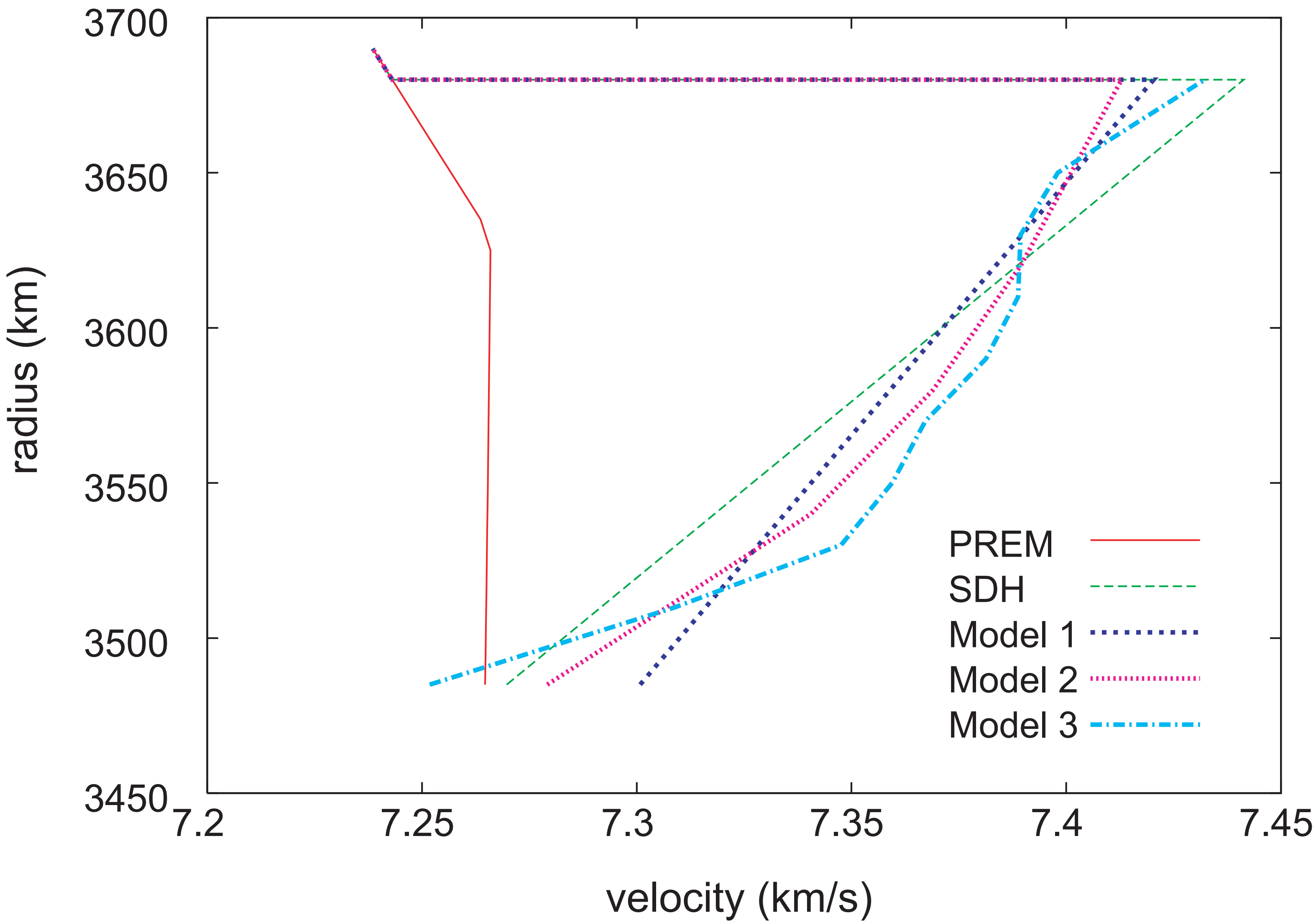
Initial model : SDH (Ding & Helmberger 1997)
Source mechanism : Harvard CMT
Used events : 8
Used passbands : 4 (8-12.5s; 12.5-25s; 25-50s; 50-100s)
Used time windows : 1188



Empirical source time function - isolated phase (Inversion ver. 0)



Preliminary models (Inversion ver. 0)



