TIME REVERSAL and SEISMIC SOURCE IMAGING

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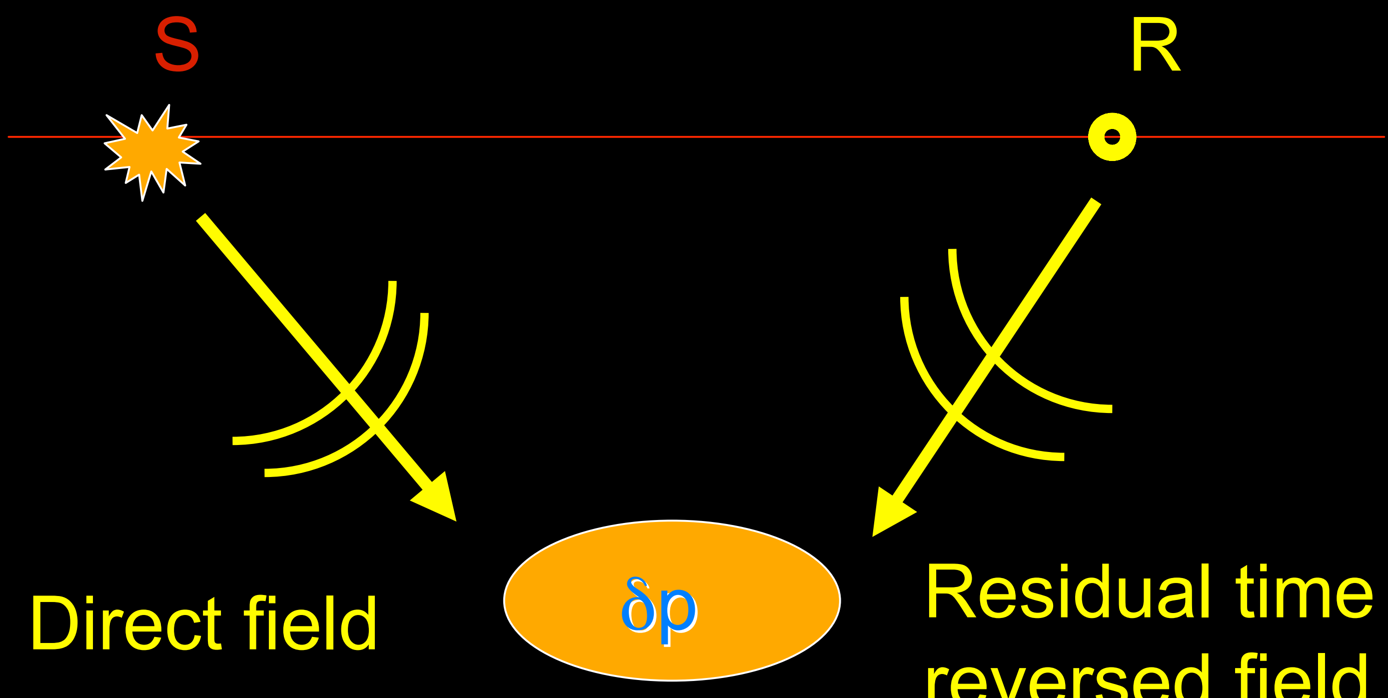
Time Reversal- Adjoint Tomography:

-Source focusing (Green function known)

-Adjoint Tomography => Structure
(source known)



Background

-Time reversal experiments (Fink’s group) => focusing

- Problem of Green function?

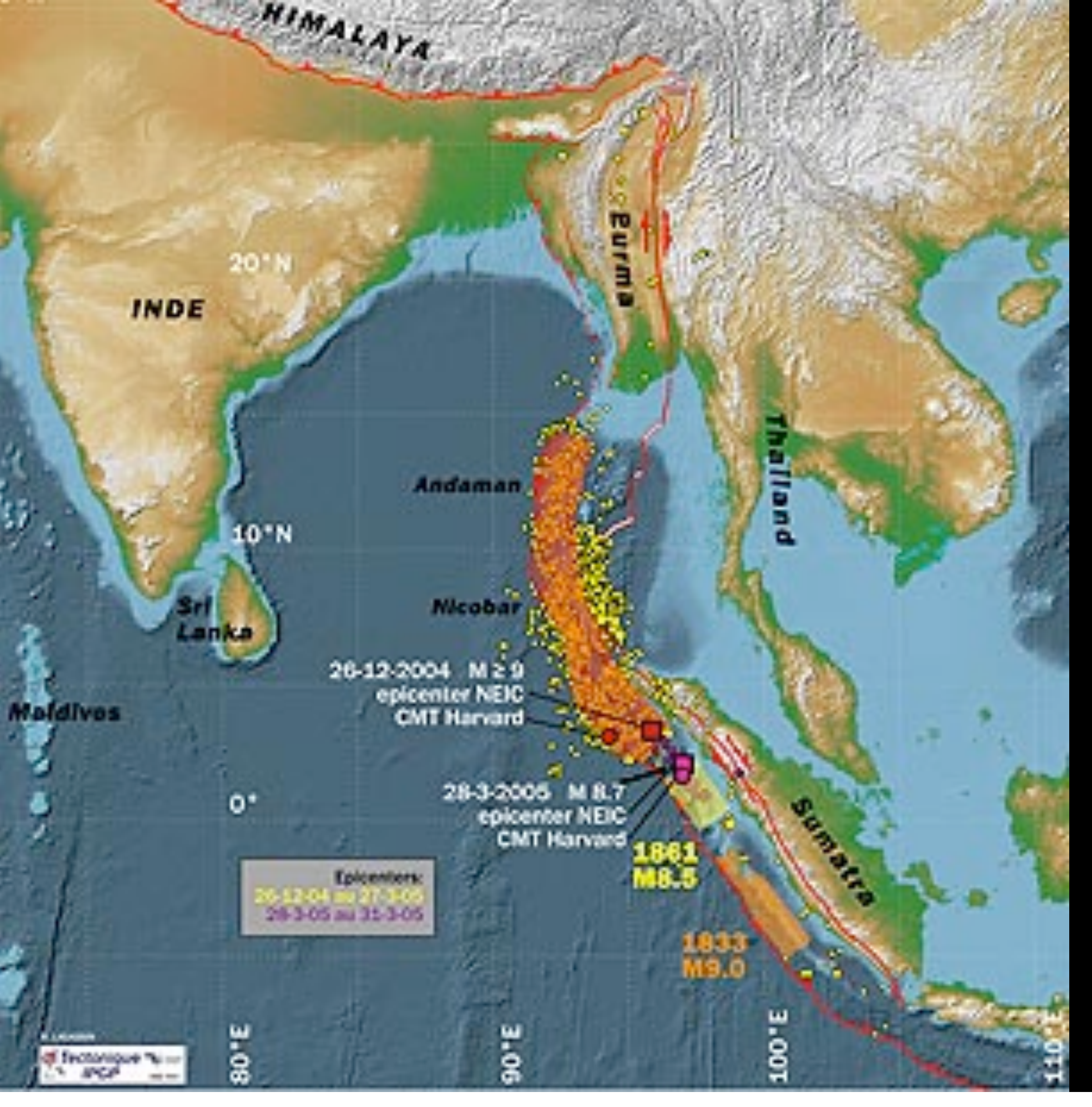
- Seismology: development of numerical techniques

SEM-CSEM (Komatitsch, Tromp, Vilotte, Capdeville,…)

Accurate Green functions?

- Motivation: Big Sumatra-Andaman earthquake

Rupture length 1200km



Time reversal Concept

Elasto-dynamics equation, for seismic displacement field **u**(r,t)

$$\partial^2 u/\partial t^2 = H.u$$

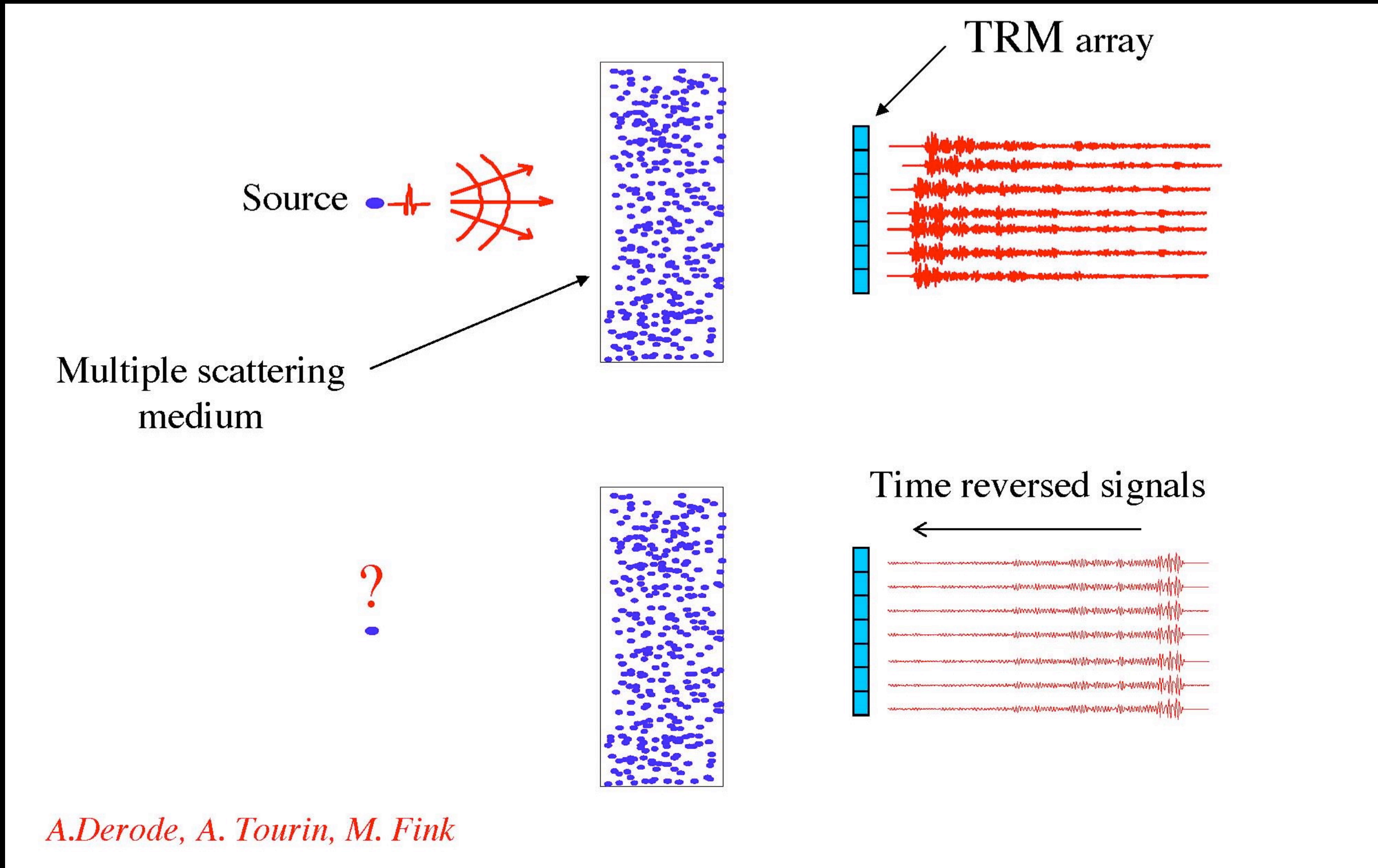
In the absence of attenuation, rotation, time invariance and spatial reciprocity

if u(t) is a solution, u(-t) is also a solution.

We can send back waves with reversed time: how to get a good focusing?

Basic Principle for acoustics:

- Acoustic Source -> receivers
- Existence of transducers being at the same time recorders and emitters



Refocusing at the source location by sending back signal (- t) through the SAME medium from a small number of emitters

Seismic Source Imaging by time reversal

Method Principle:

- Acoustic Source -> receivers
- Existence of transducers at the same time recorders and emitters sending back signal in the same medium

How to apply this concept to seismic waves within the Earth?

1C (scalar) ->3C (elastic case)?

Limited number of receivers?

Realistic Propagating Medium? 1D-3D Earth models

(Green functions)

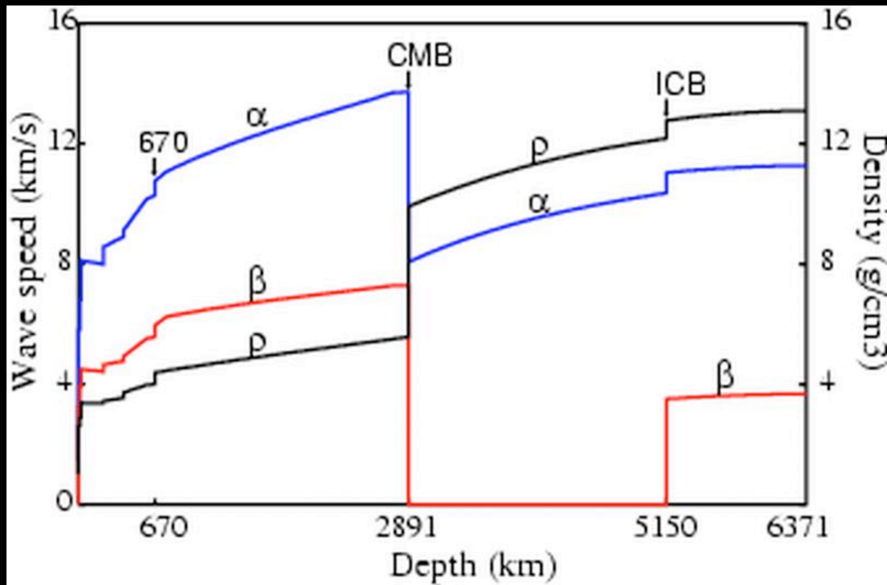
Time reversal

Seismic displacement field **u**(r,t) can be calculated everywhere by the Normal modes or CSEM- method

