Seismic wave Propagation and Imaging in Complex media: a European network

Saulo Pomponet Oliveira
Post-doctoral researcher

Host Institution: OGS Trieste
Place of Origin: Salvador, Brazil
Appointment Time: January 2007

Project: High-order and multiple-grid finite element schemes for seismic wave propagation

Task Groups: TG Numerical Methods

Project Scope

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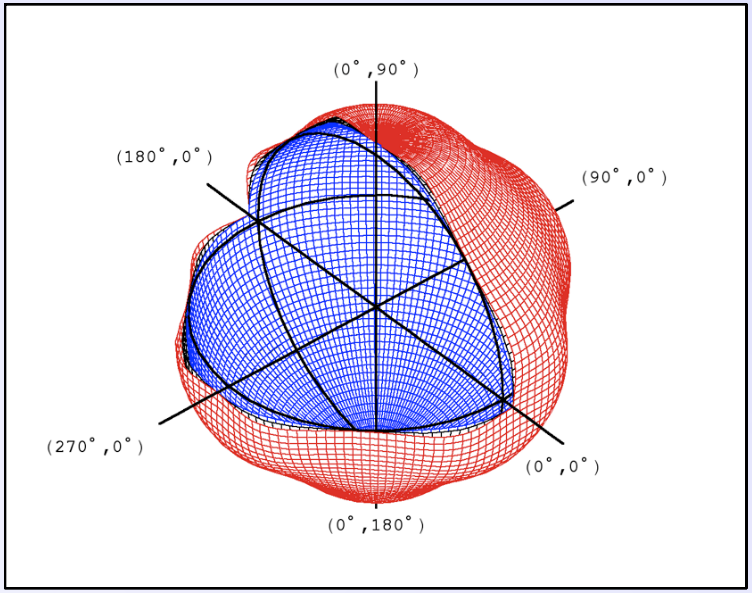
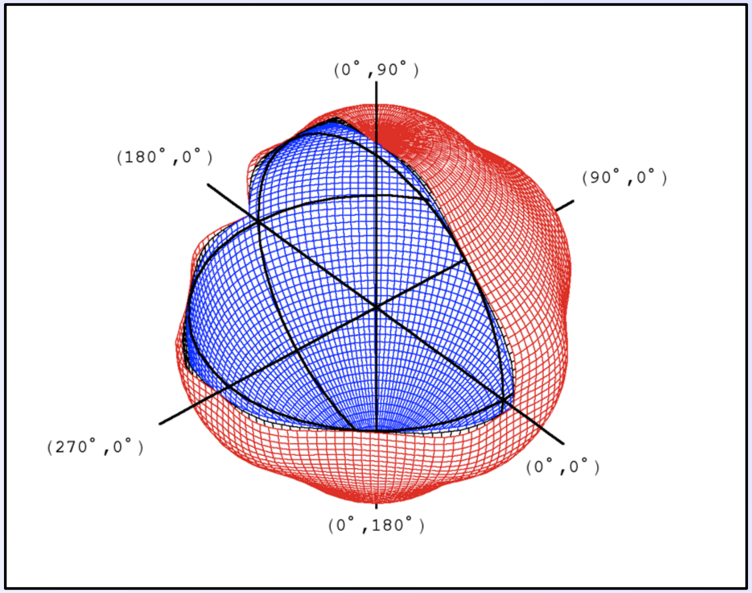
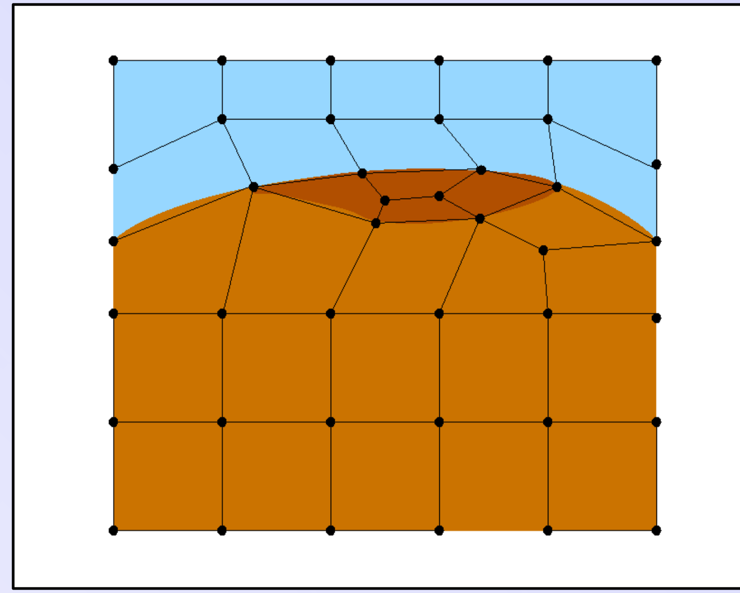