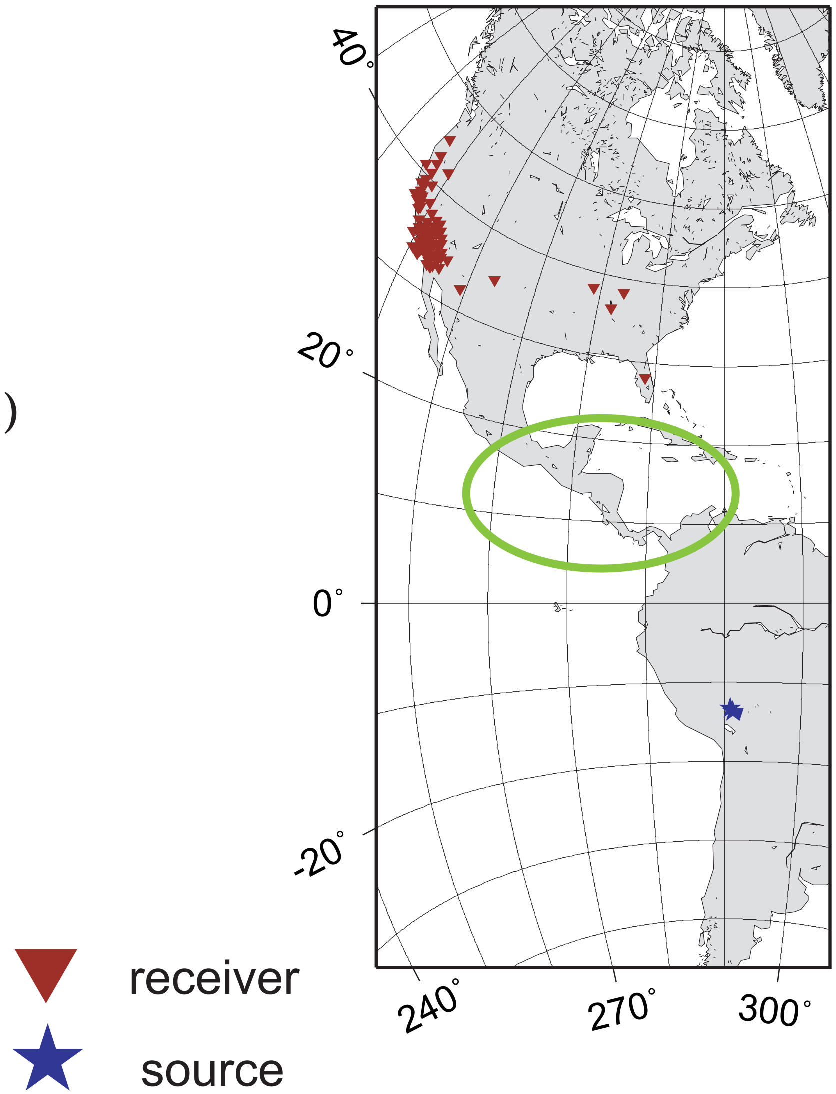
Lesson and problems from version 0

1) Inversion method
   - effective choice of parameterization
   -> use singular value decomposition (SVD)
2) Alignment of data & synthetics
   -> static correction using "sub-array"
3) Source time function
   -> use stacked isolated phases (beam)

Waveform inversion - version 1

We conduct waveform inversion for local structure using body waves.

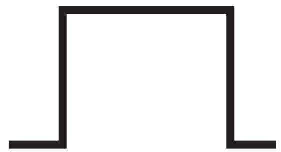
Initial model : PREM (Dziewonski & Anderson 1981)
Source mechanism : Harvard CMT
Used events : 2
Used passbands : 2 (25-50s; 50-100s)



Source time function (1)

[ver. -1] Half duration of Harvard CMT solutions

(e.g. Tajima and Grand, 1998)



[ver. 0] Empirical source time function (1)

- isolated displacement phases

(e.g. Ding & Helmberger 1997)

[ver. 1] Empirical source time function (2)

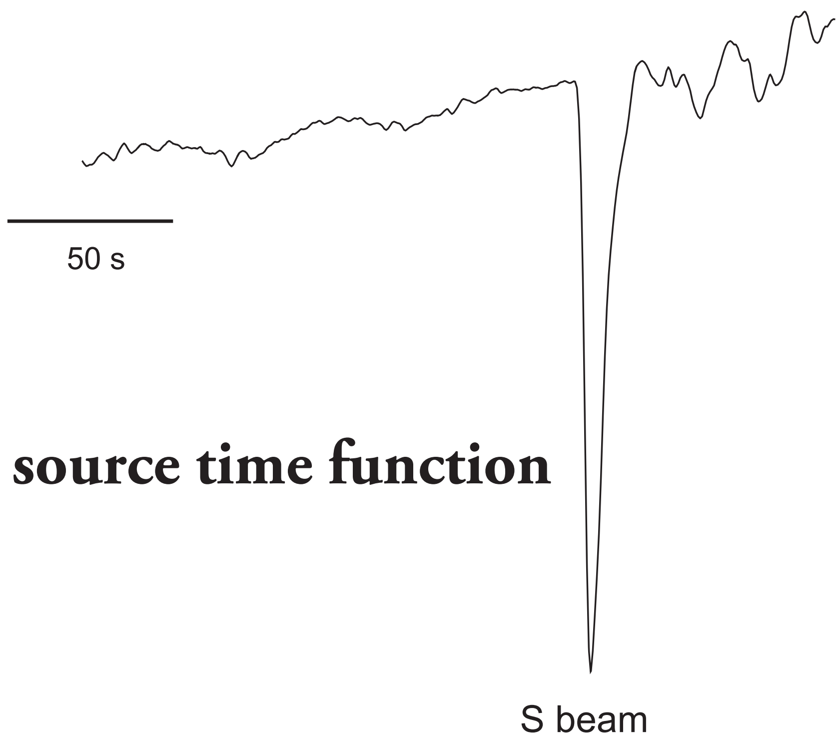
- beam method (stacked waveforms)

(e.g. Lay et al., 2004; Kito et al., 2004)