It is possible to backpropagate **u(-t)**

- Vertical component
- Very long periods T> 150s

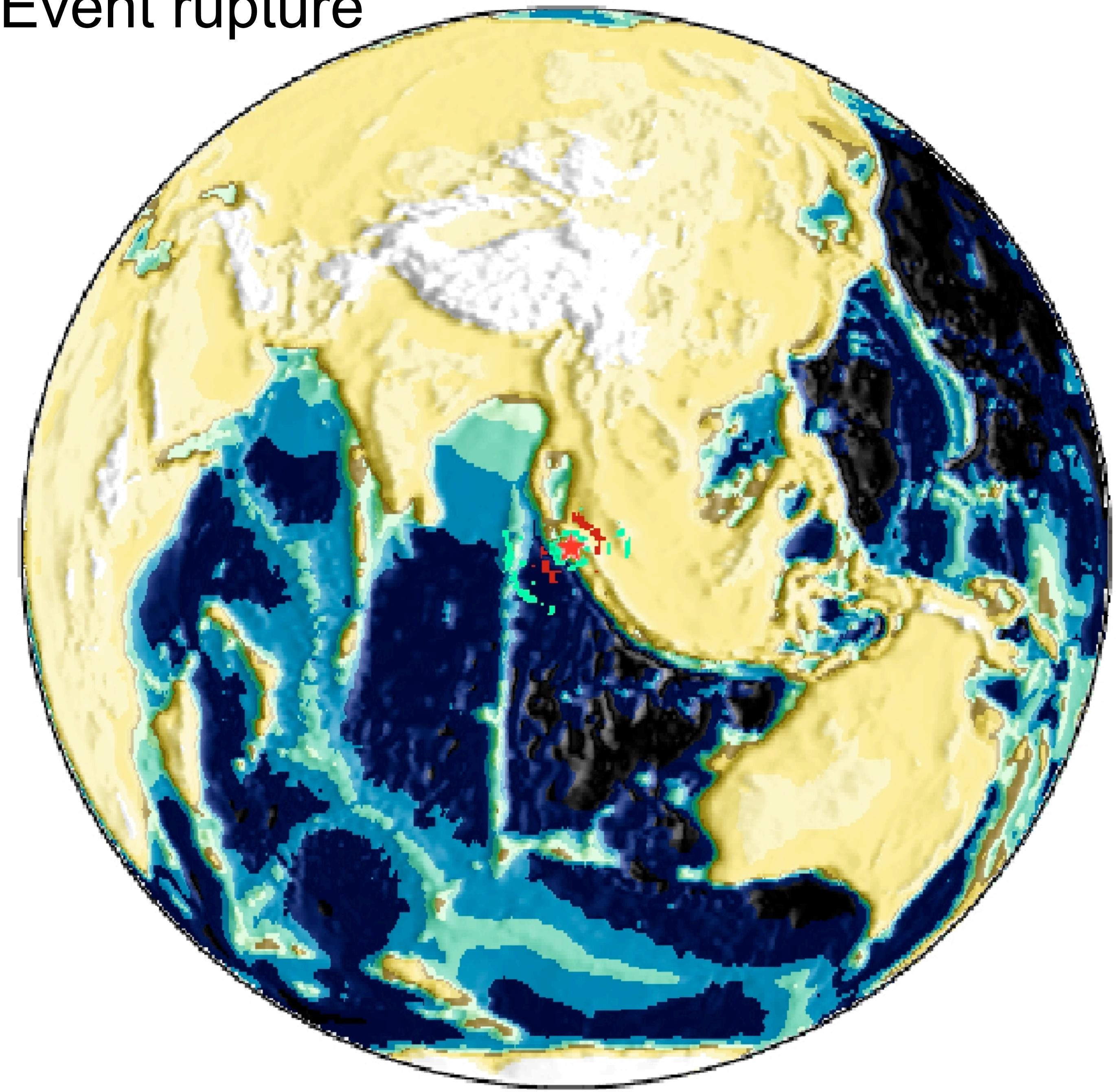
- 1D PREM



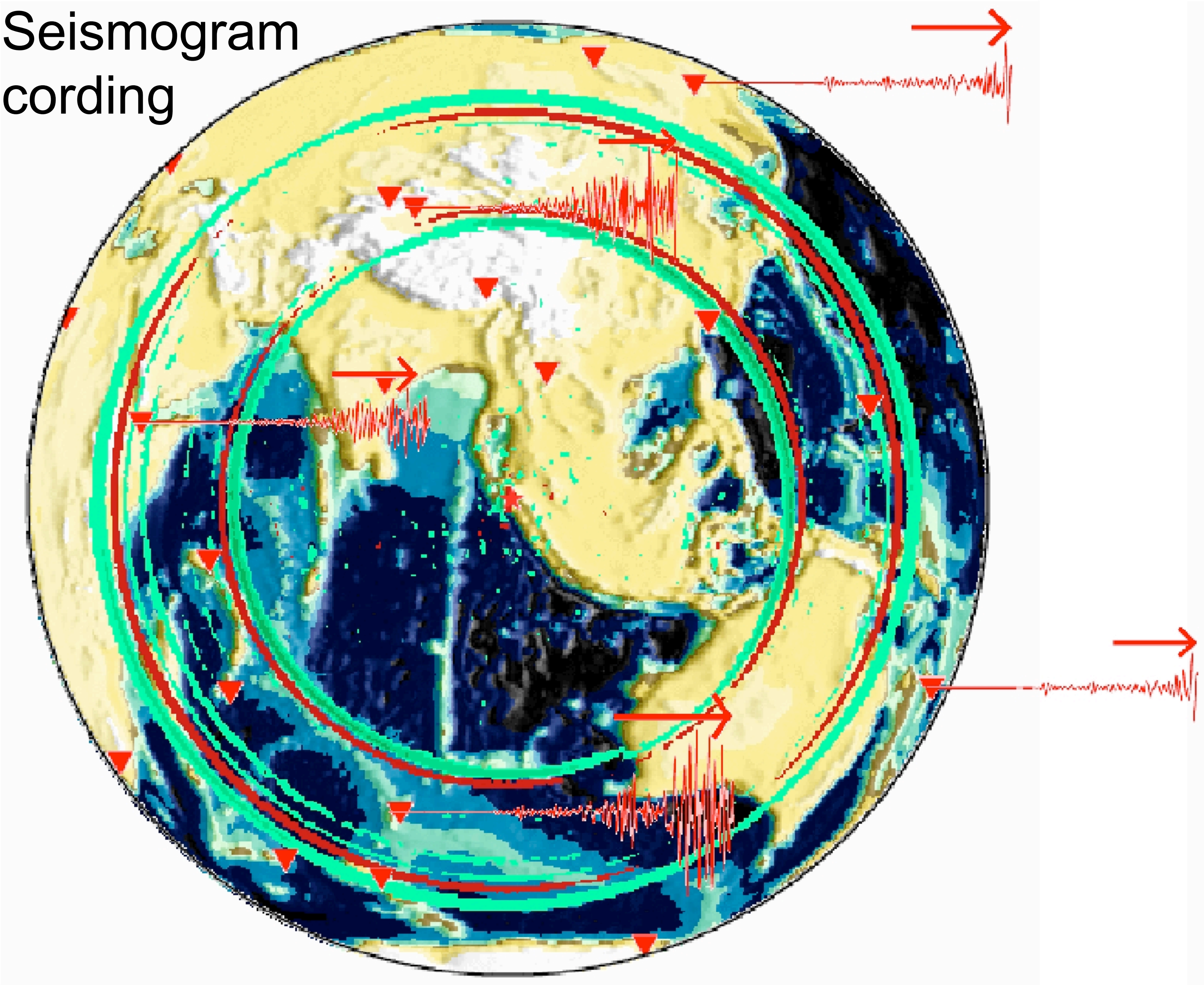
3D Model

Larmat et al., 2006

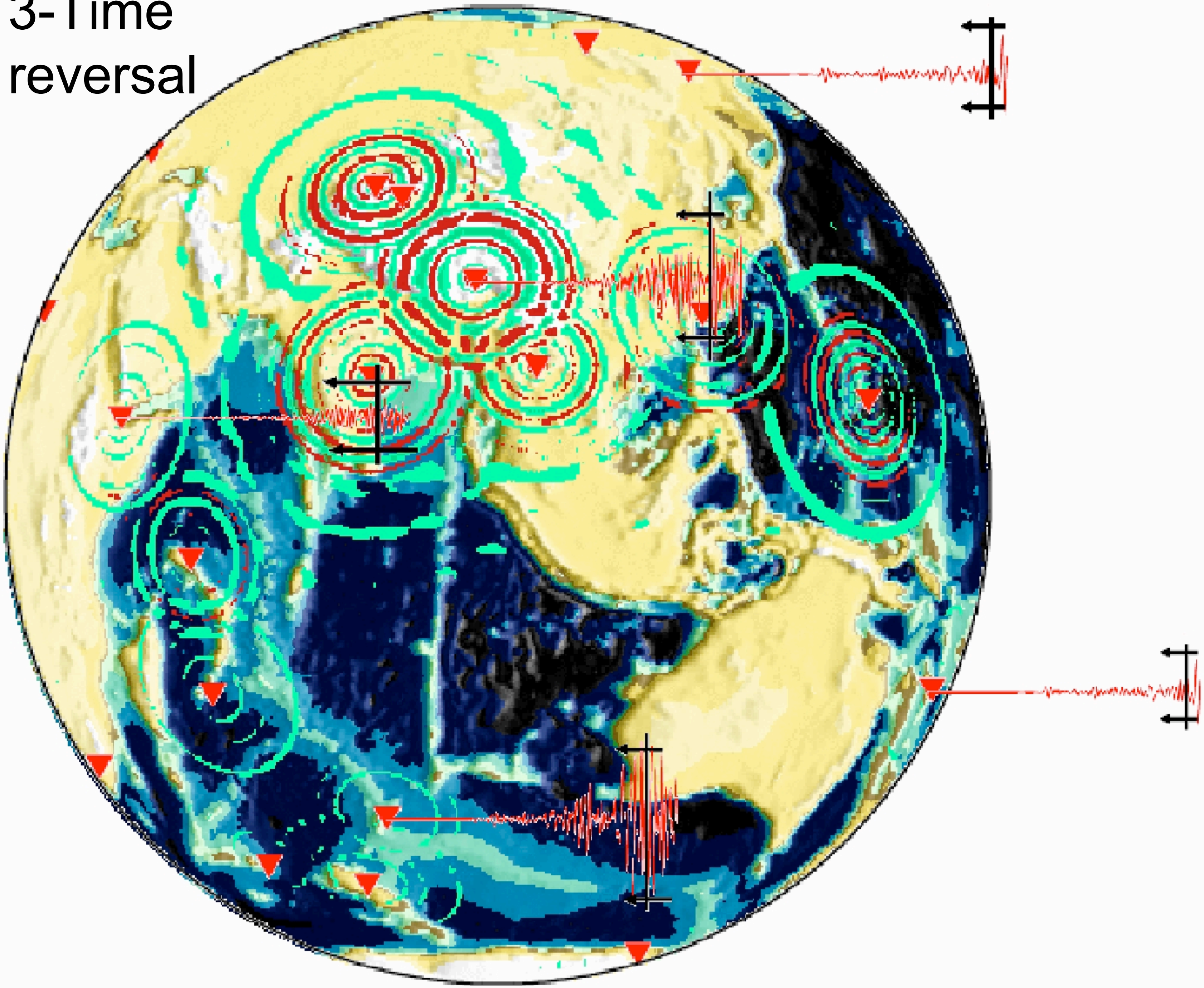
1-Event rupture



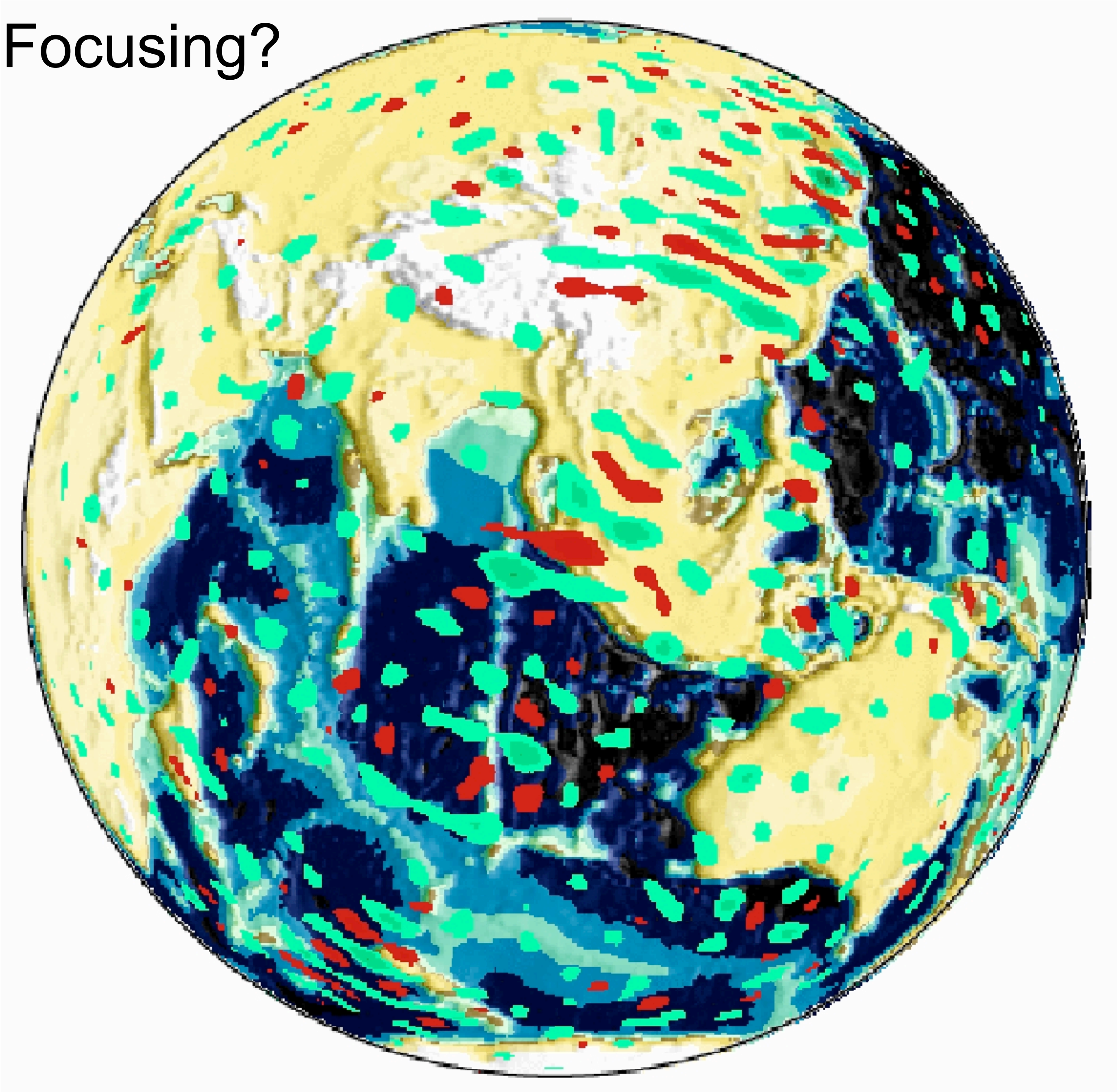
2-Seismogram recording



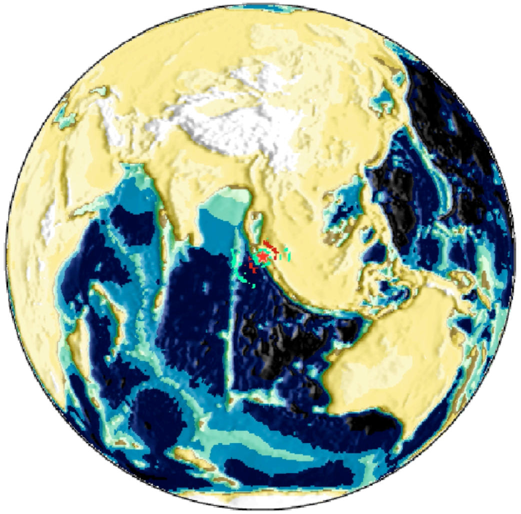
3-Time reversal



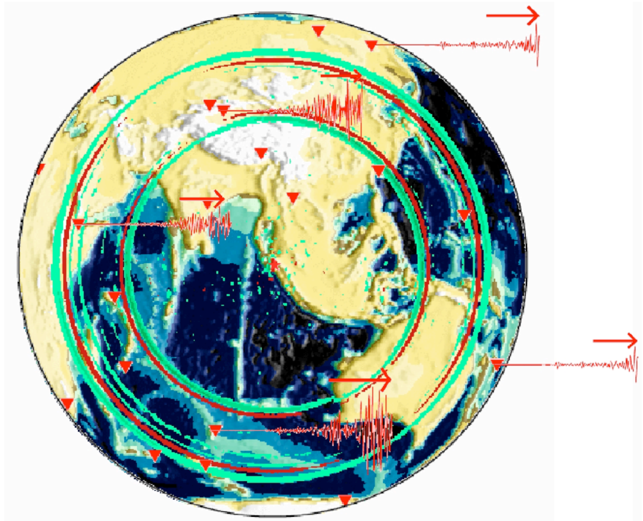
4- Focusing?



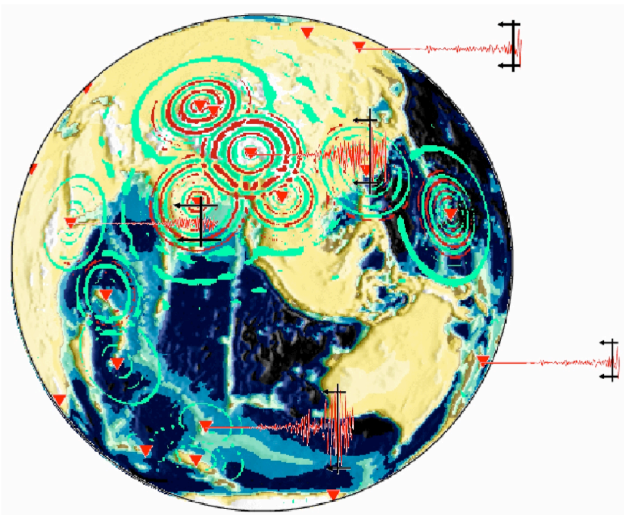
1- Event rupture



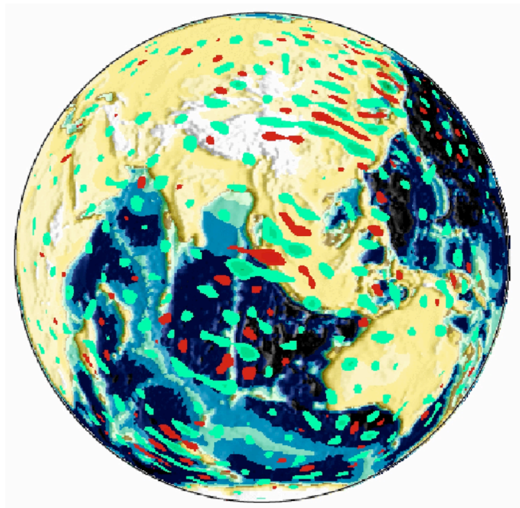
2-Seismogram recording



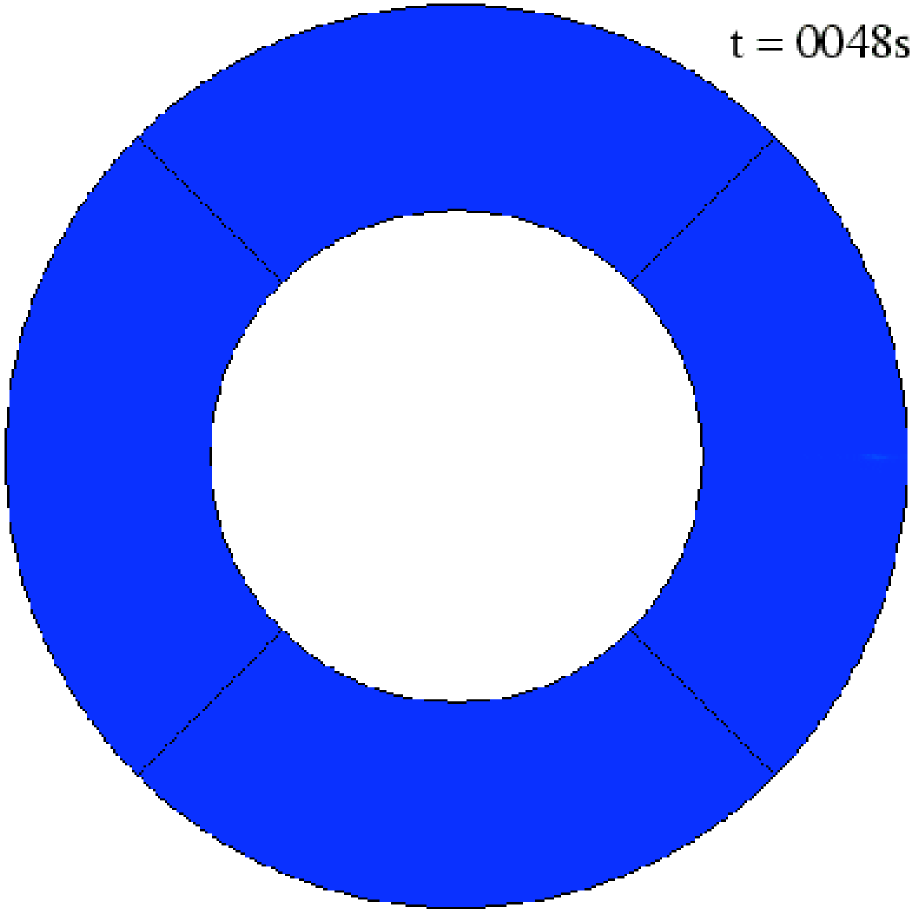
3-Time reversal experiment



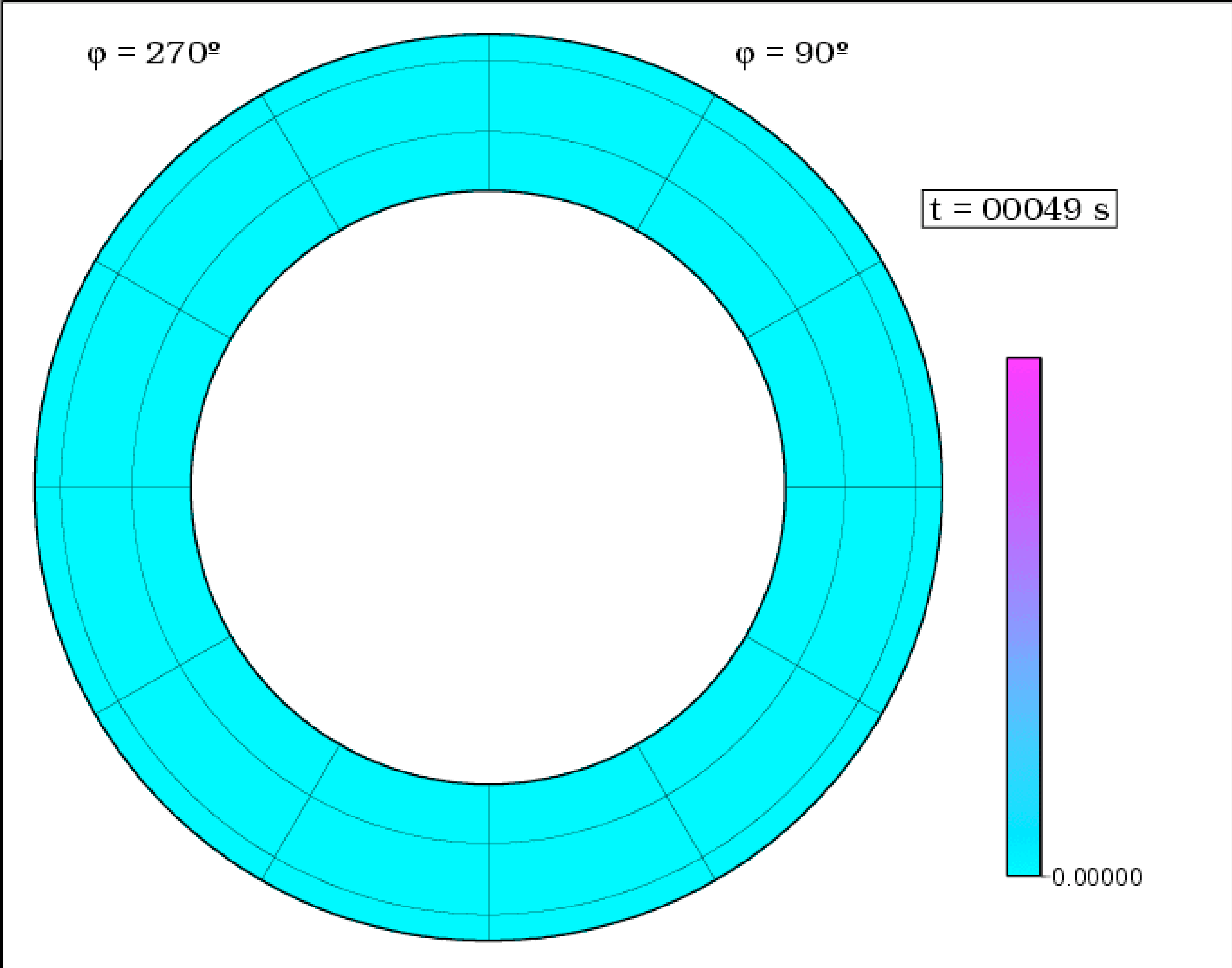
4- Focusing



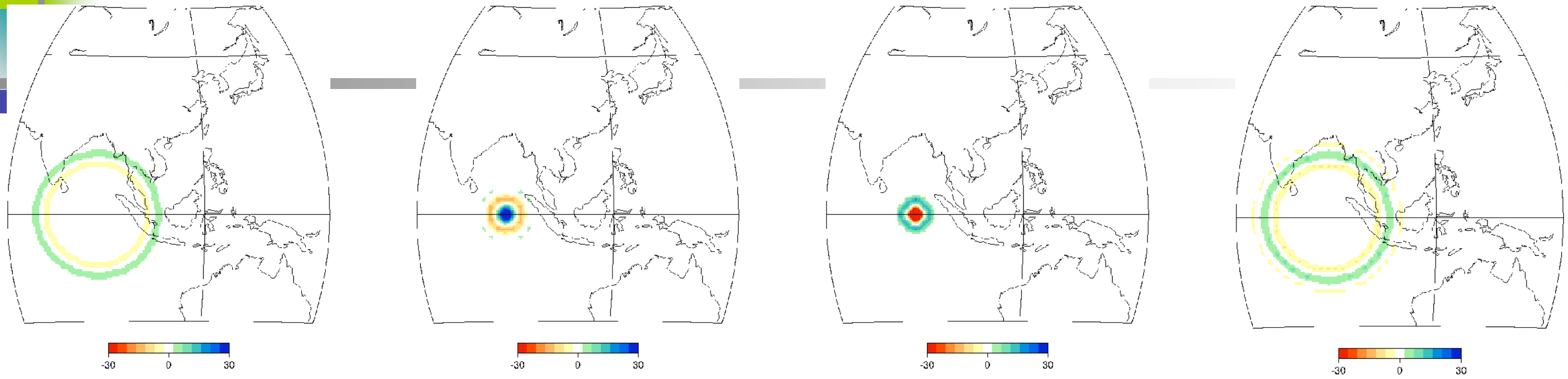
Synthetic Test: Point Source: Forward problem