Dispersion Analysis of spectral element methods

Project Scope

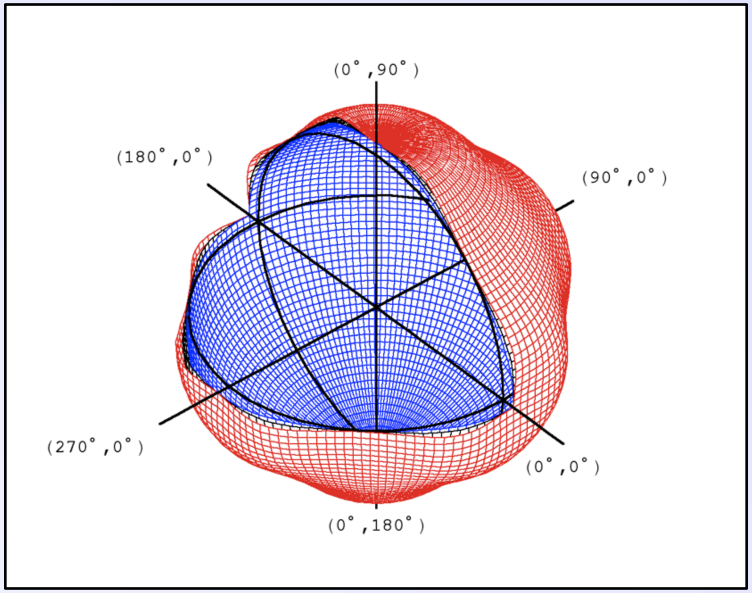




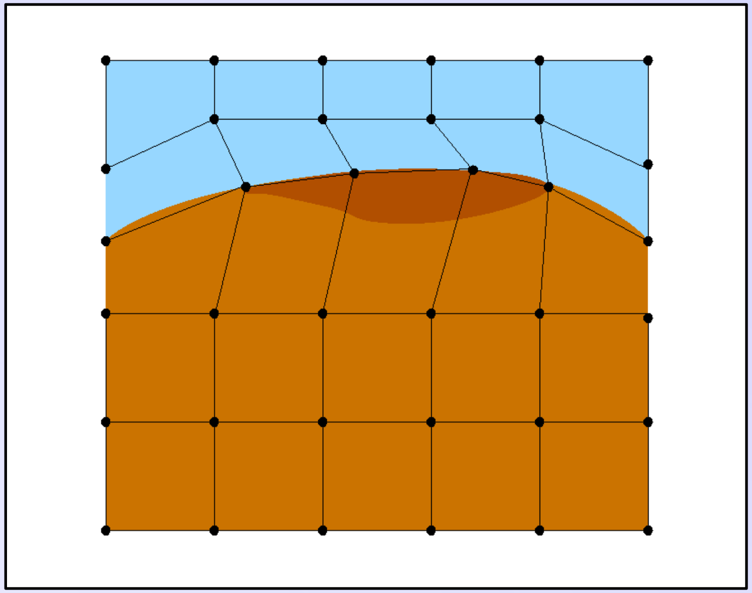
Dispersion Analysis of spectral element methods

Independent sampling of wavefield and material properties

Project Scope



Dispersion Analysis of spectral element methods



Independent sampling of wavefield and material properties

DFT modal analysis of spectral element methods for the elastic wave equation

Saulo Pomponet Oliveira and Géza Seriani
OGS, Trieste - Italy

Theoretical estimate for the

dispersion error with respect to

number of grid points per wavelength (G)

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✓ Arbitrary degree, 1D to 3D

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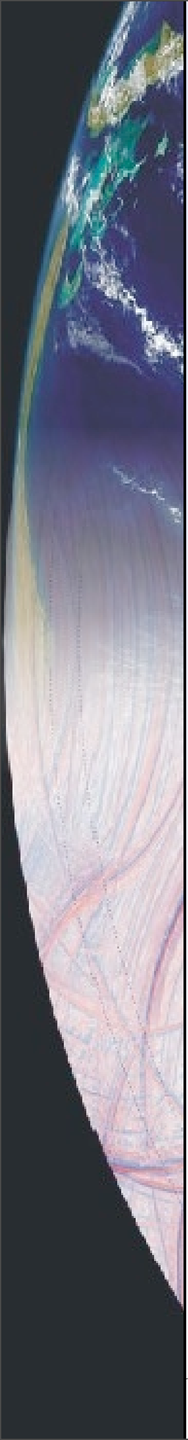