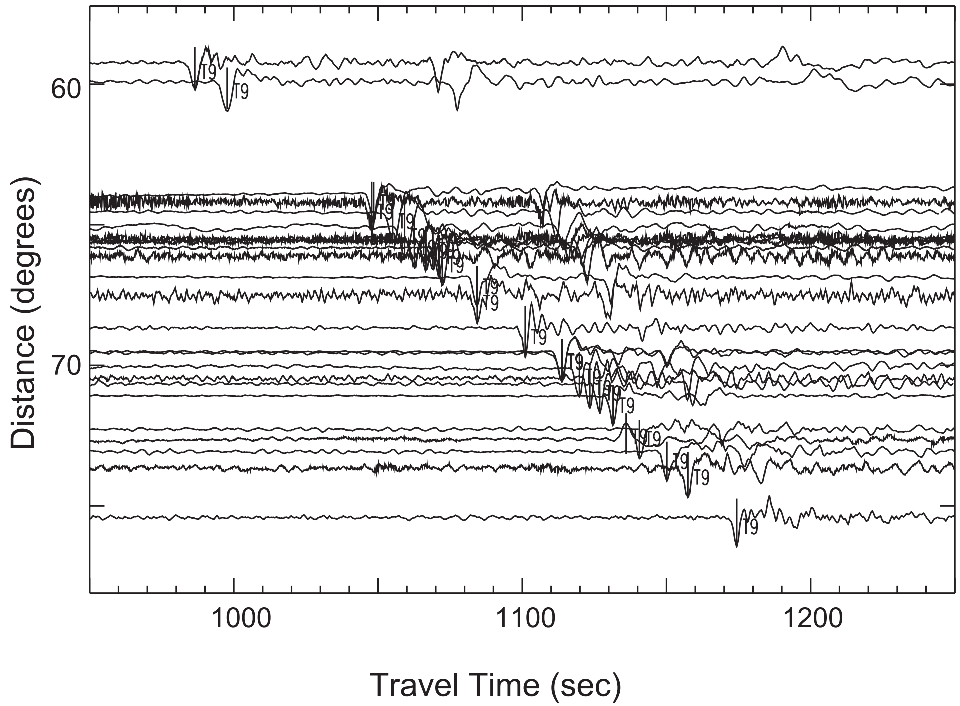
Source time function (2)

Beam method

hand-pick peaks or onsets of isolated phases for each event and then stack them in order to make avaraged source time functions (e.g. Lay et al. 2004; Kito et al. 2004)

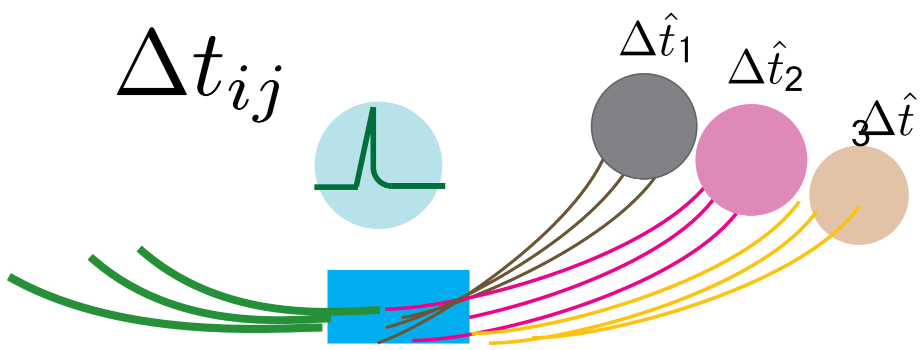


averaged source time function



Static correction (1) - alignment of data

- Align observed and synthetics using theoretical travel-times
- Stack both observed and synthetics for each subarray and each event in order to make beams (stacked waveforms)
- Then, cross-correlate them