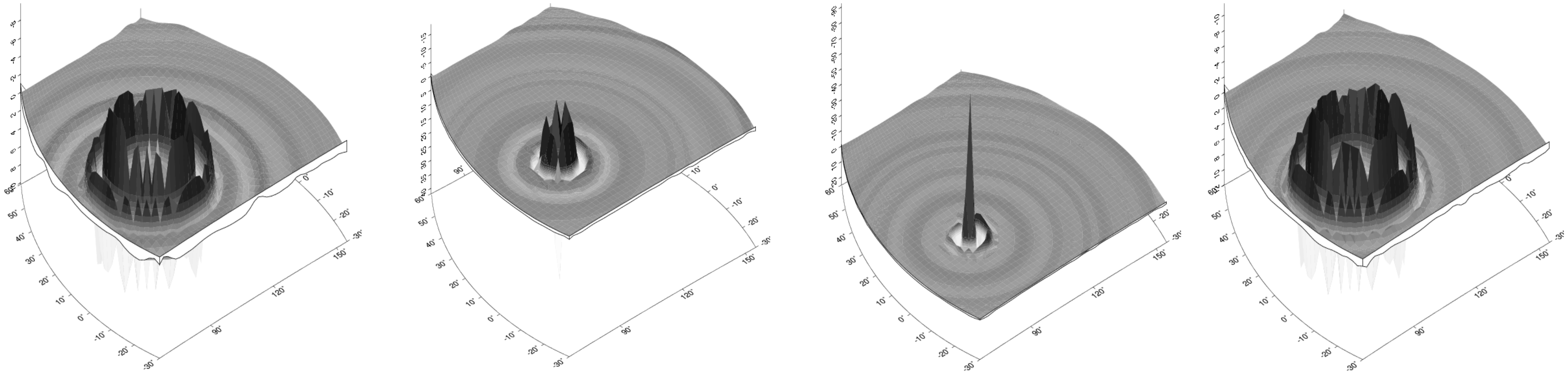
Time reversal



Homogeneous azimuthal distribution

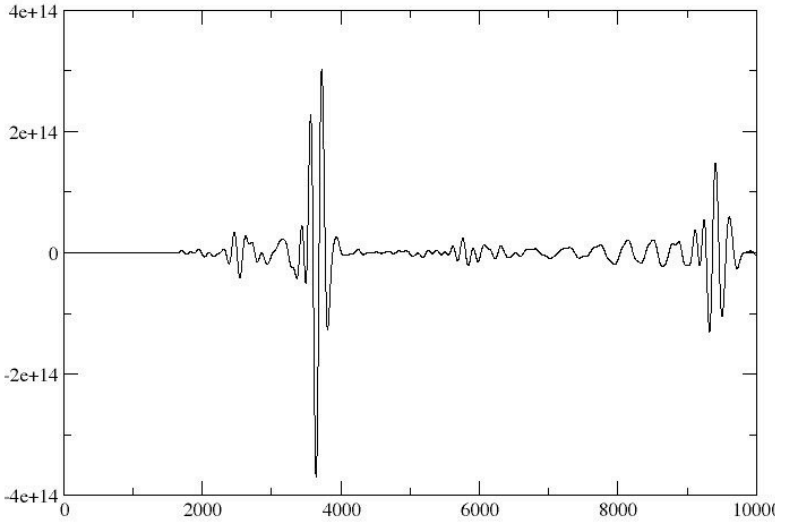


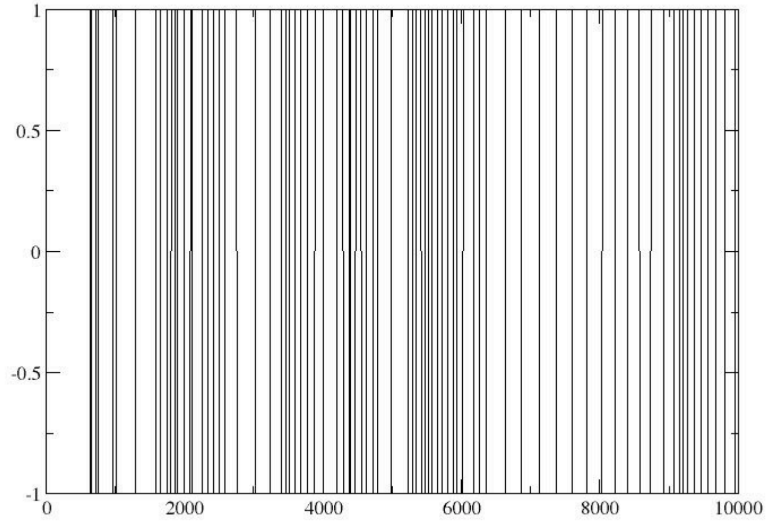
t = -500s t = -100s t = 0s t = 500s

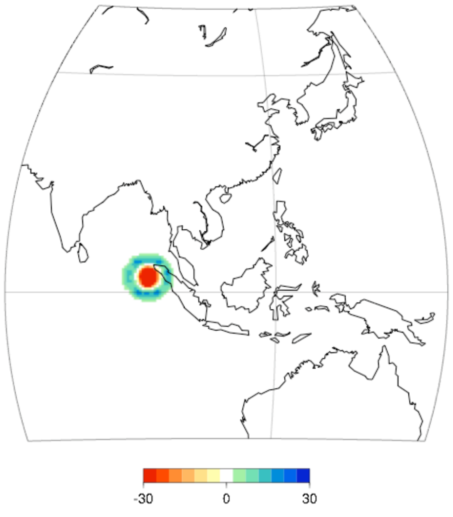


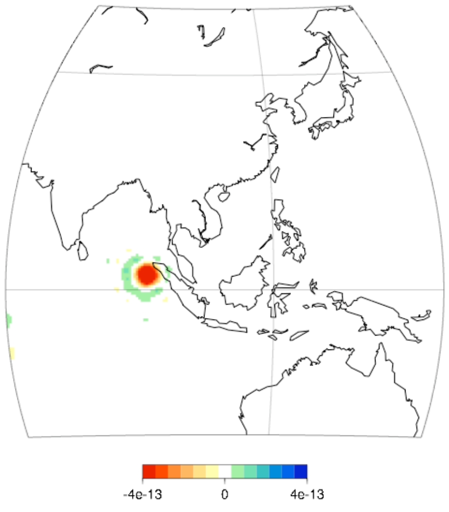
Complete seismogram

normalized

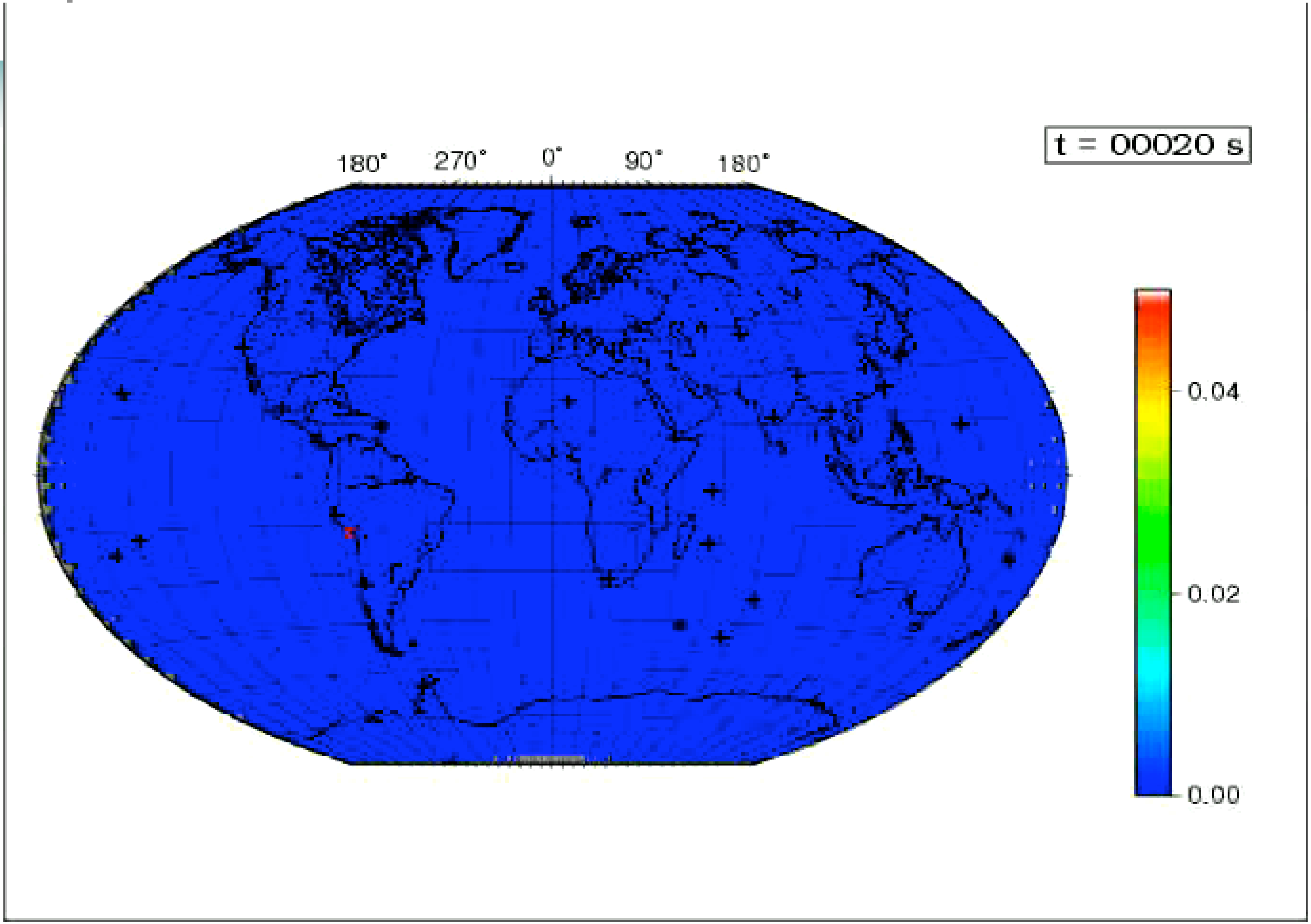






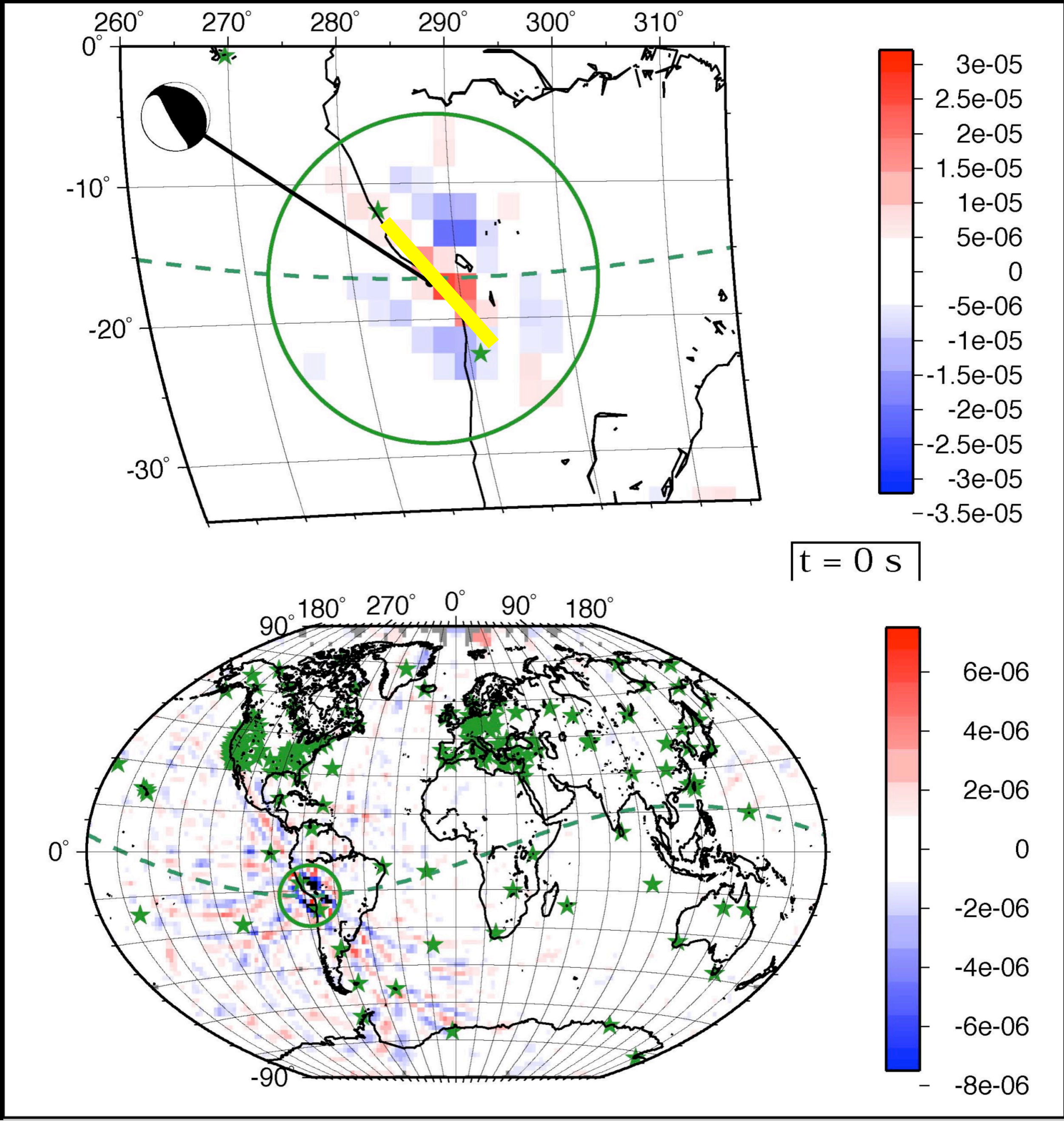


DATA: Peru Earthquake (23-06-2001) Mw= 8.4