DFT modal analysis of spectral element methods for the elastic wave equation

Saulo Pomponet Oliveira and Géza Seriani
OGS, Trieste - Italy

Theoretical estimate for the
dispersion error with respect to
number of grid points per wavelength (G)

- ✓ Arbitrary degree, 1D to 3D
- ✓ DFT + index reordering and Rayleigh quotient approximation
- ✓ Benefits from tensor representation of spectral elements

Methodology

Isotropic, elastic wave equation

$$\rho \frac{\partial^2}{\partial t^2}\mathbf{u} - (\lambda + \mu)\nabla(\nabla \cdot \mathbf{u}) - \mu \nabla \cdot \nabla \mathbf{u} = \mathbf{0}$$

Methodology

Isotropic, elastic wave equation

$$\rho \frac{\partial^2}{\partial t^2}\mathbf{u} - (\lambda + \mu)\nabla(\nabla \cdot \mathbf{u}) - \mu \nabla \cdot \nabla \mathbf{u} = \mathbf{0}$$

Spectral elements/ 2D square meshes:

$$\begin{cases} \rho \mathbf{M} \frac{\partial^2}{\partial t^2} \mathbf{u}_1^*(t) + \mathbf{K}_1 \mathbf{u}_1^*(t) + \mathbf{K}_2 \mathbf{u}_2^*(t) = \mathbf{0} \\ \rho \mathbf{M} \frac{\partial^2}{\partial t^2} \mathbf{u}_2^*(t) + \mathbf{K}_2^T \mathbf{u}_1^*(t) + \mathbf{K}_3 \mathbf{u}_2^*(t) = \mathbf{0} \end{cases}$$

Element matrices

$$\begin{aligned}
\mathbf{M}^e &= (\Delta x \Delta y/4)\mathbf{A} \otimes \mathbf{A}, \\
\mathbf{K}_1^e &= E(\Delta y/\Delta x)\mathbf{A} \otimes \mathbf{B} + \mu(\Delta x/\Delta y)\mathbf{B} \otimes \mathbf{A}, \\
\mathbf{K}_2^e &= \lambda(\mathbf{C})^T \otimes \mathbf{C} + \mu \mathbf{C} \otimes (\mathbf{C})^T, \\
\mathbf{K}_3^e &= \mu(\Delta y/\Delta x)\mathbf{A} \otimes \mathbf{B} + E(\Delta x/\Delta y)\mathbf{B} \otimes \mathbf{A}
\end{aligned}$$

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\end{aligned}
$$

$$A_{i,j} = \int_{-1}^{1} \phi_j(z)\phi_i(z)\, dz$$

$$B_{i,j} = \int_{-1}^{1} \frac{\partial \phi_j}{\partial z}(z)\frac{\partial \phi_i}{\partial z}(z)\, dz \qquad i, j = 0, ..., N^I$$

$$C_{i,j} = \int_{-1}^{1} \phi_j(z)\frac{\partial \phi_i}{\partial z}(z)\, dz$$

DFT modal analysis:

$$u_1^*(t) = R_1 \left(e^{i(2\pi \mathbf{k}\cdot\mathbf{x}_1 - \omega^* t)}, \ldots, e^{i(2\pi \mathbf{k}\cdot\mathbf{x}_{\mathcal{N}-1} - \omega^* t)} \right)$$

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Substitute **u***, take DFT:

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$$u_1^*(t) = R_1 e^{-i\omega^* t}\mathbf{v}[k_2] \otimes \mathbf{v}[k_1]$$