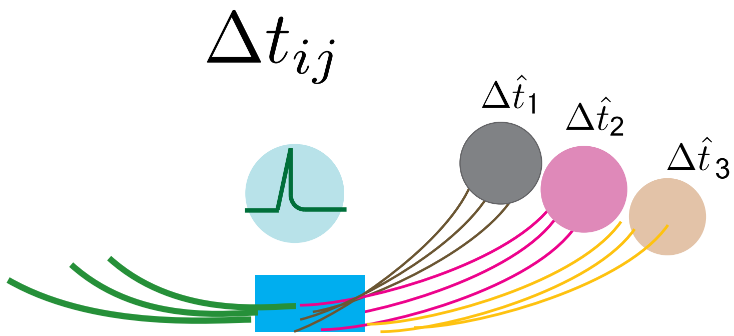
... which we call static correction

Static correction (2) - subarray correction

Subarray correction

$$\Delta\bar{t}_j = \frac{\sum_i \Delta t_{ij}}{n}.$$

$$\Delta\hat{t}_i = \frac{\sum_j (\Delta t_{ij} - \Delta\bar{t}_j)}{m},$$



Δt_{ij} : time shift of the *i*-th subarray for the *j*-th event

$\Delta\bar{t}_j$: average time shift for the *j*-th event

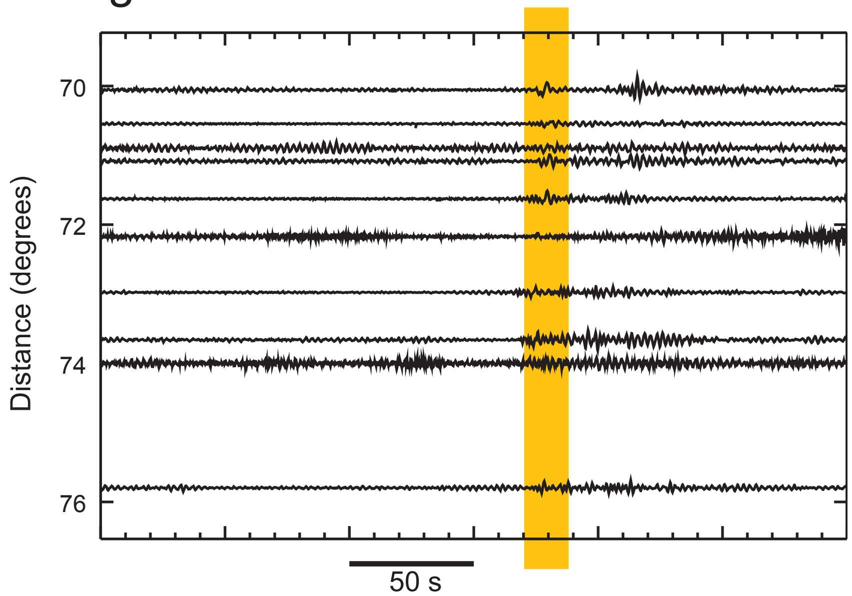
$\Delta\hat{t}_i$: avarage of the differential time shift for the *i*-th subarray