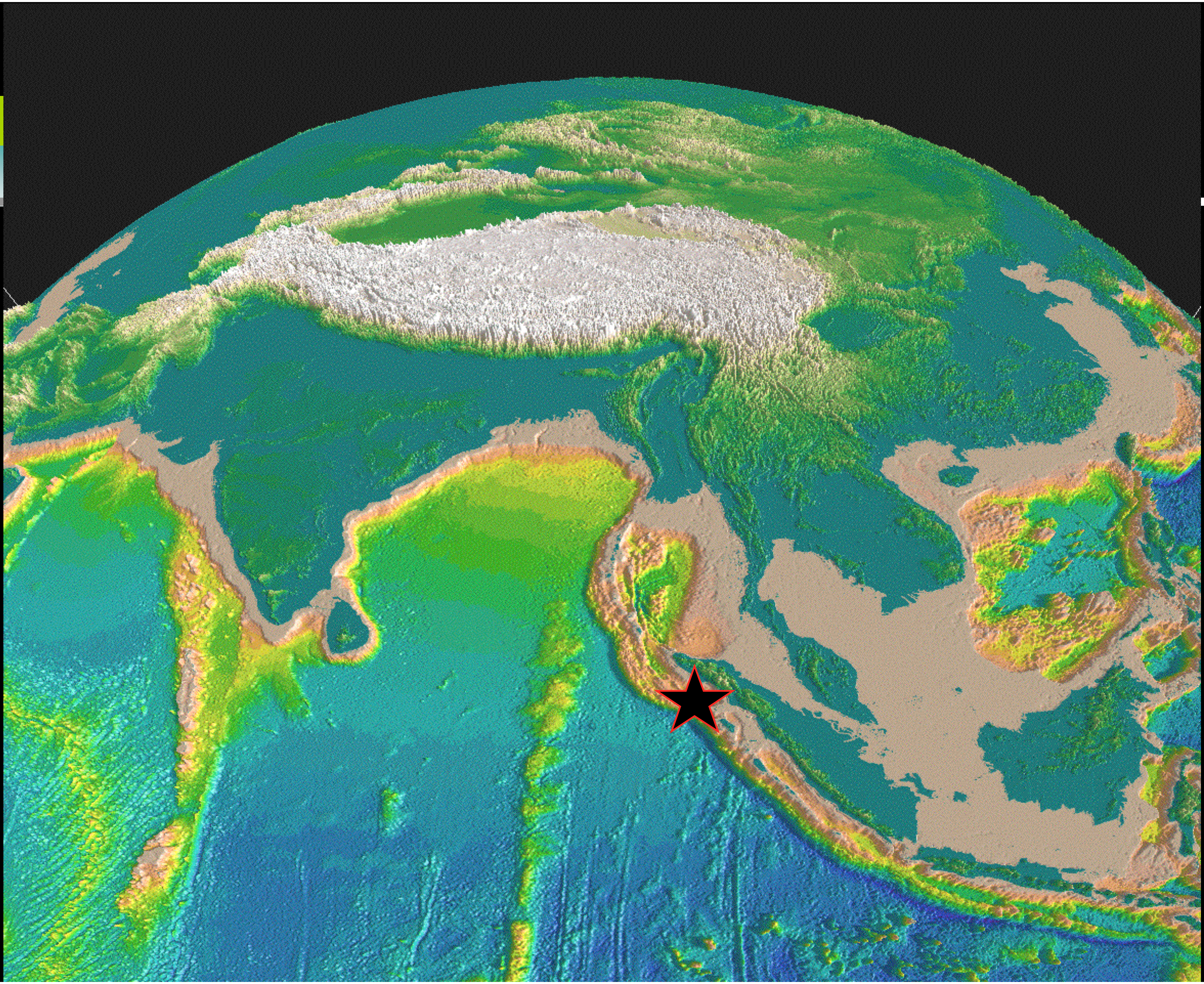


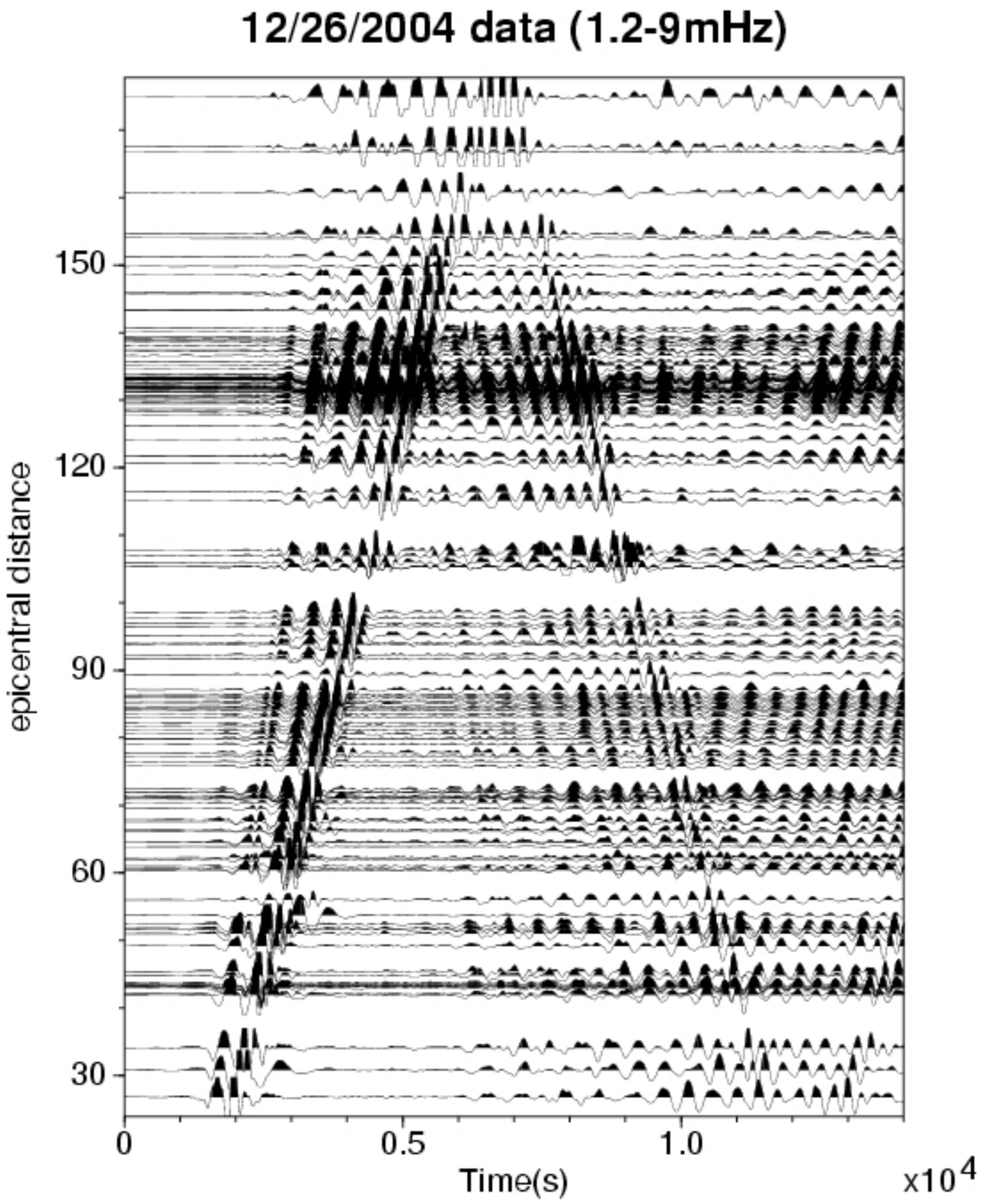
PERU 23 June 2001 - 8.4

Fault Plane



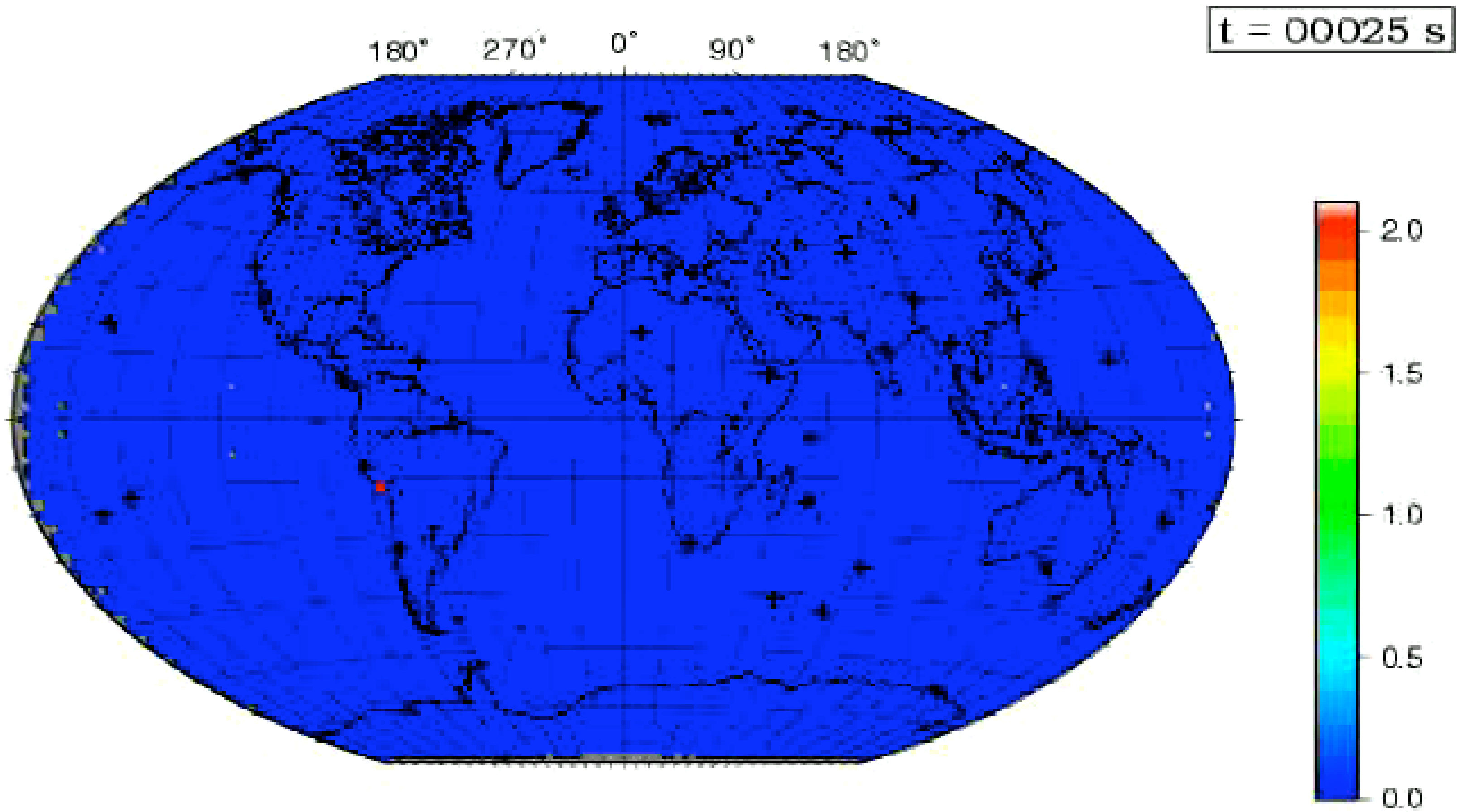
C. Larmat





The 121 real records we work with in this experiment (#11).

Sumatra Earthquake 26/12/04 -NM-SEM



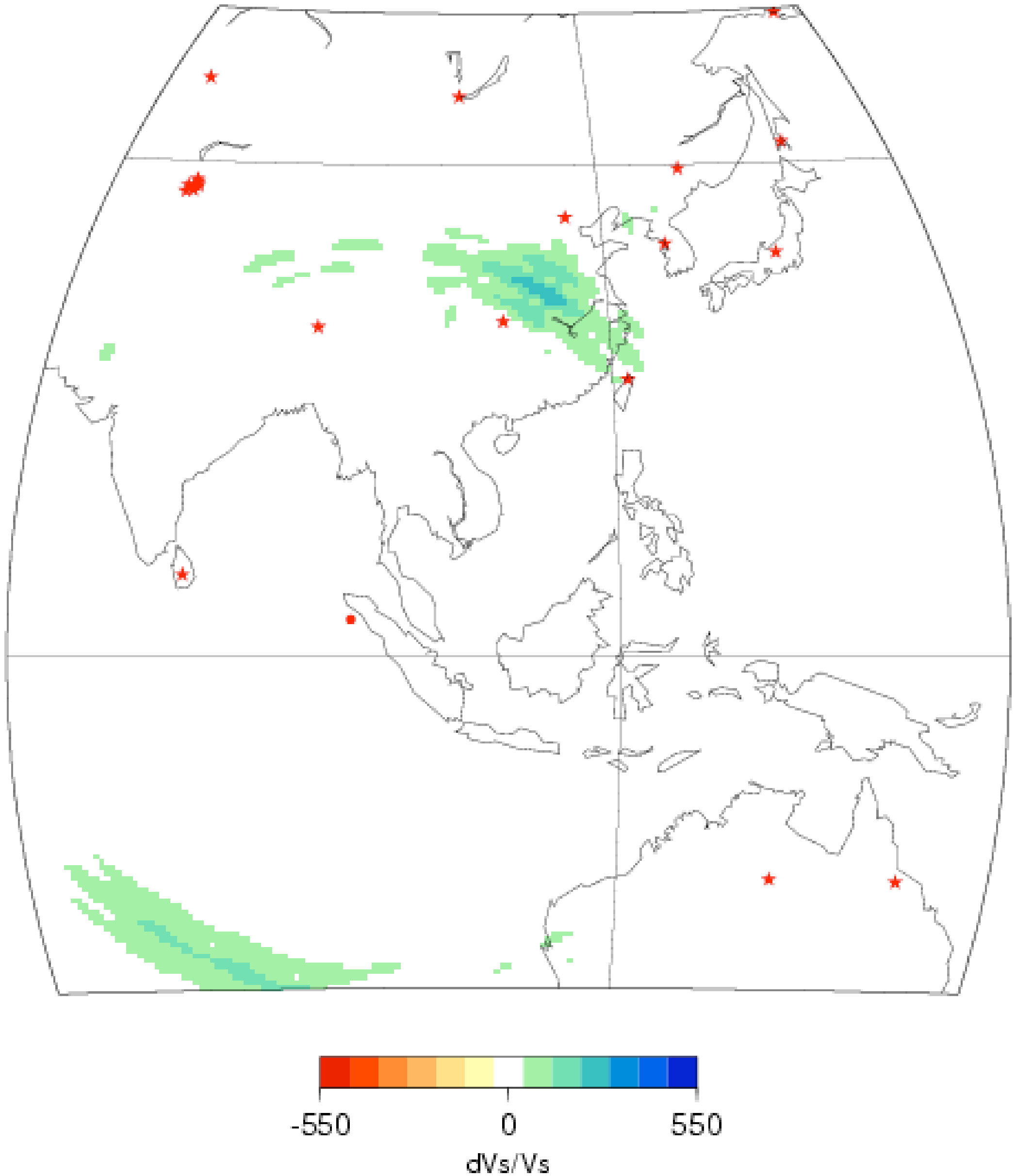
Sumatra

1D earth model

Synthetic Test
Normal modes:

Point Source

Y. Capdeville

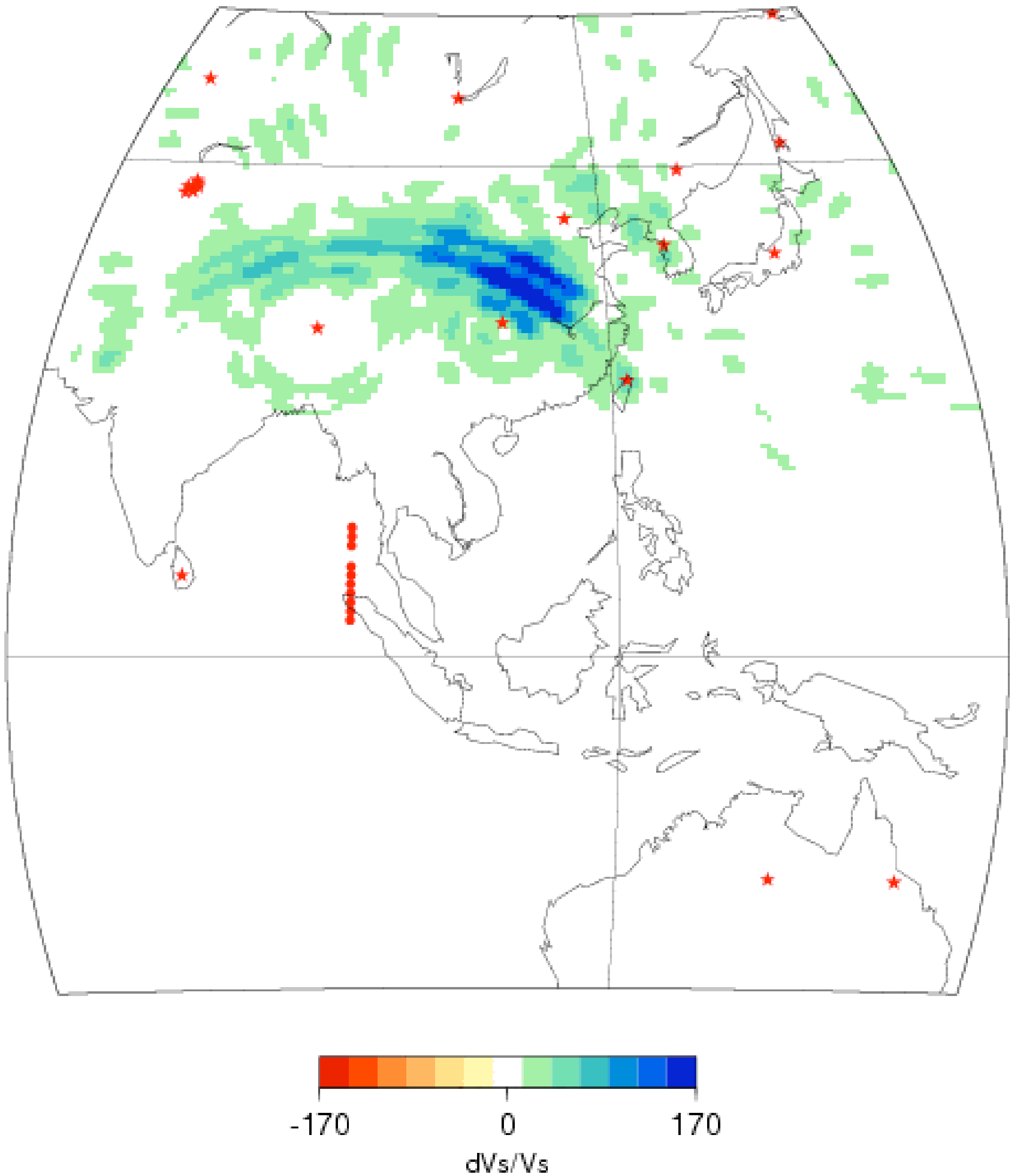


Sumatra

Synthetic test

Normal modes:

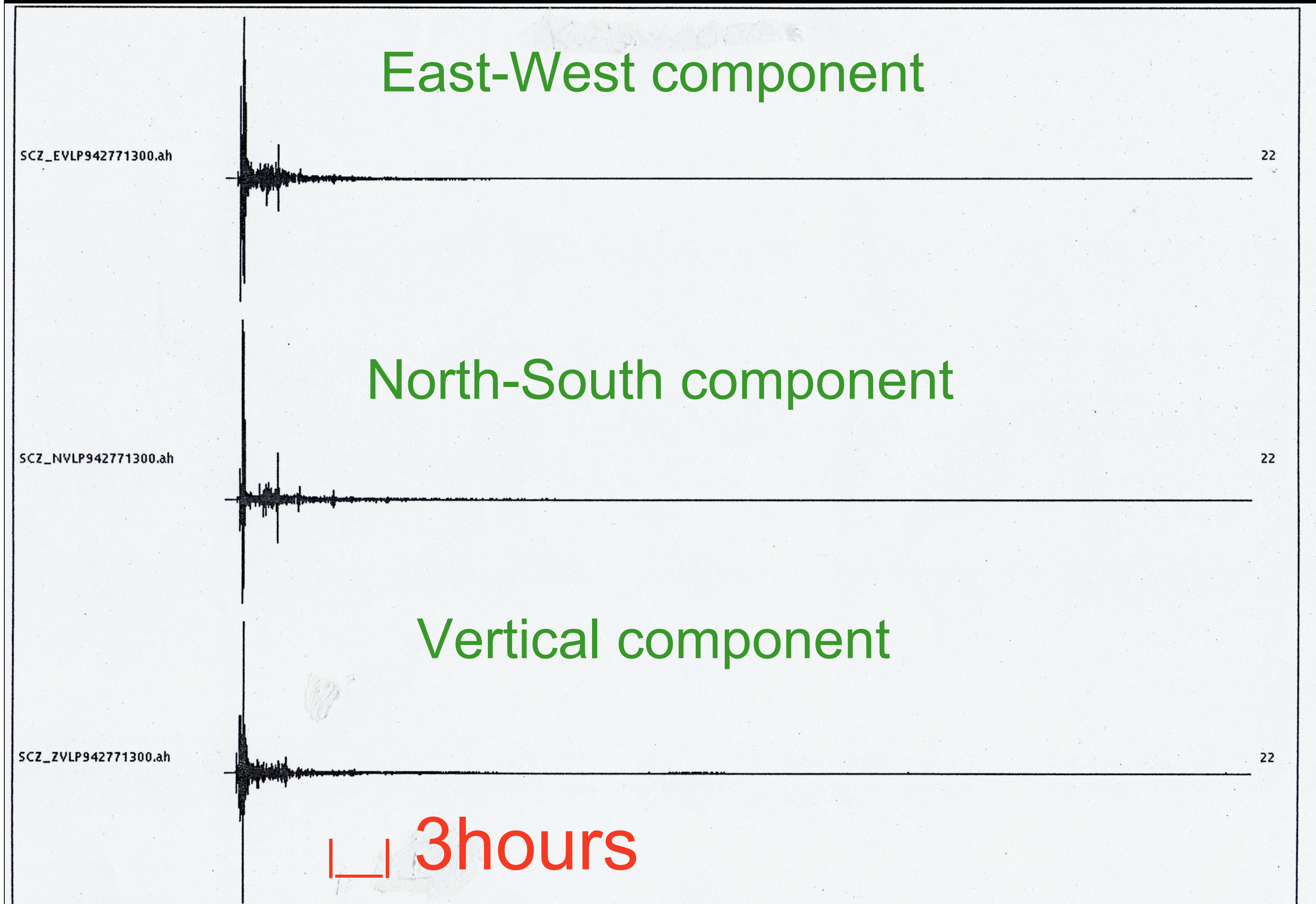
Extended source



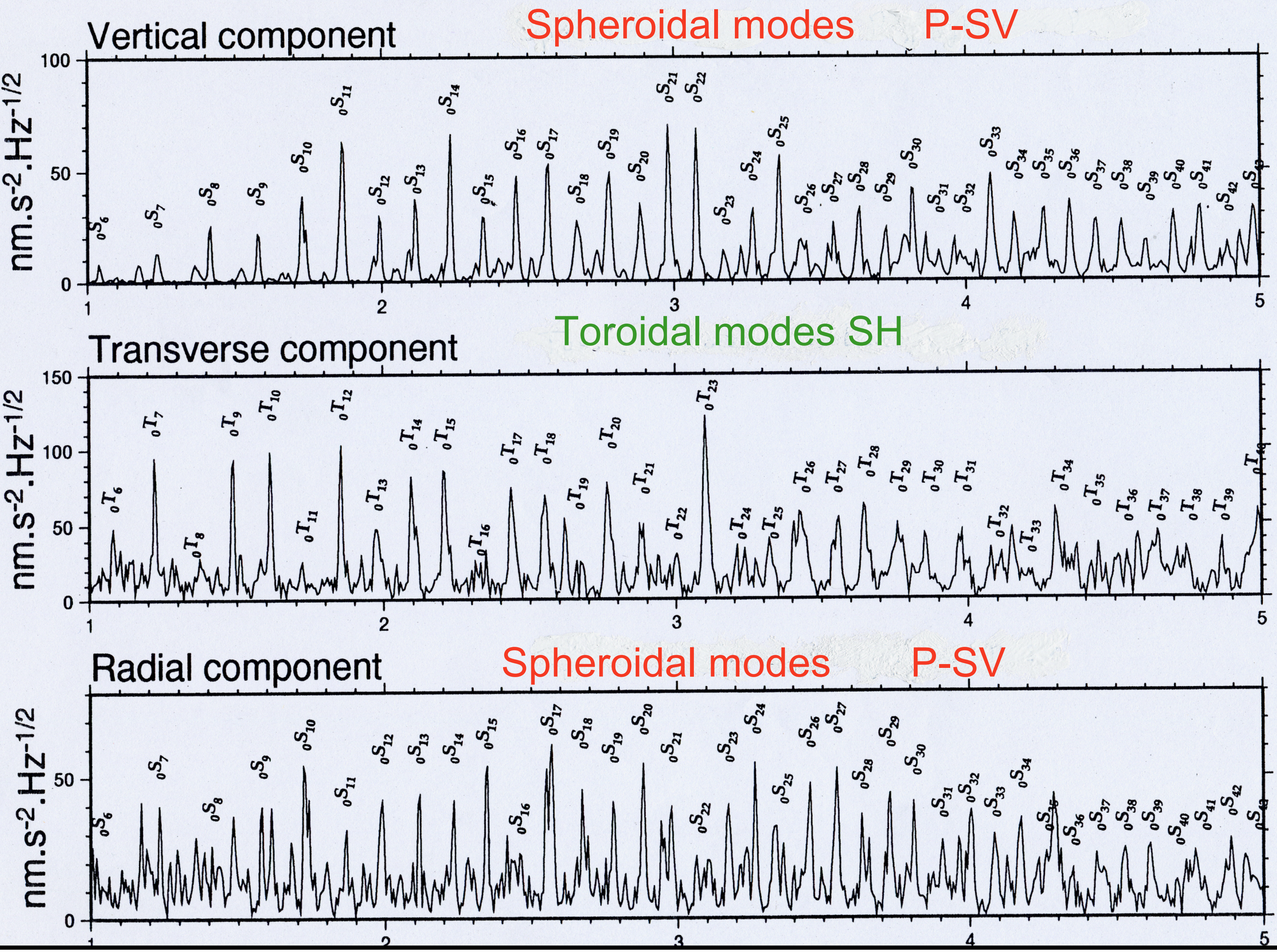
WHY does Time Reversal work when applied to seismic waves ?

Normal Mode Approach of Draeger & Fink (1999)
Cavity, Scalar case

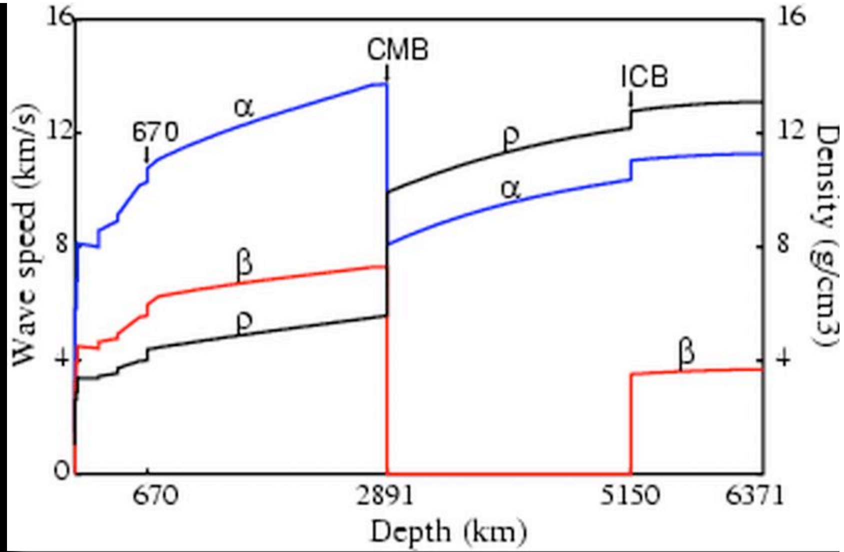
Kurils islands 1994-277 Ms=8.3



KURIL 94 277 - SCZ VLP - 36h.



1D-Reference Earth Model:
$M_0(r)$, $\rho(r)$, $V_P(r)$, $V_s(r)$
(PREM, Dziewonski and Anderson, 1981)



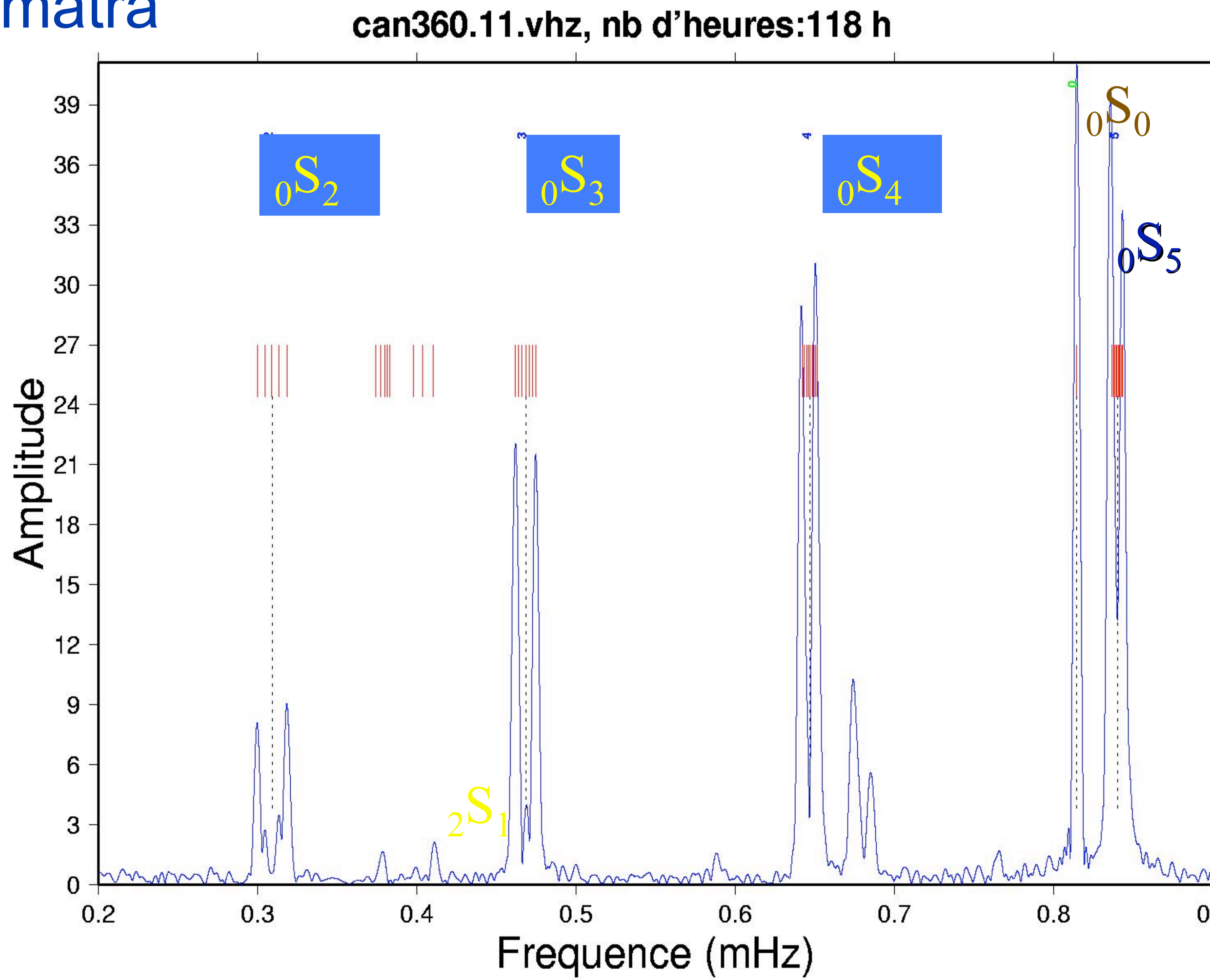
$\rho\partial_{tt}u_0 + \mathbf{H_0 u_0 = 0}$

Eigenfrequencies: ${}_n\omega_l$
Eigenfunctions: ${}_nu_l^m\ (r,t) = |n,l,m>$ (complete basis)

2 kinds of modes: Toroïdal ${}_nT_l$, Spheroïdal ${}_nS_l$

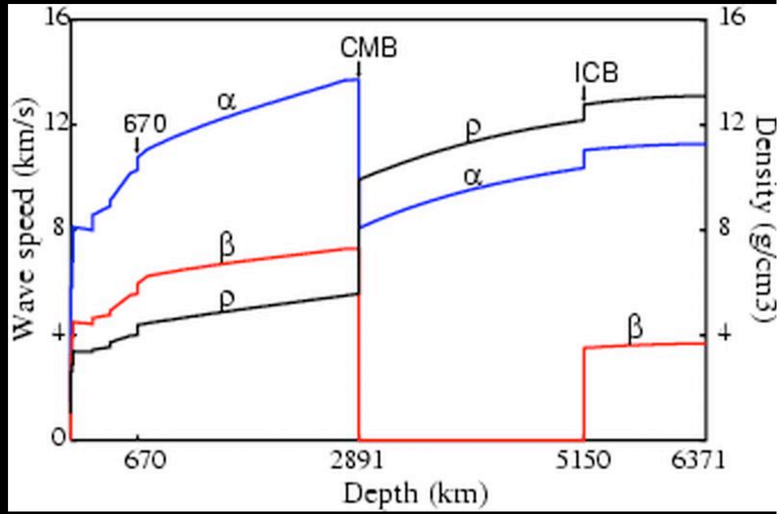
Degeneracy of eigenfrequencies ${}_n\omega_l$: 2 l +1
for radially symmetric models

Sumatra



Roult et al, 2005

1D- Reference Earth Model



Seismic Source

$\rho\partial_{tt}\mathbf{u} + \mathbf{H_0u} = \mathbf{F_s}$

- Synthetic Seismograms by normal mode summation (k={n,l,m}).

Displacement at point **r** at time t due to a force system **F** at point source $\mathbf{r_s}$

$\mathbf{u}(\mathbf{r},t)= \Sigma_k -(\mathbf{u_k.F})_\mathbf{S}\, u_k(r)\cos \omega_k t /\omega_k^2 \exp(-\omega_k t/2Q_k)$

Source Term $(\mathbf{u_k}\, \mathbf{.F})_\mathbf{S} = (\mathbf{M}:\varepsilon)_S$

M Seismic moment tensor, ε deformation tensor

Why does time reversal works when applied to seismic waves?

$\mathbf{r_R}$ receiver, $\mathbf{r_S}$ source location

$$\mathbf{u}(\mathbf{r_R},t) = \Sigma_k -(\mathbf{F}.\mathbf{u_k})_\mathbf{S} \cos \omega_k t / \omega_k^2 \, u_k(\mathbf{r_R})$$

$$\mathbf{u}(\mathbf{r_R},t) = \mathbf{F_S}(t) * G_{SR}(t)$$

Time reversed seismogram in r_R : $\mathbf{F_S}(-t) * G_{SR}(-t)$

in r_M: $\mathbf{v}(\mathbf{r_M},t) = \mathbf{F_S}(-t) * G_{SR}(-t) * G_{RM}(t)$

Why does time reversal works when applied to seismic waves?

for a point source,

If M in S, autocorrelation:

$$\mathbf{v}(\mathbf{r_S},t) = G_{SR}(-t) * G_{RS}(t) = \int G_{SR}(t+\tau)G_{RS}(\tau)d\tau$$

If M not in S, cross-correlation:

$$\mathbf{v}(\mathbf{r_M},t) = G_{SR}(-t) * G_{RM}(t) = \int G_{SR}(t+\tau)G_{RM}(\tau)d\tau$$

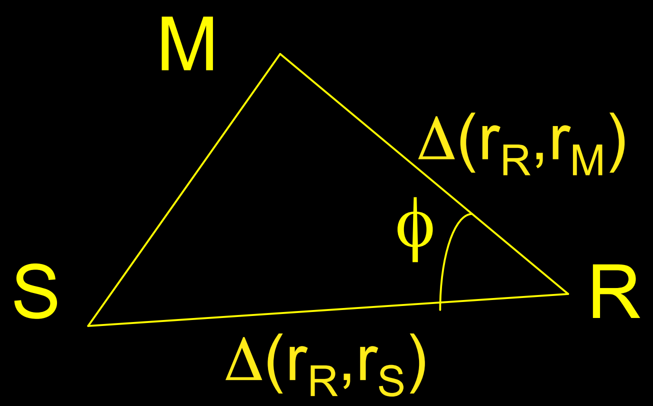
Why does time reversal works when applied to seismic waves?

$$\mathbf{v}(\mathbf{r_M},t) = \Sigma_k \Sigma_{k'} \mathbf{u_{k'}}(\mathbf{r_R}) u_k(\mathbf{r_S}) F_S u_k(\mathbf{r_R}) \mathbf{u_{k'}} (\mathbf{r_M}) \int g(t,\tau)d\tau$$

k multiplet: {n,l,m} $\quad u_k(\mathbf{r_R}) = {}_nD_l(r_R)\, Y_l^m(\theta,\phi)$

Addition theorem: $\Sigma_k Y_l^m(\theta_1,\phi_1) Y_l^m(\theta_2,\phi_2) = P_l^0(\cos \Delta(r_1,r_2))$

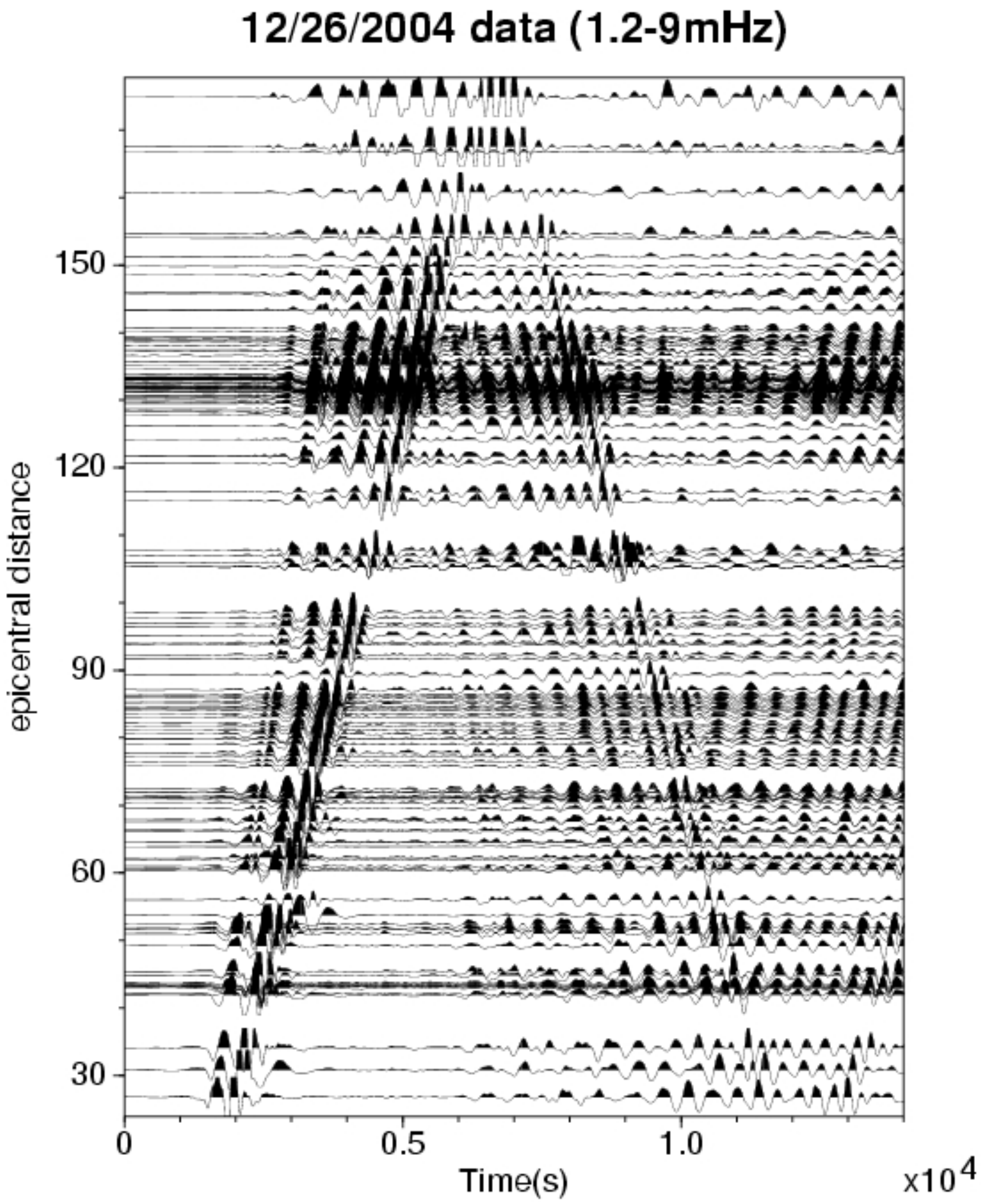
$$\mathbf{v}(\mathbf{r_M},t)=\Sigma_{n,l} \Sigma_{n',l'} {}_{n'}D_{l'} P_{l'}^0(\cos\Delta(r_R,r_M)) F_S\, {}_nD_l P_l^0(\cos\Delta(r_R,r_S)) \int g(t,\tau)d\tau$$