Substitute **u***, take DFT:

$$-\rho(\omega^*)^2 R_1 \hat{\mathbf{M}}\hat{\mathbf{v}} + R_1 \hat{\mathbf{K}}_1 \hat{\mathbf{v}} + R_2 \hat{\mathbf{K}}_2 \hat{\mathbf{v}} = \mathbf{0}$$

$$\hat{\mathbf{v}} = \hat{\mathbf{v}}[k_2] \otimes \hat{\mathbf{v}}[k_1]$$

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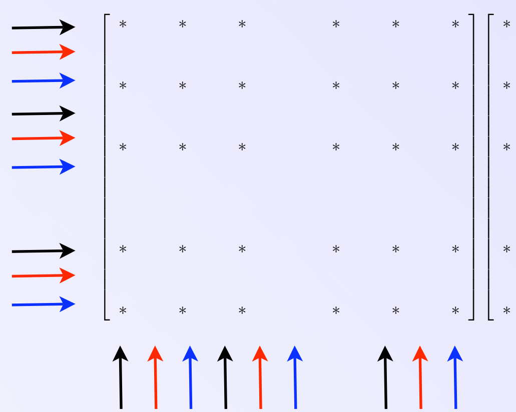

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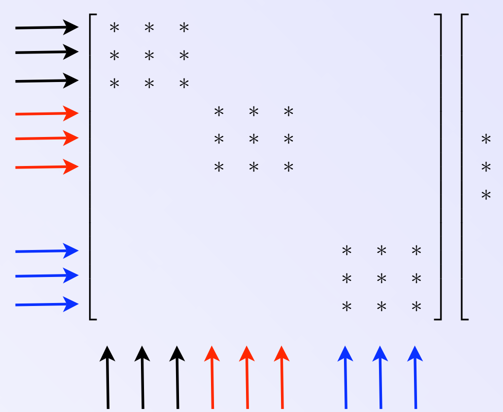

Mulder (1999)

Final eigenvalue system:

$$\begin{bmatrix} \mathbf{D}_1 & \mathbf{D}_2 \\ \mathbf{D}_2 & \mathbf{D}_3 \end{bmatrix} \begin{bmatrix} R_1 \hat{\mathbf{v}} \\ R_2 \hat{\mathbf{v}} \end{bmatrix} = (\omega^*)^2 \begin{bmatrix} \hat{\mathbf{A}} \otimes \hat{\mathbf{A}} & 0 \\ 0 & \hat{\mathbf{A}} \otimes \hat{\mathbf{A}} \end{bmatrix} \begin{bmatrix} R_1 \hat{\mathbf{v}} \\ R_2 \hat{\mathbf{v}} \end{bmatrix}$$

$$\begin{aligned} \mathbf{D}_1 &= \frac{E}{\rho} \frac{4}{\Delta x^2} \hat{\mathbf{A}} \otimes \hat{\mathbf{B}} + \frac{\mu}{\rho} \frac{4}{\Delta y^2} \hat{\mathbf{B}} \otimes \hat{\mathbf{A}}, \\ \mathbf{D}_2 &= -\frac{\lambda + \mu}{\rho} \frac{4}{\Delta x \Delta y} \hat{\mathbf{C}} \otimes \hat{\mathbf{C}}, \\ \mathbf{D}_3 &= \frac{\mu}{\rho} \frac{4}{\Delta y^2} \hat{\mathbf{A}} \otimes \hat{\mathbf{B}} + \frac{E}{\rho} \frac{4}{\Delta x^2} \hat{\mathbf{B}} \otimes \hat{\mathbf{A}}. \end{aligned}$$

Final eigenvalue system:

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NO SOLUTION!

$$\mathbf{D}_1 = \frac{E}{\rho}\frac{4}{\Delta x^2}\hat{\mathbf{A}} \otimes \hat{\mathbf{B}} + \frac{\mu}{\rho}\frac{4}{\Delta y^2}\hat{\mathbf{B}} \otimes \hat{\mathbf{A}},$$

$$\mathbf{D}_2 = -\frac{\lambda+\mu}{\rho}\frac{4}{\Delta x \Delta y}\hat{\mathbf{C}} \otimes \hat{\mathbf{C}},$$

$$\mathbf{D}_3 = \frac{\mu}{\rho}\frac{4}{\Delta y^2}\hat{\mathbf{A}} \otimes \hat{\mathbf{B}} + \frac{E}{\rho}\frac{4}{\Delta x^2}\hat{\mathbf{B}} \otimes \hat{\mathbf{A}}.$$

Final eigenvalue system:

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$$\begin{aligned} \mathbf{D}_1 &= \frac{E}{\rho} \frac{4}{\Delta x^2} \hat{\mathbf{A}} \otimes \hat{\mathbf{B}} + \frac{\mu}{\rho} \frac{4}{\Delta y^2} \hat{\mathbf{B}} \otimes \hat{\mathbf{A}}, \\ \mathbf{D}_2 &= -\frac{\lambda+\mu}{\rho} \frac{4}{\Delta x \Delta y} \hat{\mathbf{C}} \otimes \hat{\mathbf{C}}, \\ \mathbf{D}_3 &= \frac{\mu}{\rho} \frac{4}{\Delta y^2} \hat{\mathbf{A}} \otimes \hat{\mathbf{B}} + \frac{E}{\rho} \frac{4}{\Delta x^2} \hat{\mathbf{B}} \otimes \hat{\mathbf{A}}. \end{aligned}$$