n : number of subarrays

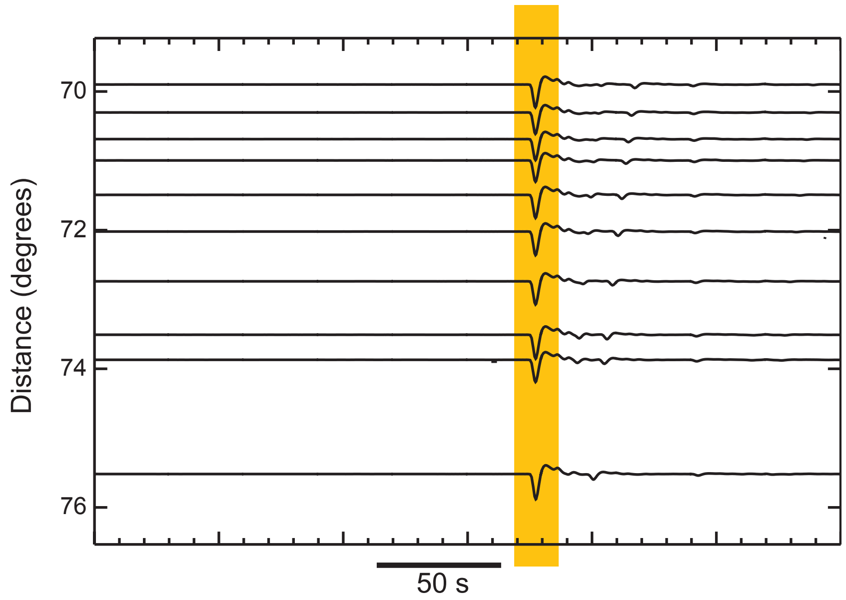
m : number of events

Static correction (3) - Cross-correlation

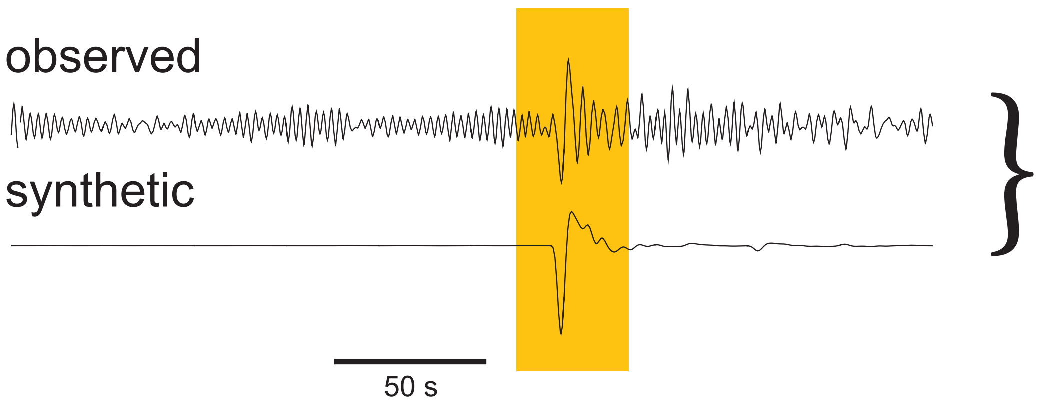
aligned observed waveforms