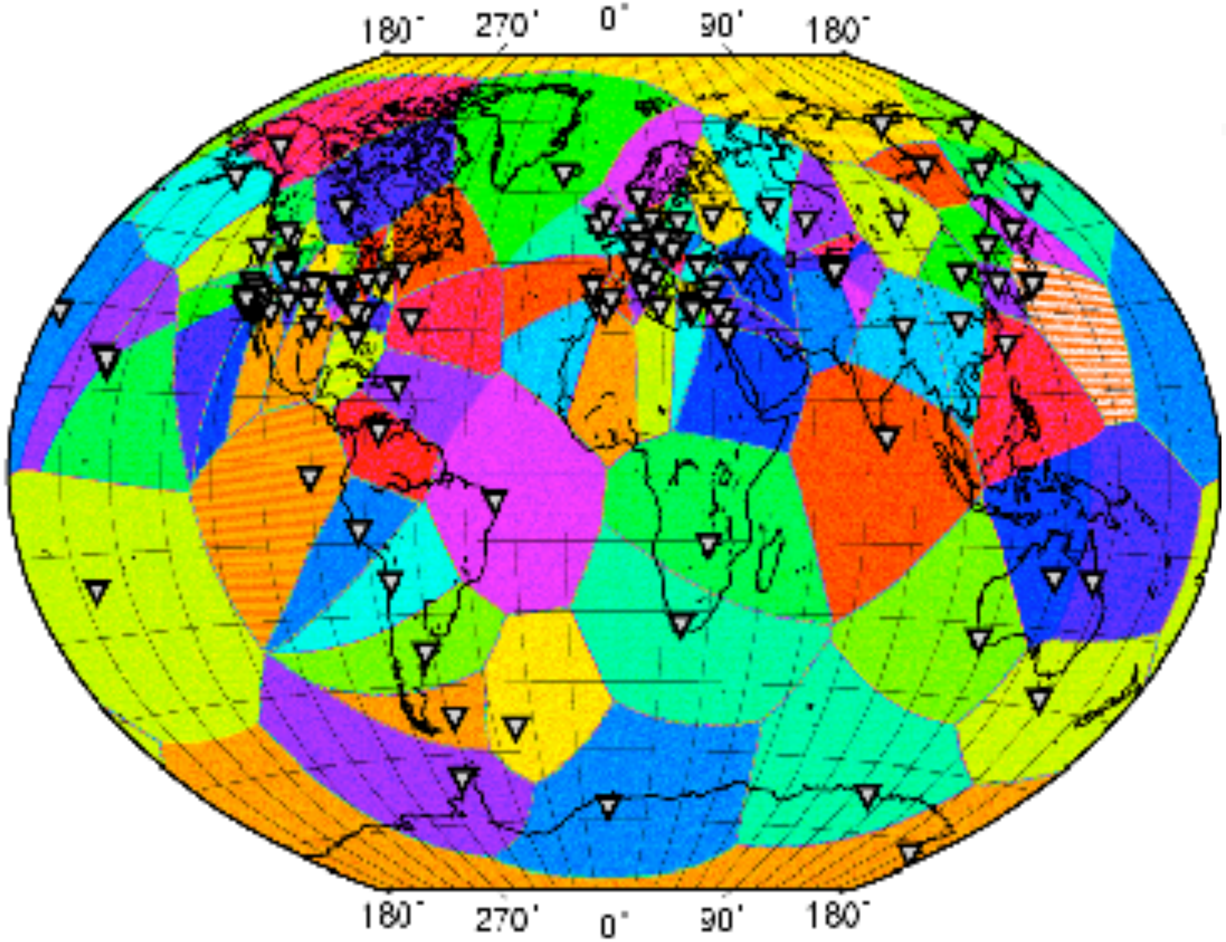
=>Max if $\phi = 0$ or π

(Romanowicz, Snieder, …)

Sumatra-Andaman Earthquake



The 121 real records we work with in this experiment (#11).

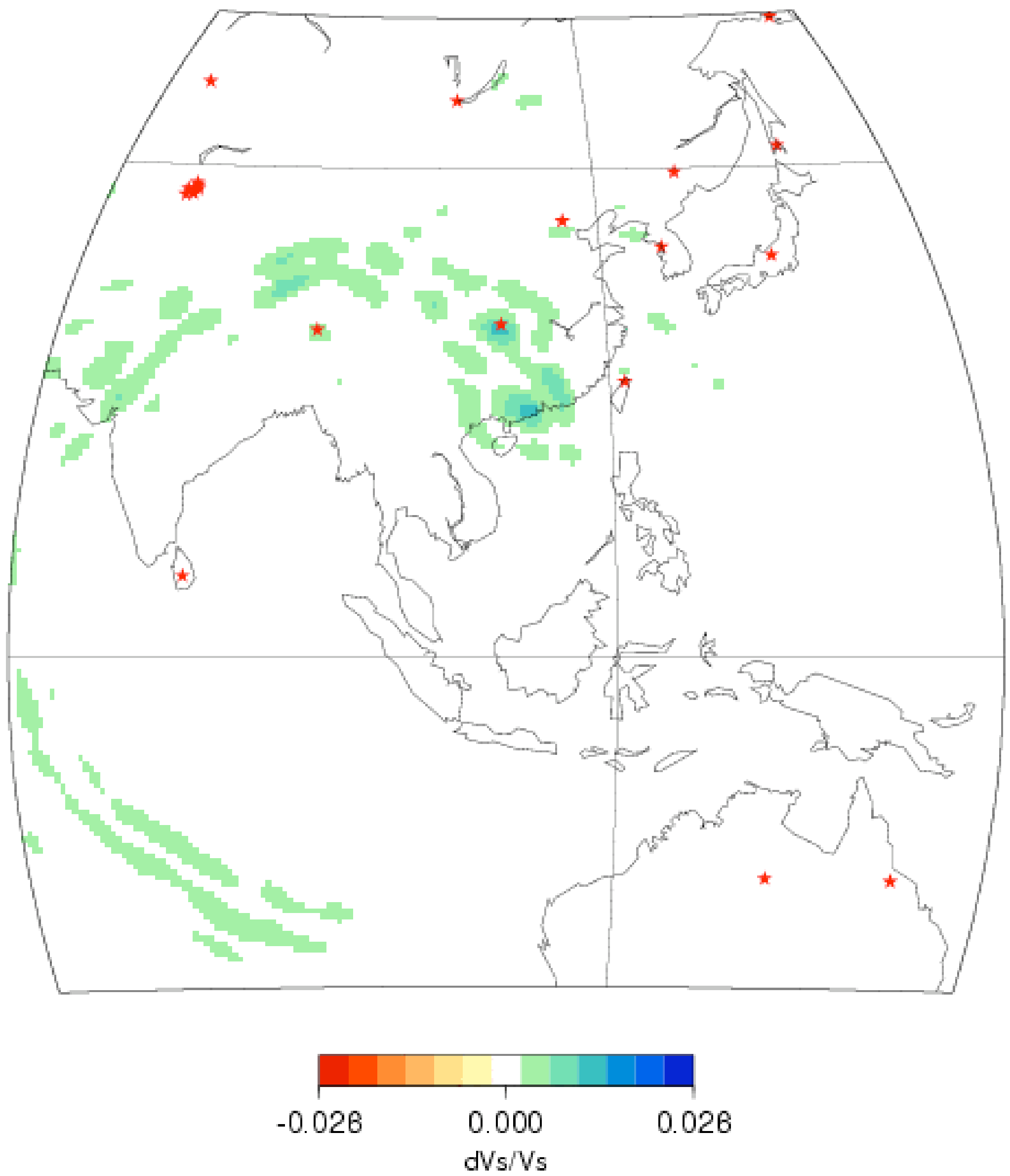


b) Stations network in a seismic TR experiment

Sumatra

Normal mode
Time reversal

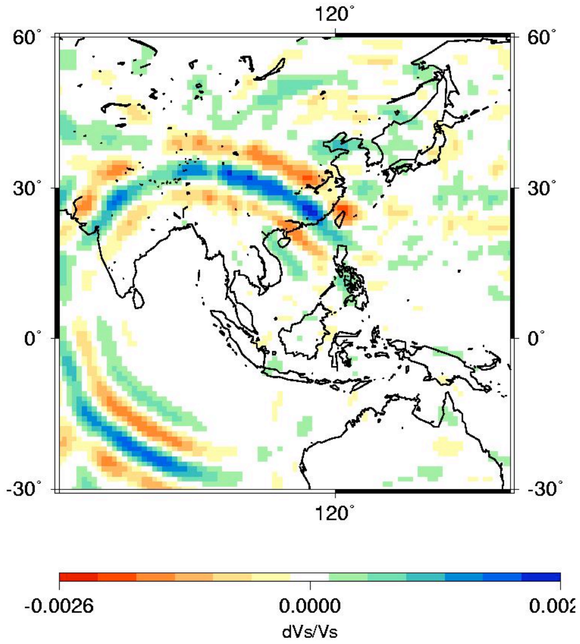
Real Data



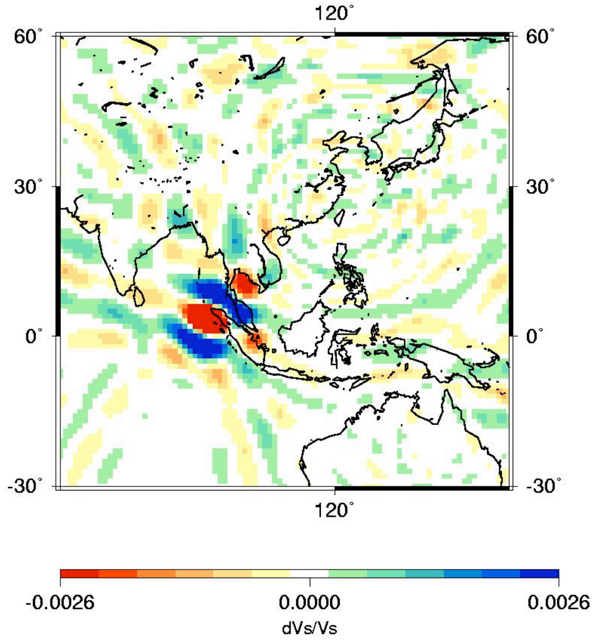
First conclusions

-Time reversal focus at the right time
($t_0 \approx$ -7000s)
- and at the right place

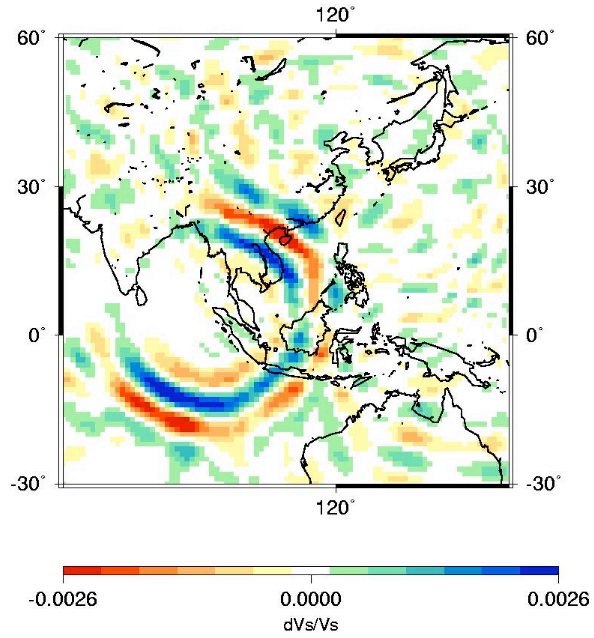
t= -6000s



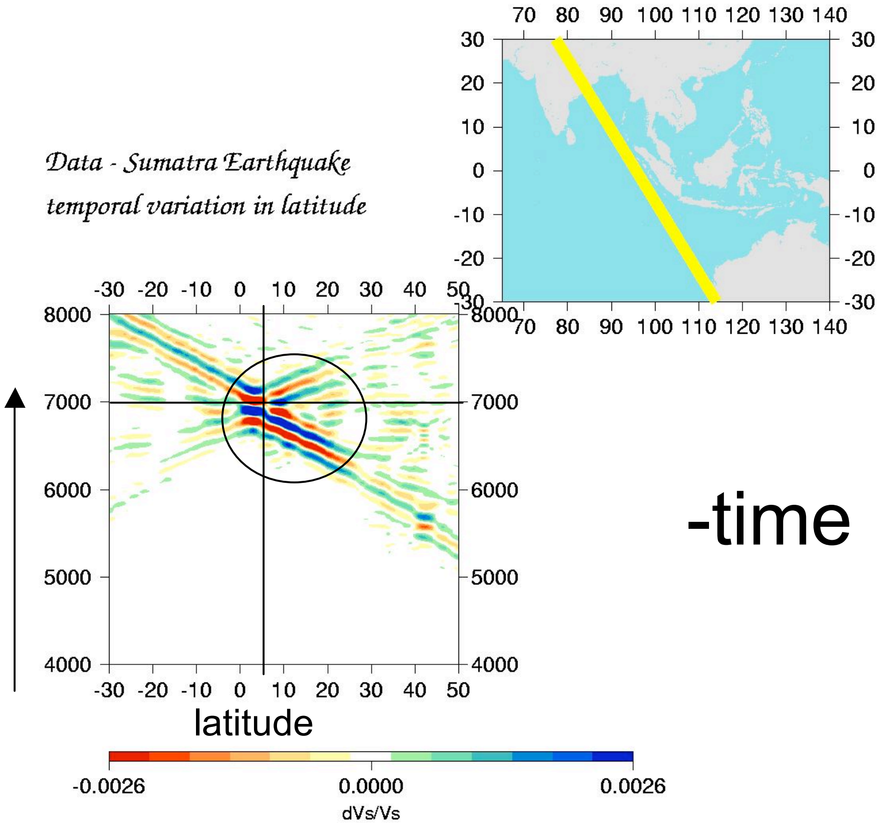
t= -7000s



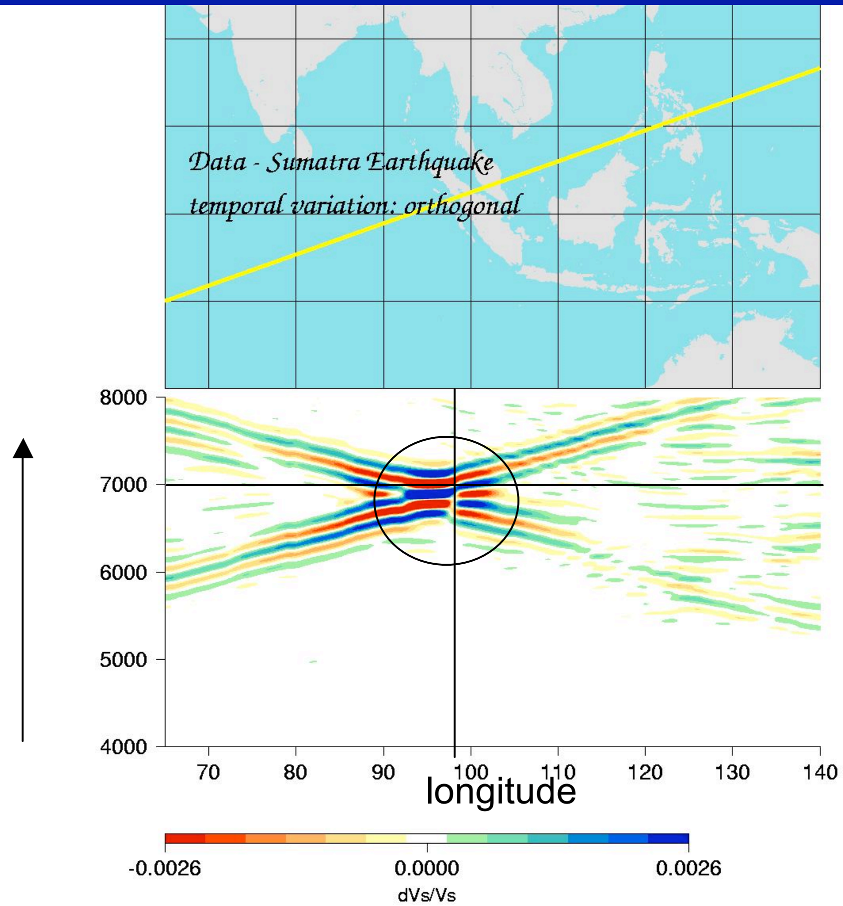
t= -7500s



Can we get information about the history Of the seismic rupture?

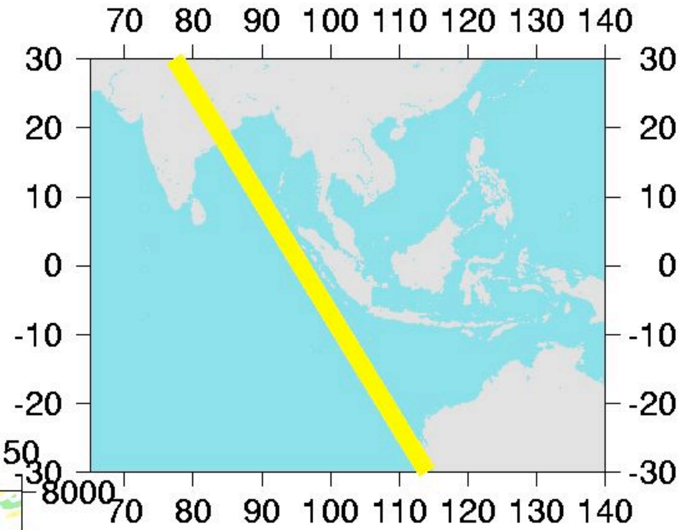


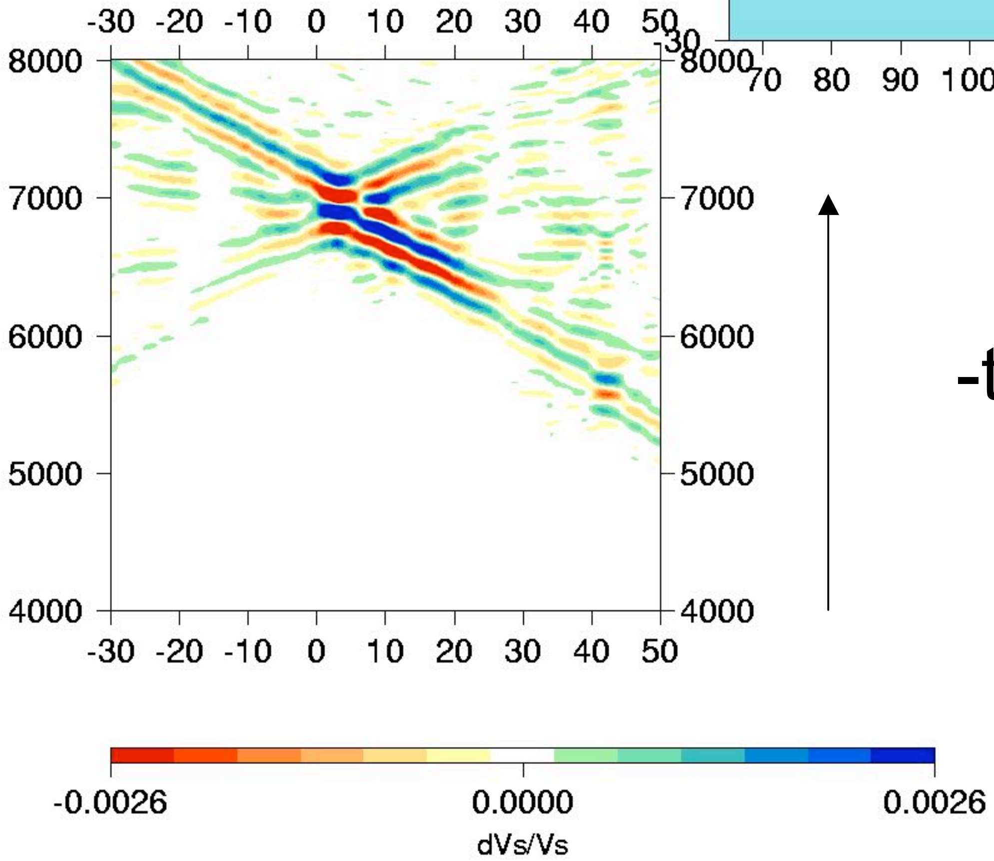
-time



Data - Sumatra Earthquake

temporal variation in latitude





-time

Source Rupture Imaging

$$\mathbf{u_Z}(\mathbf{r},t) = \Sigma_k -u_k (\mathbf{r}) \cos \omega_k t / \omega_k^2 \exp(-\omega_k t/2Q_k) (\mathbf{u_k.F})_\mathbf{S}$$