Rayleigh quotient approximation:

$$\begin{bmatrix} d_1 & d_2 \\ d_2 & d_3 \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} = (\omega^*)^2 \begin{bmatrix} R_1 \\ R_2 \end{bmatrix}, \quad d_i = \frac{\hat{\mathbf{v}}^T \mathbf{D}_i \hat{\mathbf{v}}}{\hat{\mathbf{v}}^T \hat{\mathbf{A}} \otimes \hat{\mathbf{A}} \hat{\mathbf{v}}}$$

3D

$$\begin{bmatrix} d_1 & d_2 & d_3 \\ d_2 & d_4 & d_5 \\ d_3 & d_5 & d_6 \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} = (\omega^*)^2 \begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix}, \quad d_i = \frac{\hat{\mathbf{v}}^T \mathbf{D}_i \hat{\mathbf{v}}}{\hat{\mathbf{v}}^T \hat{\mathbf{A}} \otimes \hat{\mathbf{A}} \otimes \hat{\mathbf{A}} \hat{\mathbf{v}}}$$

$$\mathbf{D}_1 = \frac{E}{\rho}\frac{4}{\Delta x^2}\hat{\mathbf{A}} \otimes \hat{\mathbf{A}} \otimes \hat{\mathbf{B}} + \frac{\mu}{\rho}\frac{4}{\Delta y^2}\hat{\mathbf{A}} \otimes \hat{\mathbf{B}} \otimes \hat{\mathbf{A}} + \frac{\mu}{\rho}\frac{4}{\Delta z^2}\hat{\mathbf{B}} \otimes \hat{\mathbf{A}} \otimes \hat{\mathbf{A}},$$

$$\mathbf{D}_2 = -\frac{\lambda+\mu}{\rho}\frac{4}{\Delta x \Delta y}\hat{\mathbf{A}} \otimes \hat{\mathbf{C}} \otimes \hat{\mathbf{C}},$$

$$\mathbf{D}_3 = -\frac{\lambda+\mu}{\rho}\frac{4}{\Delta x \Delta z}\hat{\mathbf{C}} \otimes \hat{\mathbf{A}} \otimes \hat{\mathbf{C}},$$

$$\mathbf{D}_4 = \frac{\mu}{\rho}\frac{4}{\Delta x^2}\hat{\mathbf{A}} \otimes \hat{\mathbf{A}} \otimes \hat{\mathbf{B}} + \frac{E}{\rho}\frac{4}{\Delta y^2}\hat{\mathbf{A}} \otimes \hat{\mathbf{B}} \otimes \hat{\mathbf{A}} + \frac{\mu}{\rho}\frac{4}{\Delta z^2}\hat{\mathbf{B}} \otimes \hat{\mathbf{A}} \otimes \hat{\mathbf{A}},$$

$$\mathbf{D}_5 = -\frac{\lambda+\mu}{\rho}\frac{4}{\Delta y \Delta z}\hat{\mathbf{C}} \otimes \hat{\mathbf{C}} \otimes \hat{\mathbf{A}},$$

$$\mathbf{D}_6 = \frac{\mu}{\rho}\frac{4}{\Delta x^2}\hat{\mathbf{A}} \otimes \hat{\mathbf{A}} \otimes \hat{\mathbf{B}} + \frac{\mu}{\rho}\frac{4}{\Delta y^2}\hat{\mathbf{A}} \otimes \hat{\mathbf{B}} \otimes \hat{\mathbf{A}} + \frac{E}{\rho}\frac{4}{\Delta z^2}\hat{\mathbf{B}} \otimes \hat{\mathbf{A}} \otimes \hat{\mathbf{A}}$$

Time discretization

$$\rho \mathbf{M} \frac{\mathbf{u}_1^*(t_{n+1}) - 2\mathbf{u}_1^*(t_n) + \mathbf{u}_1^*(t_{n-1})}{\Delta t^2} + \mathbf{K}_1 \mathbf{u}_1^*(t_n) + \mathbf{K}_2 \mathbf{u}_2^*(t_{n-1}) = \mathbf{0}$$