aligned synthetic seismograms

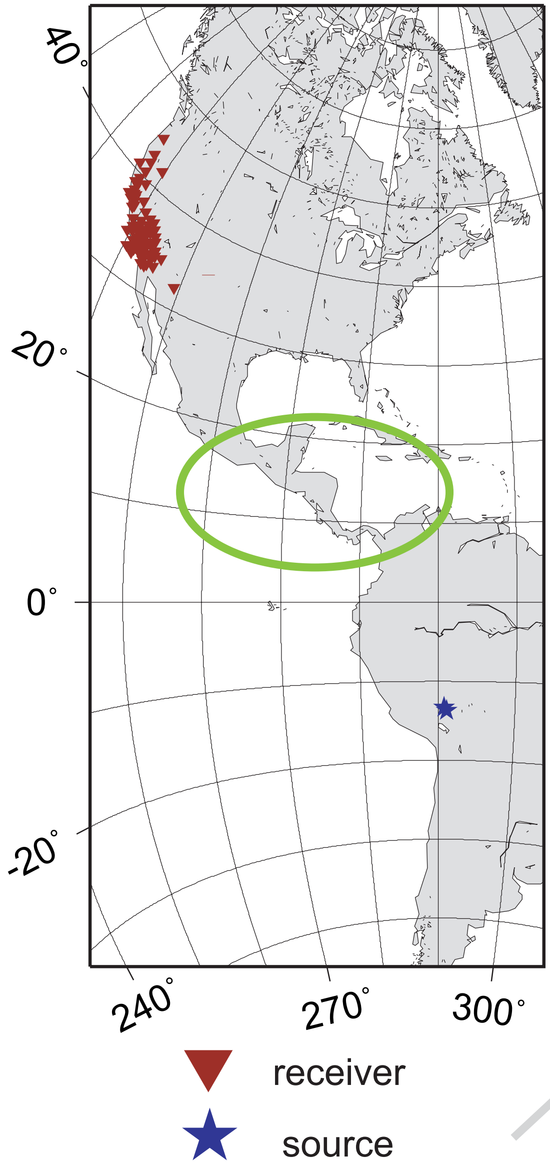


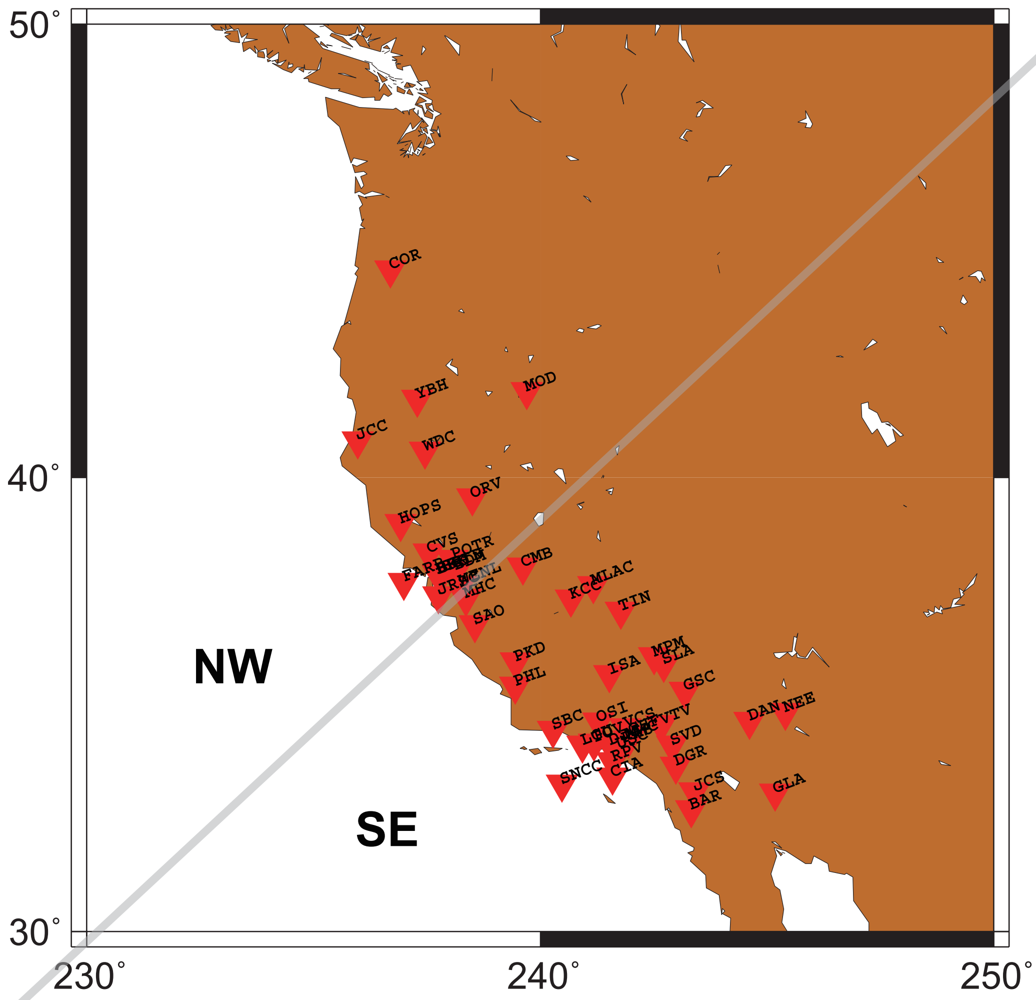
stacked waveforms



Cross-correlate

Subarrays for North America