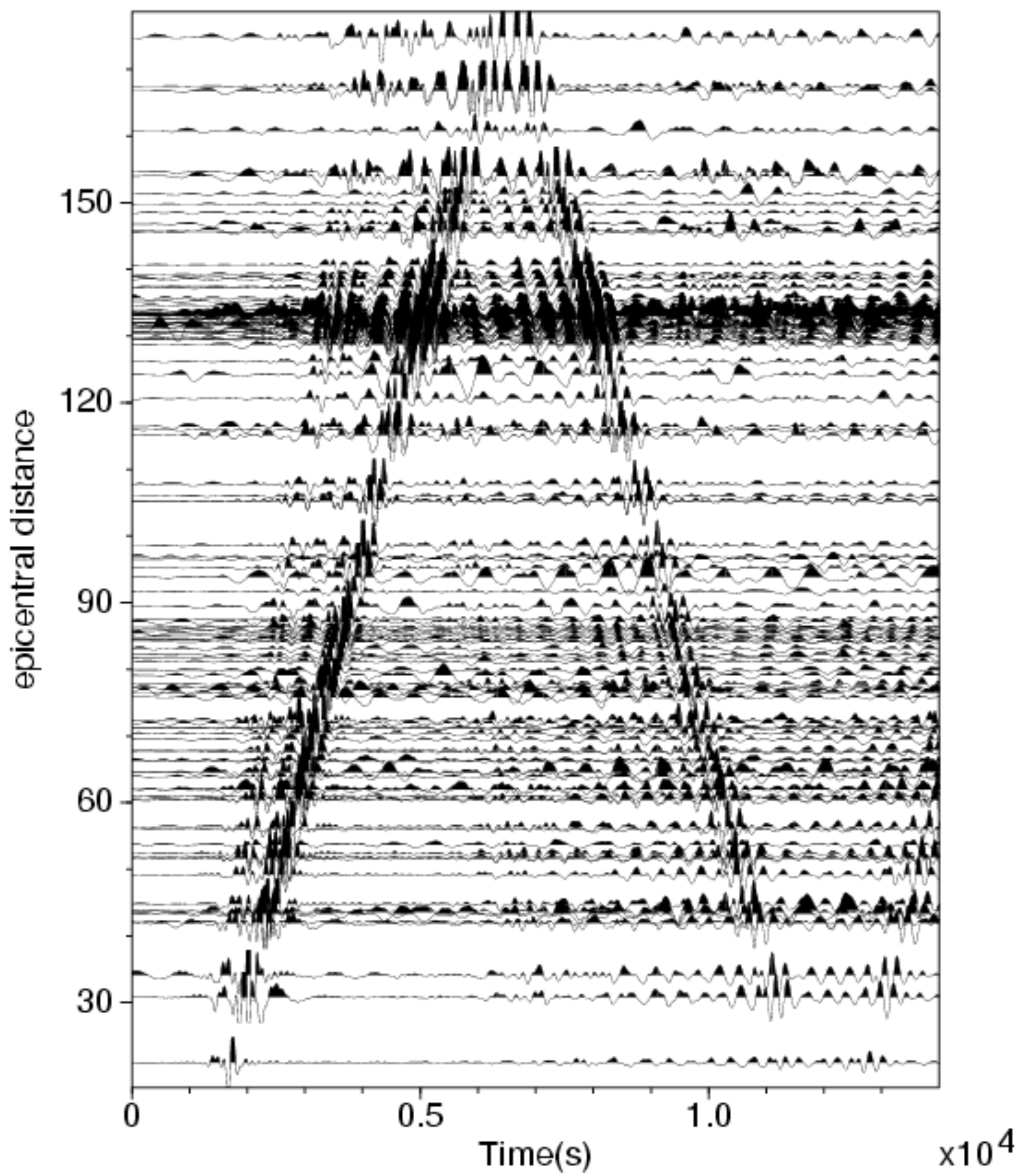
$$\mathbf{u_Z}(\mathbf{r}, \omega) = G (\mathbf{r},\mathbf{r_S}, \omega) S(\mathbf{r_S}, \omega)$$

$G (\mathbf{r},\mathbf{r_S}, \omega)$ Green Function

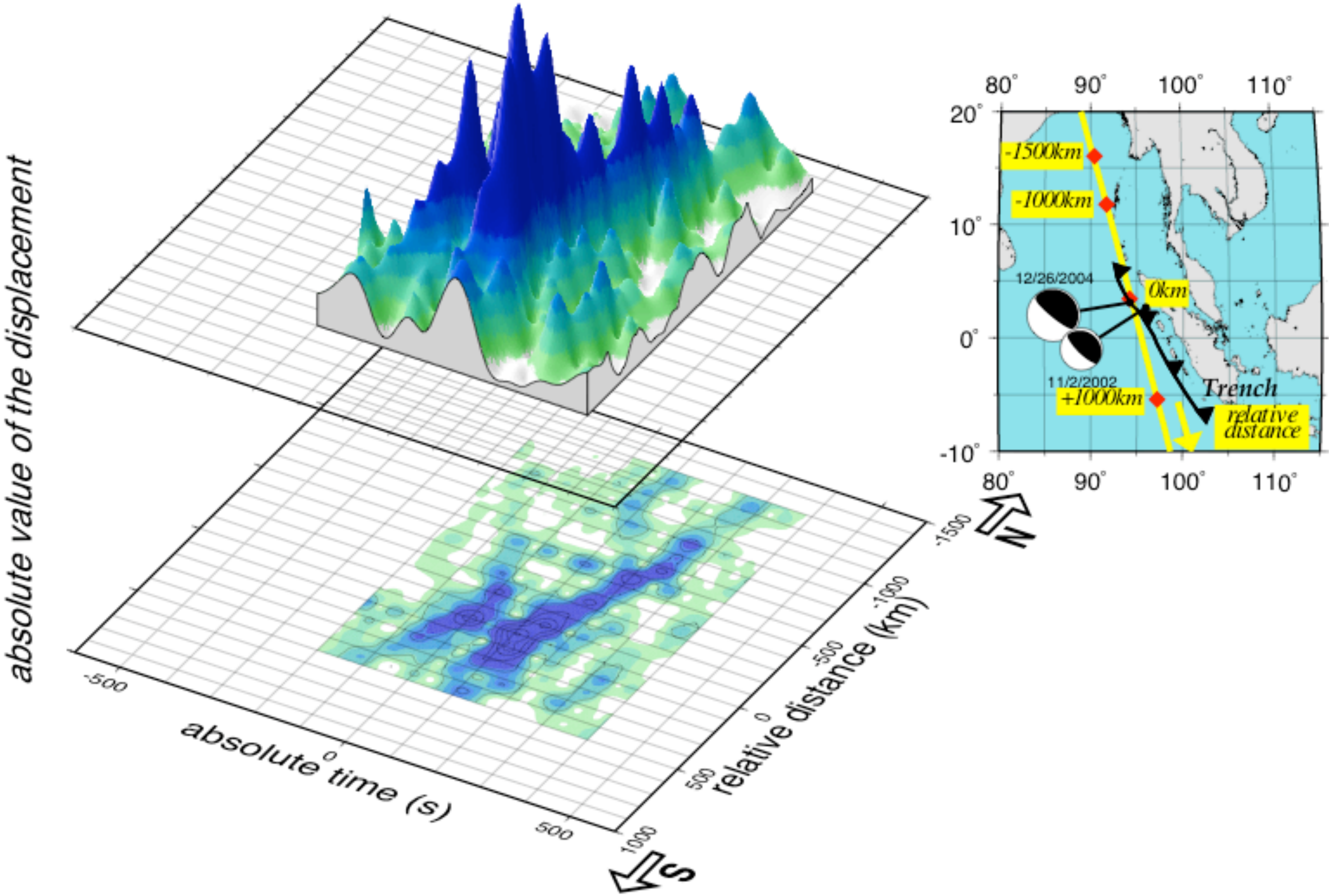
$S(\mathbf{r_S}, \omega)$ Source Function

=> Reference source: delta function?

11/02/2002 data (1.2-9mHz)

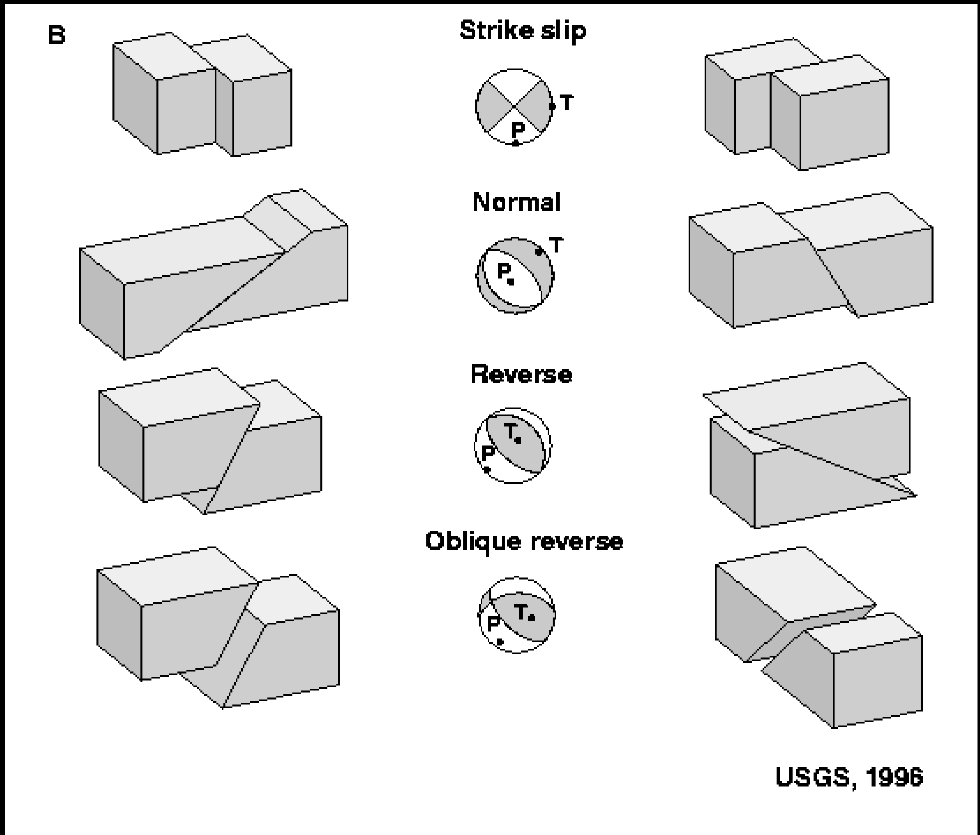


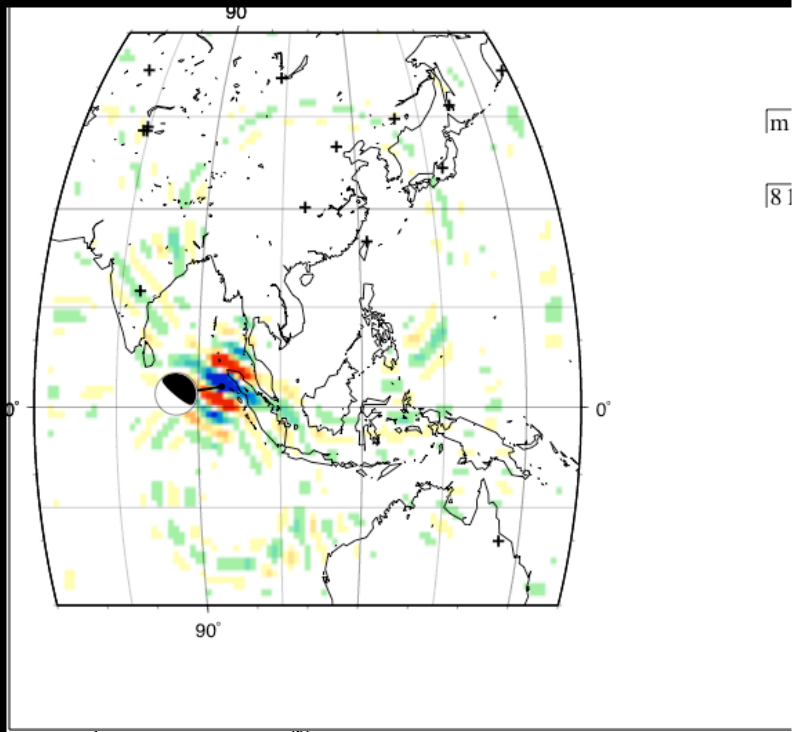
The 121 real records we work with in this experiment (#12).

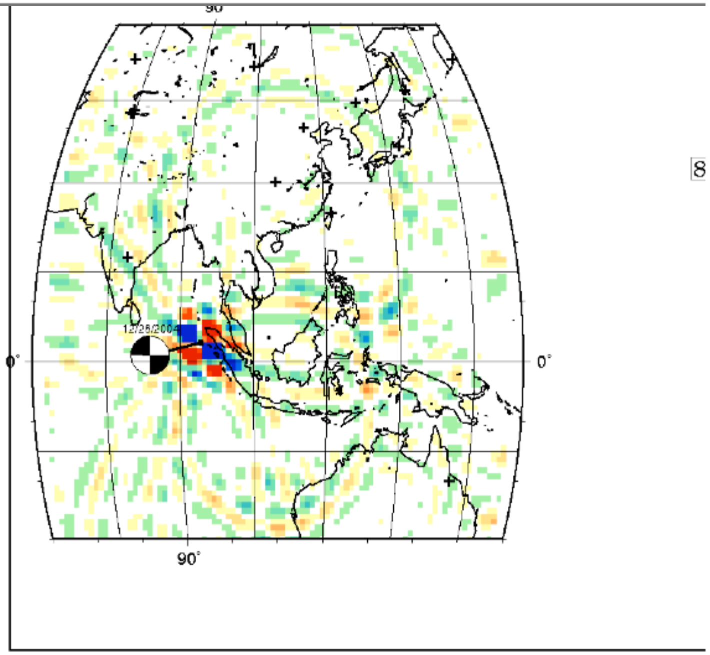


absolute value of the displacement
absolute time (s)
-500
0
500
relative distance (km)
-1500
-1000
-500
0
500
1000
N
S
80°
90°
100°
110°
20°
10°
0°
-10°
-1500km
-1000km
0km
+1000km
12/26/2004
11/2/2002
Trench
relative distance

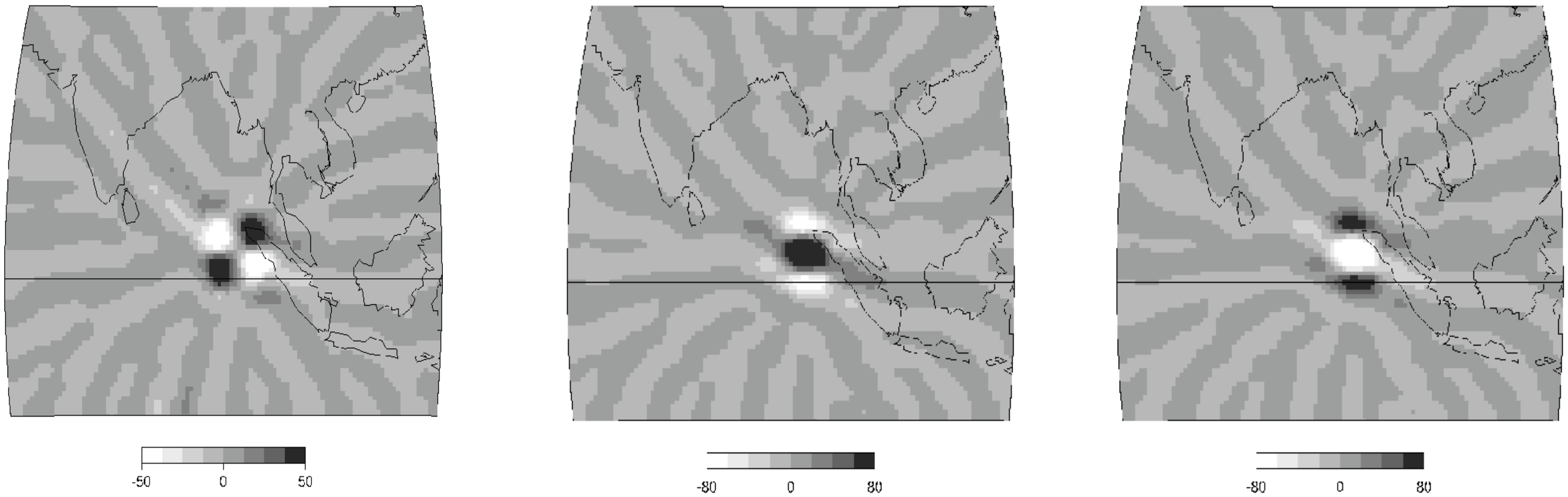
Different types of faults







Different Radiation Patterns



Mozambique 22/02/2006 at 22h 19min 15.7s (TU).