Time discretization

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Substitute **u**, take DFT:

$$\rho \frac{e^{-i\omega^* \Delta t} - 2 + e^{i\omega^* \Delta t}}{\Delta t^2} R_1 \hat{\mathbf{M}} \hat{\mathbf{v}} + R_1 \hat{\mathbf{K}}_1 \hat{\mathbf{v}} + R_2 \hat{\mathbf{K}}_2 \hat{\mathbf{v}} = \mathbf{0}$$

Time discretization

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Substitute **u**, take DFT:

$$-\rho \left(\frac{2 \sin(\omega^* \Delta t / 2)}{\Delta t} \right)^2 R_1 \hat{\mathbf{M}} \hat{\mathbf{v}} + R_1 \hat{\mathbf{K}}_1 \hat{\mathbf{v}} + R_2 \hat{\mathbf{K}}_2 \hat{\mathbf{v}} = \mathbf{0}$$

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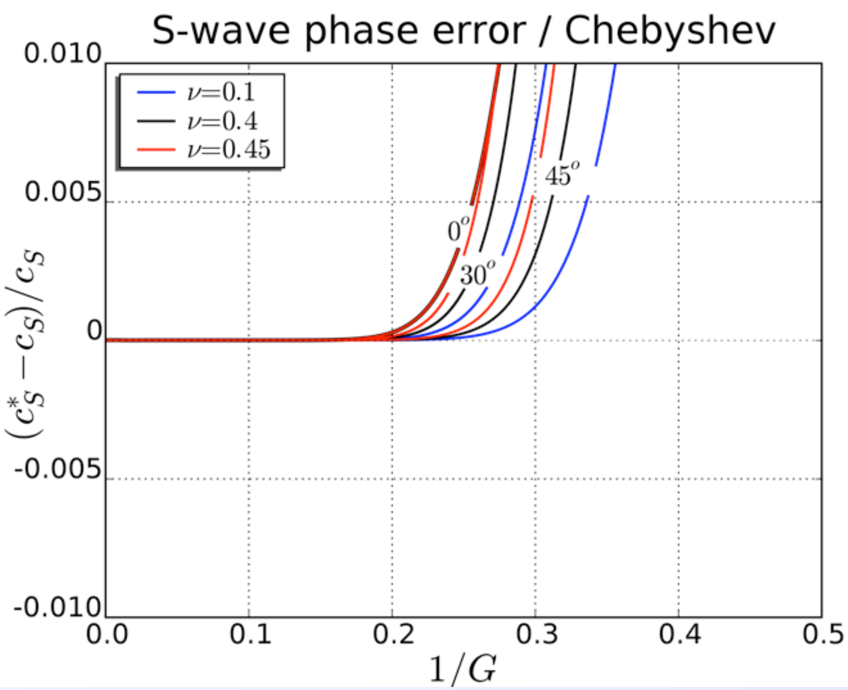
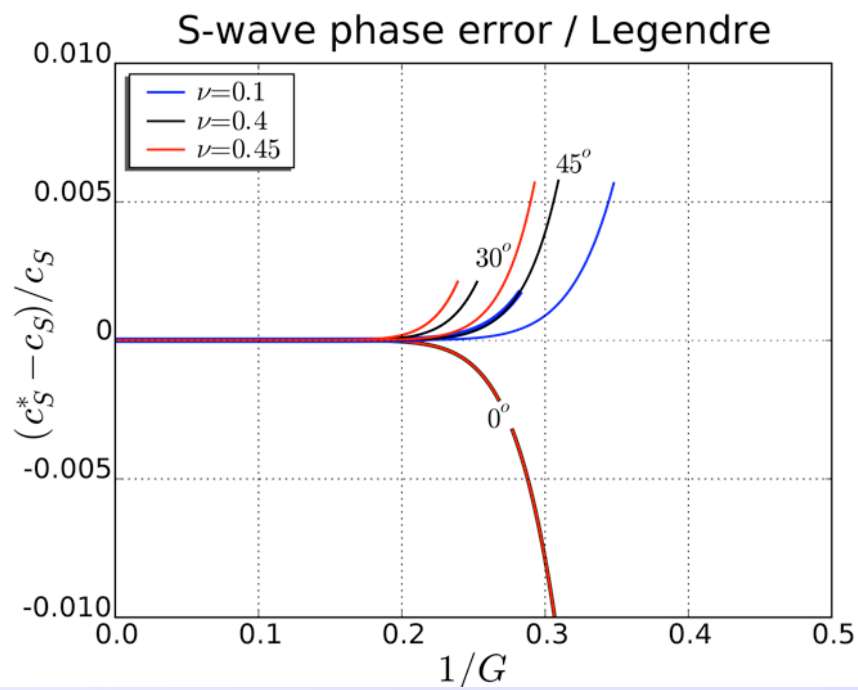
(similar to $-\rho(\omega^*)^2 R_1 \hat{\mathbf{M}} \hat{\mathbf{v}} + R_1 \hat{\mathbf{K}}_1 \hat{\mathbf{v}} + R_2 \hat{\mathbf{K}}_2 \hat{\mathbf{v}} = \mathbf{0}$)