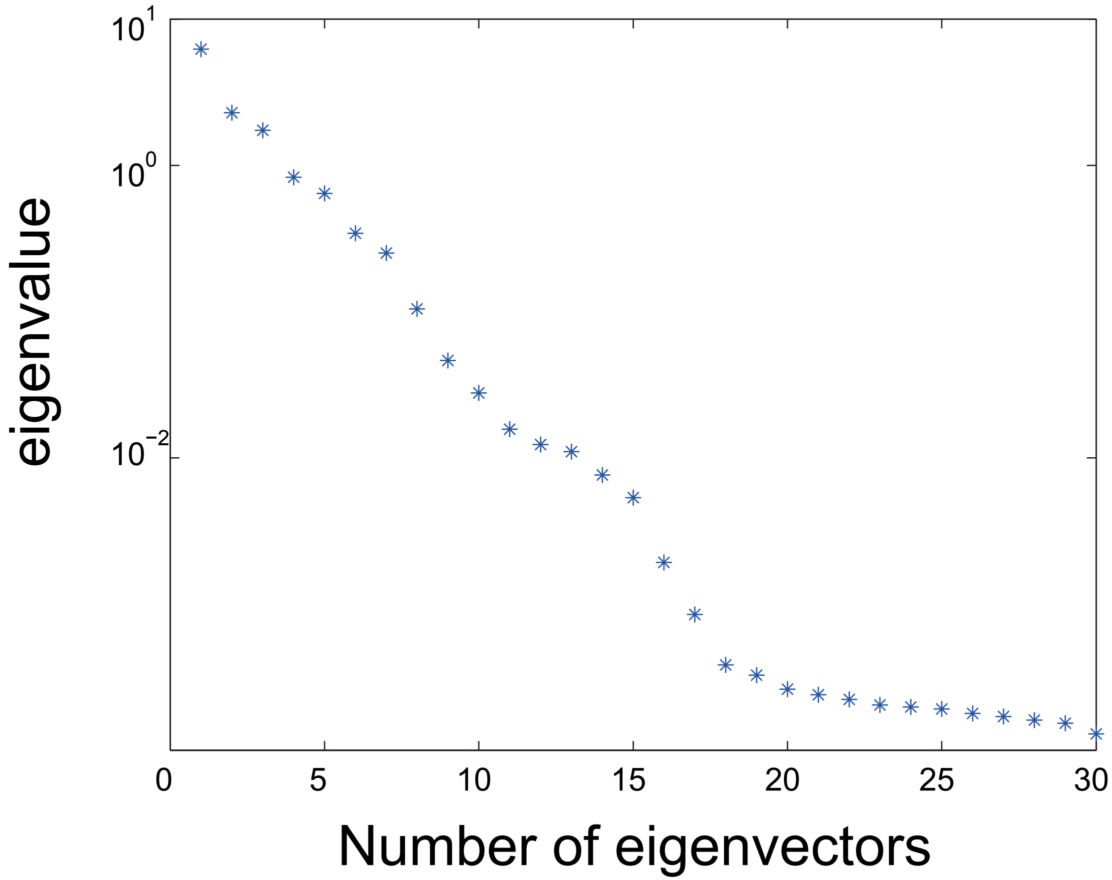




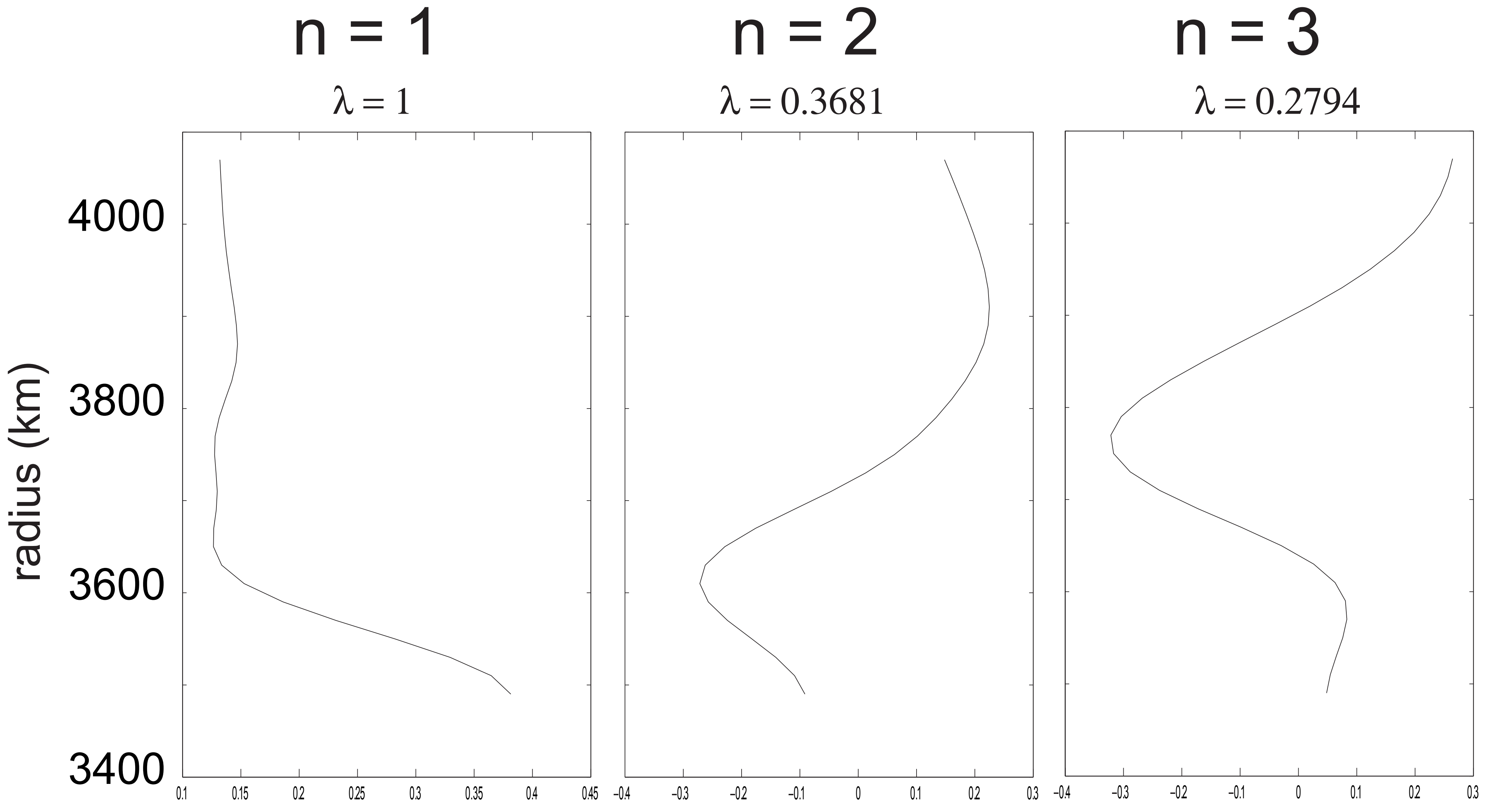
Subarray corrections

NW : 0.108 s
SE : -0.108 s

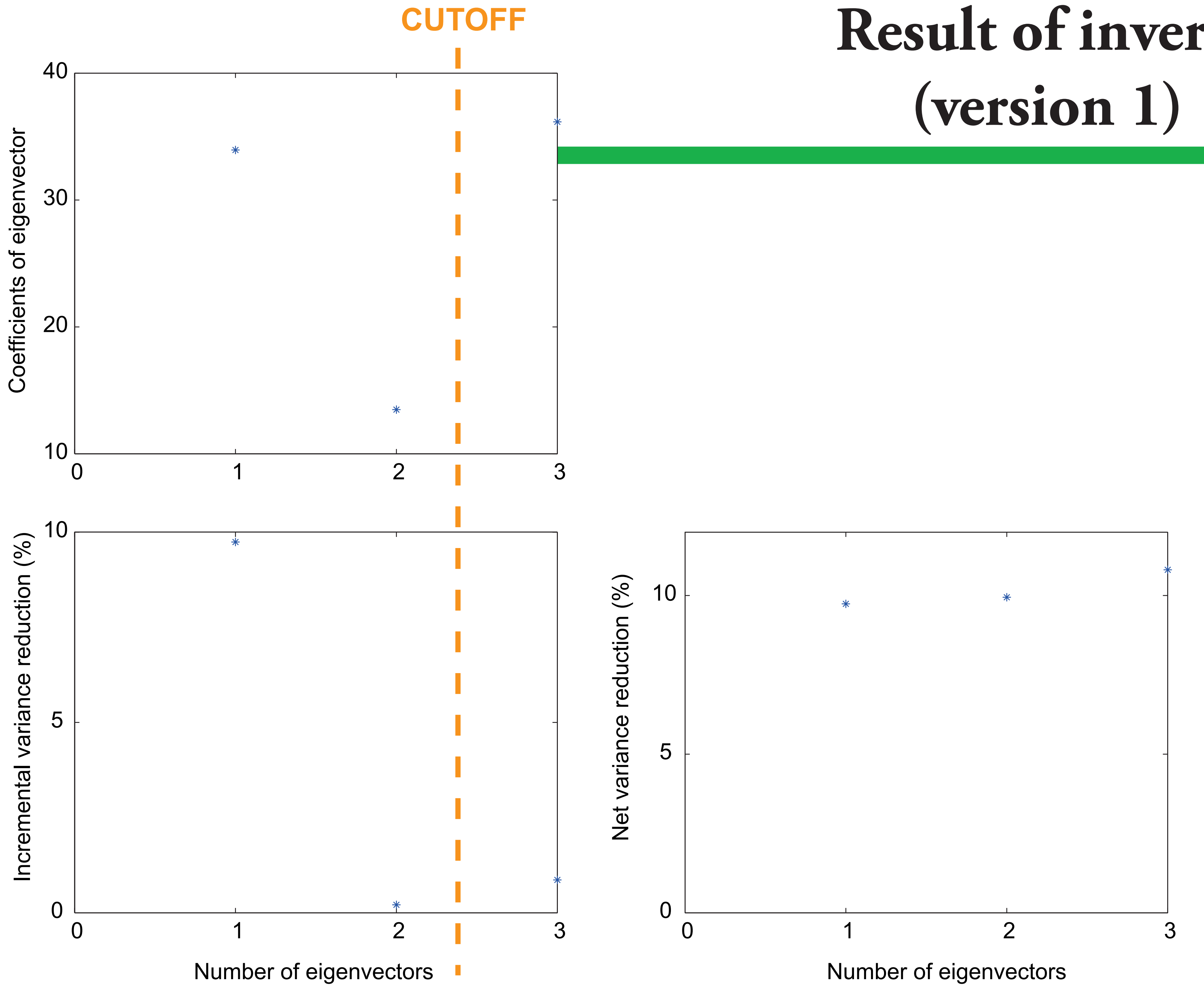
Singular value decomposition - eigenvalues