latitude : –21.32°

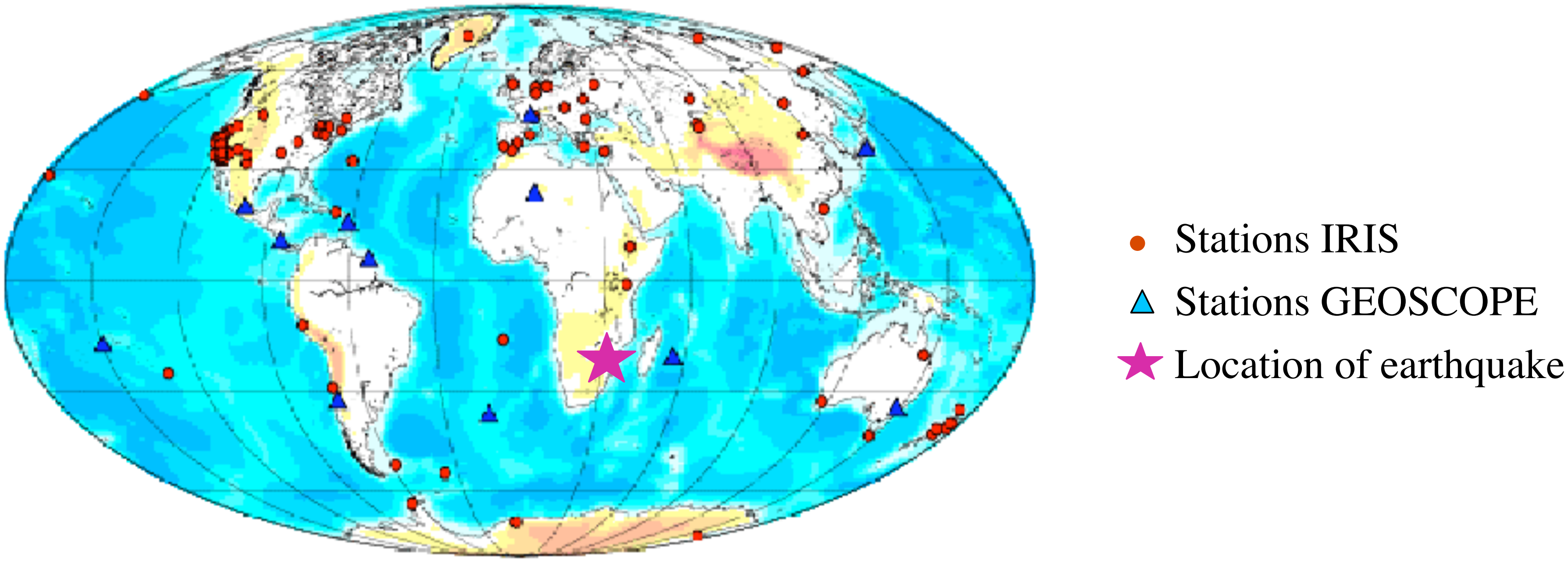
M_w = 7.5

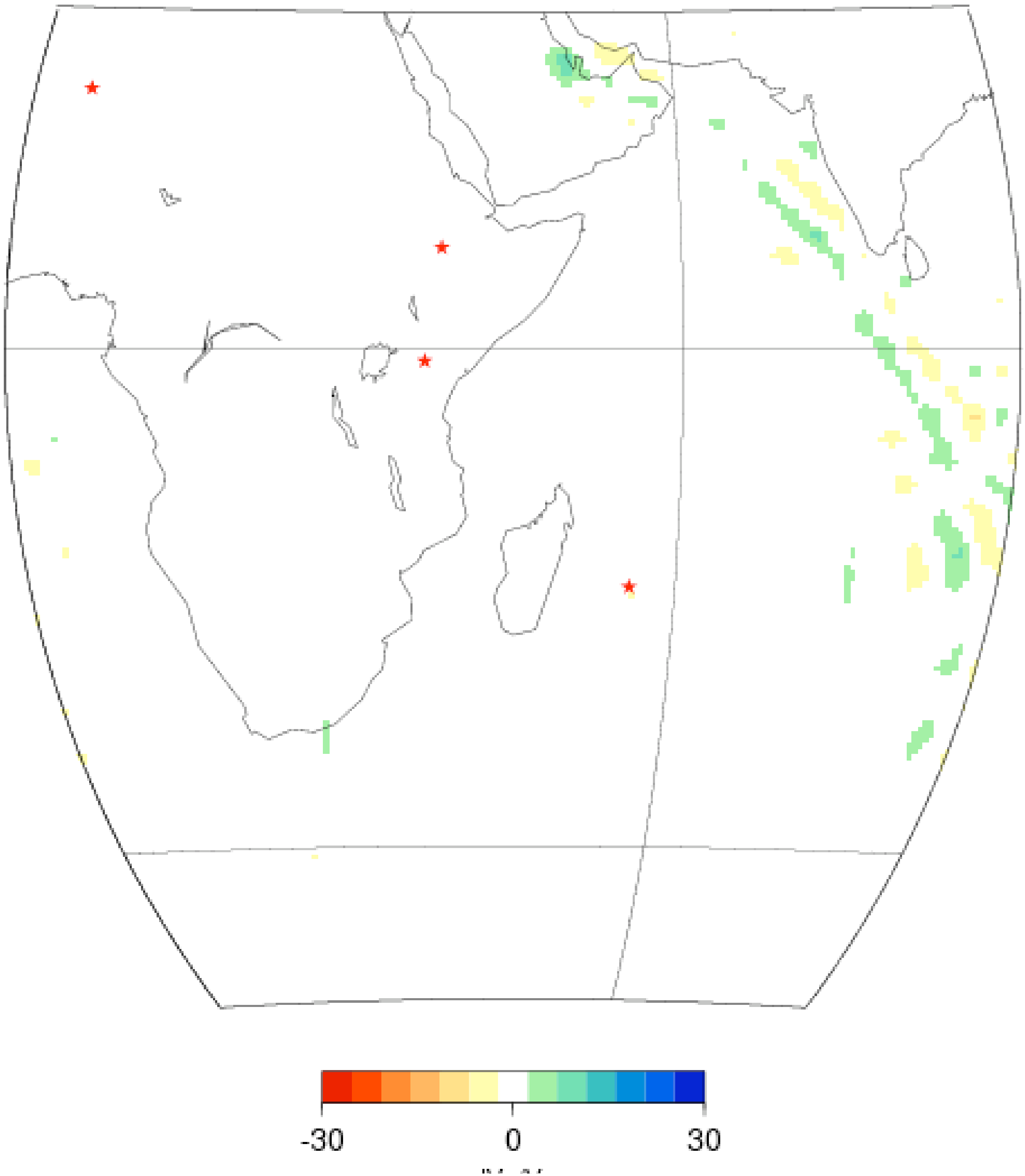
longitude : 33.37°

Normal fault N168°

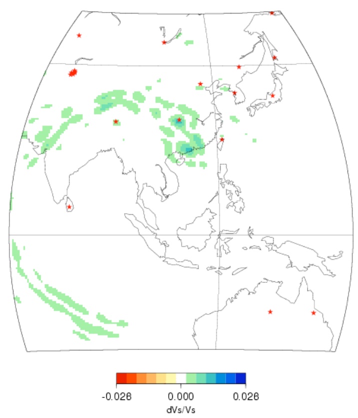
depth : 12 km

Data of 112 stations filtered between 2 et 9 mHz.




-30
0
30

TIME REVERSAL



- Application to real seismograms with broadband FDSN stations
- Automated localization in time and in spa of earthquakes
- Spatio-temporal Imaging of seismic source

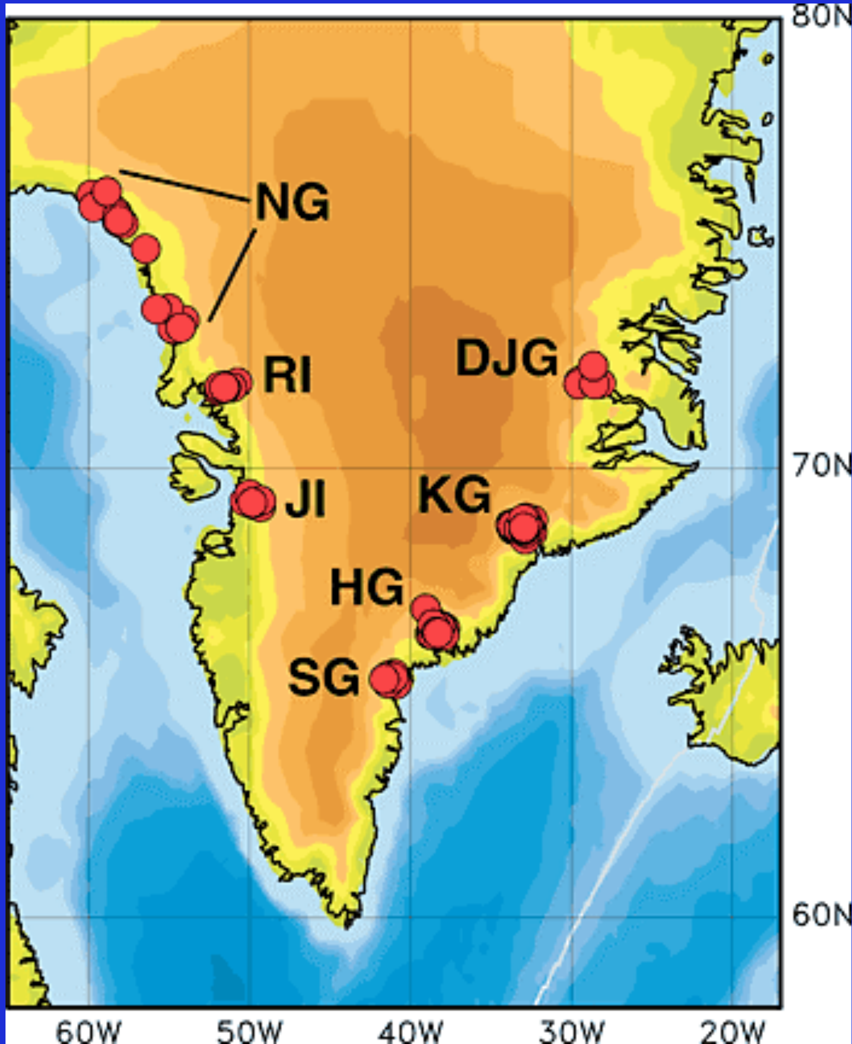
  http://www.gps.caltech.edu/~carene

  http://www.ipgp.jussieu.fr/~larmat
- Detection of unknown seismic sources

(Glacial, "quiet" earthquakes, Seismic "Hum" of the Earth)

- Applications to seismic Tomography- Detection of mantle plumes…

Glacial Earthquakes



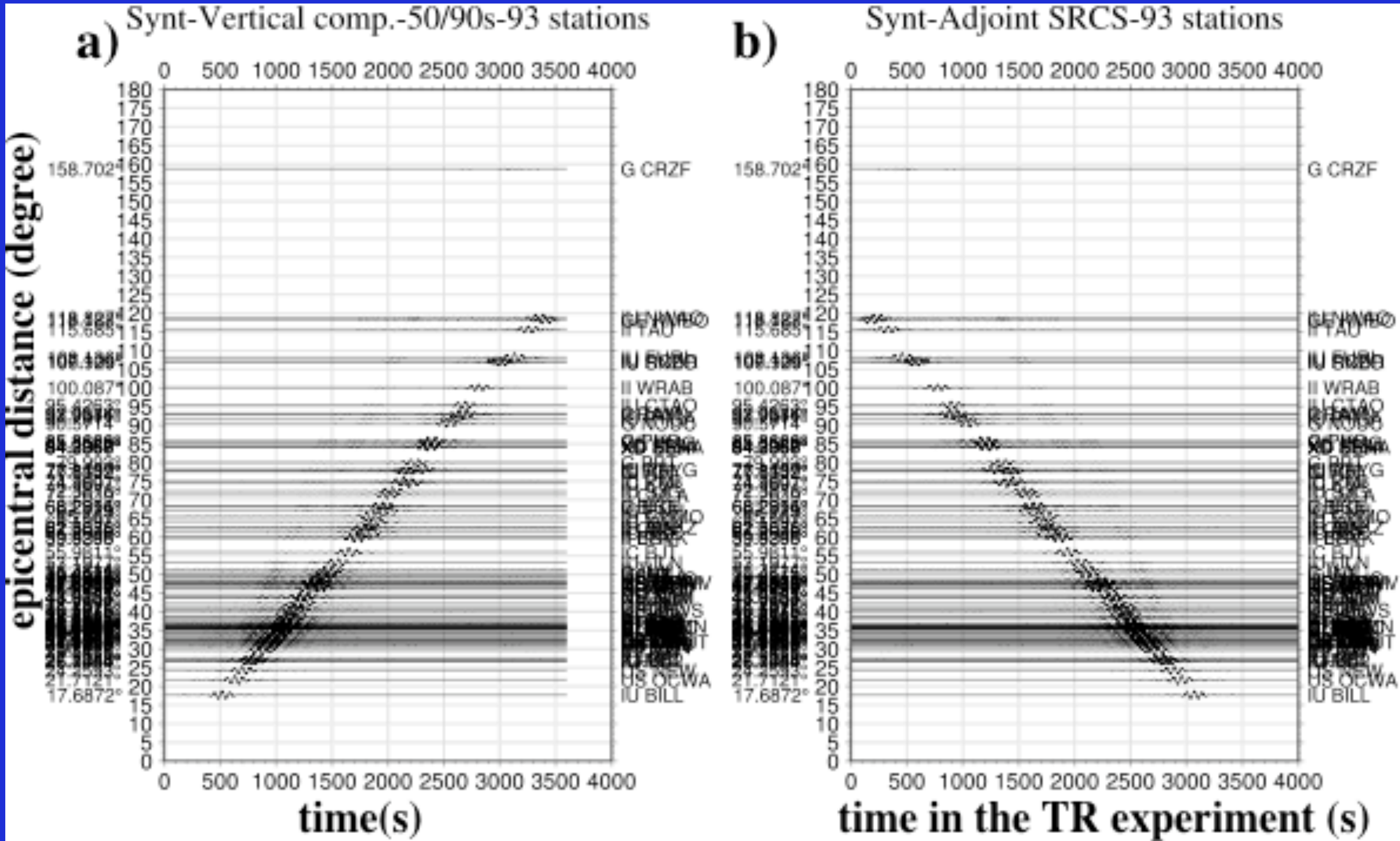
Ekstrom et al., 2003

Glacial Earthquakes

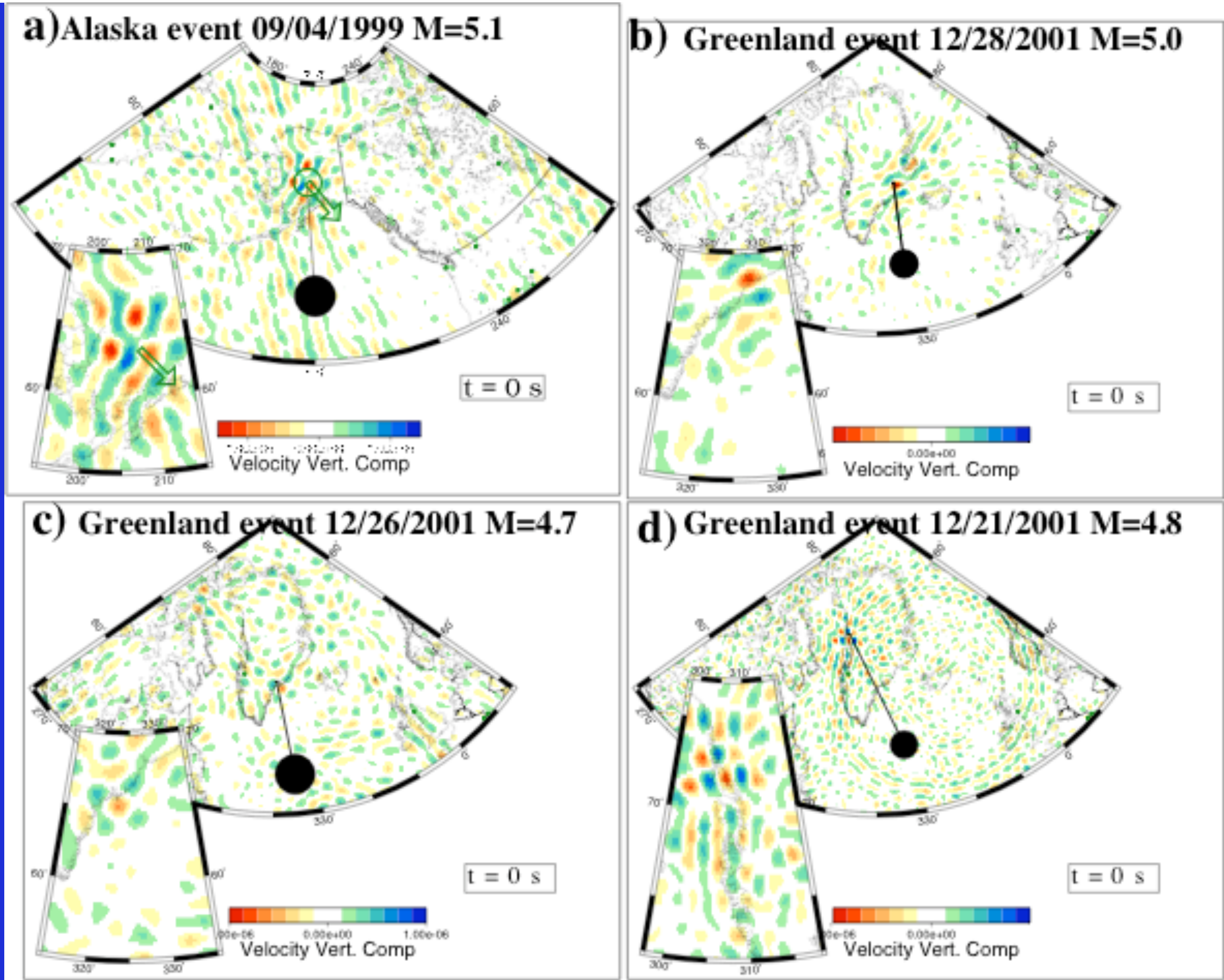
- Carene Larmat, Jeroen Tromp (CalTech)

- Can we locate glacial earthquakes by time reversal?

Glacial Earthquakes

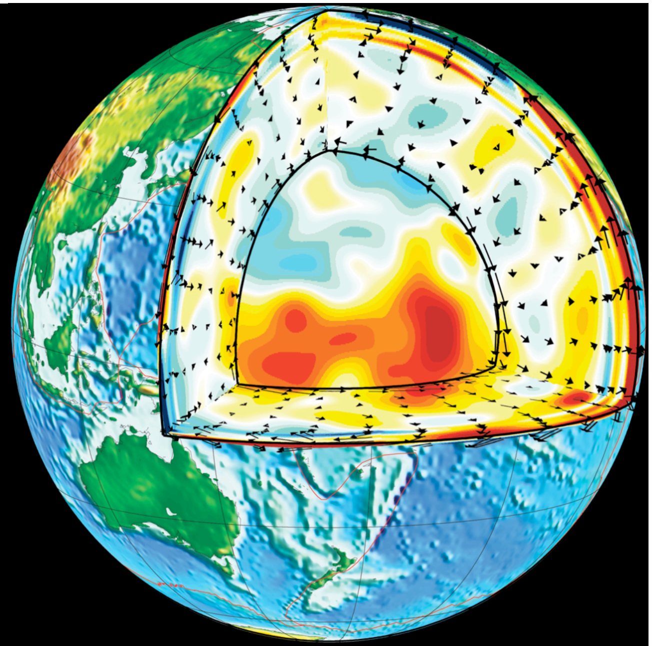


Larmat et al., 2007



Larmat et al., 2007

• Time reversal of seismic waves



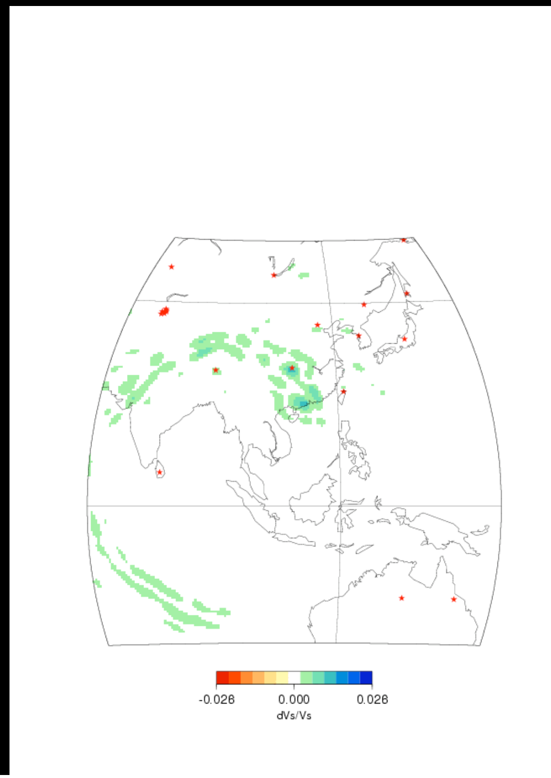
✓Scientific challenges:

-Earthquakes in 3D models (localisation, rupture, …)

- Seismic noise….

-From global to regional scale (T>200s -> 30s)

-Earth structure (Adjoint Tomo): Image geological objects (mantle plumes, slabs, …)



Tsunami of 26/12/04