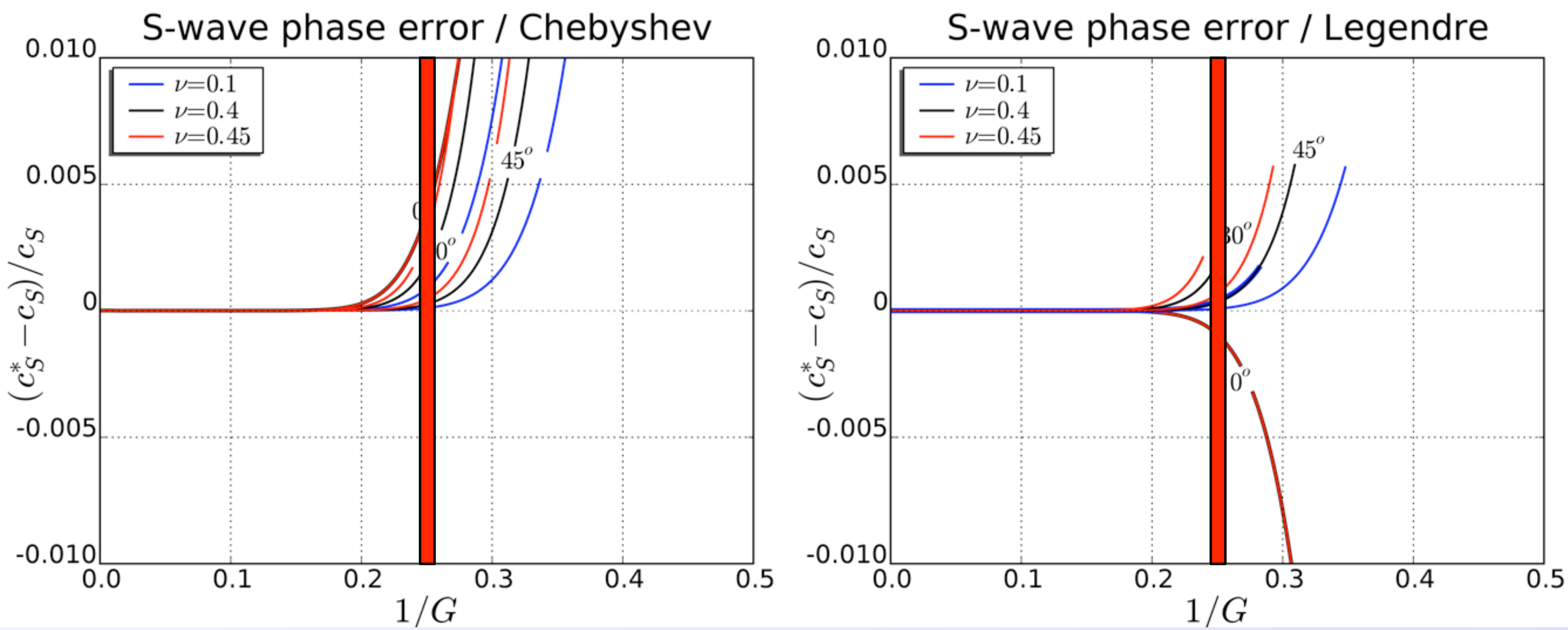
Dispersion of S-Waves, 8^{th}-degree





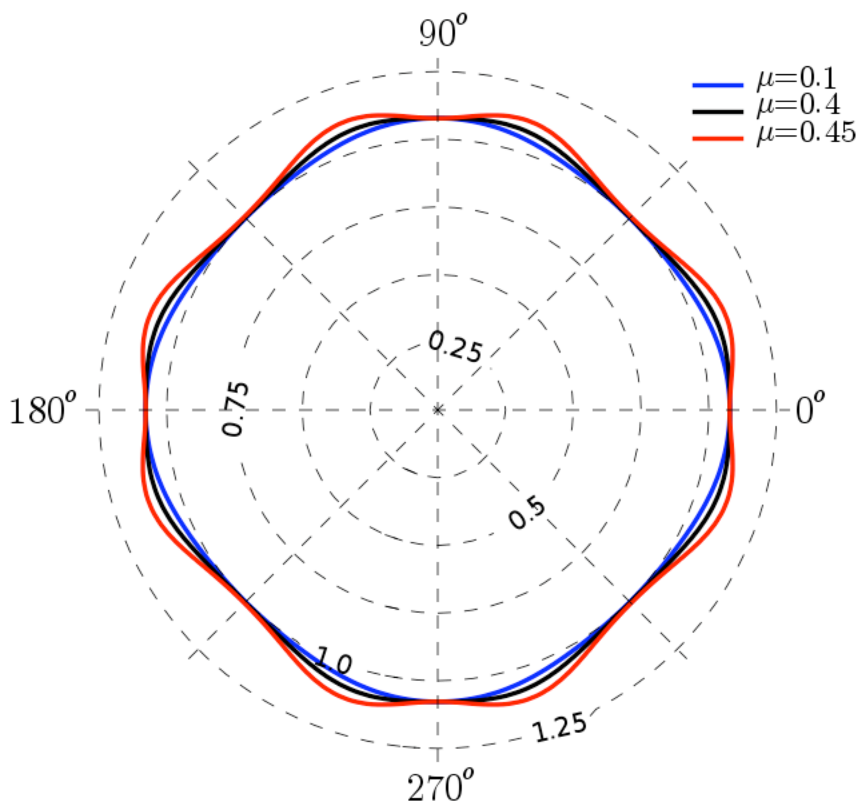
$$c_S^* = \frac{\omega_S^*}{\kappa} \qquad c_S = \sqrt{\frac{\mu}{\rho}}$$