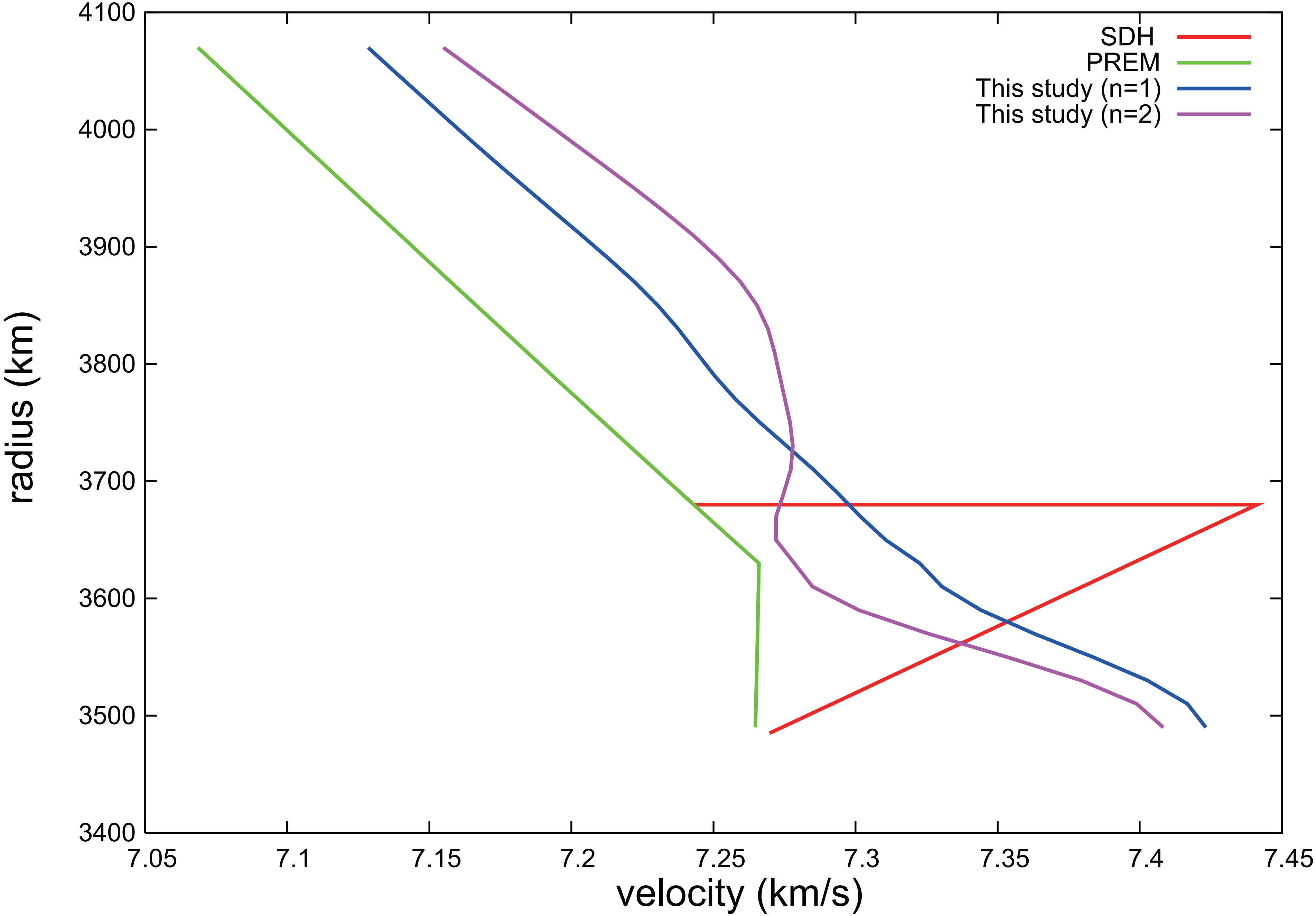
Singular value decomposition - eigenvectors



Result of inversion (version 1)



Results of inversion (version 1)



Future work

[ver. 2]

- Increase number of data
- Enhance types of data (Sdiff, ScS, ScS_2)
- Use higher frequency data
- Inversion for source parameters
- SVD cutoff criterion
- Attachment to existing model

[ver. 3]

- Anisotropy
- 3-D pixel model