Dispersion of S-Waves, 8^{th}-degree

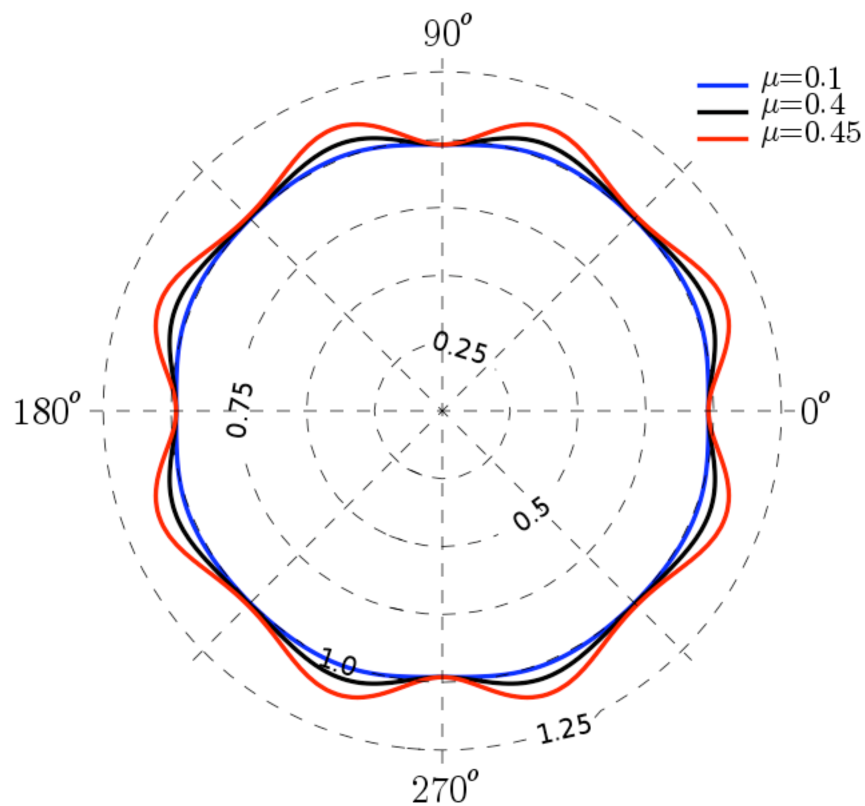


$$c_S^* = \frac{\omega_S^*}{\kappa} \qquad c_S = \sqrt{\frac{\mu}{\rho}}$$

Polar plots : $1 + 20(c^*_S - c_S)/c_S$
four grid points per wavelength

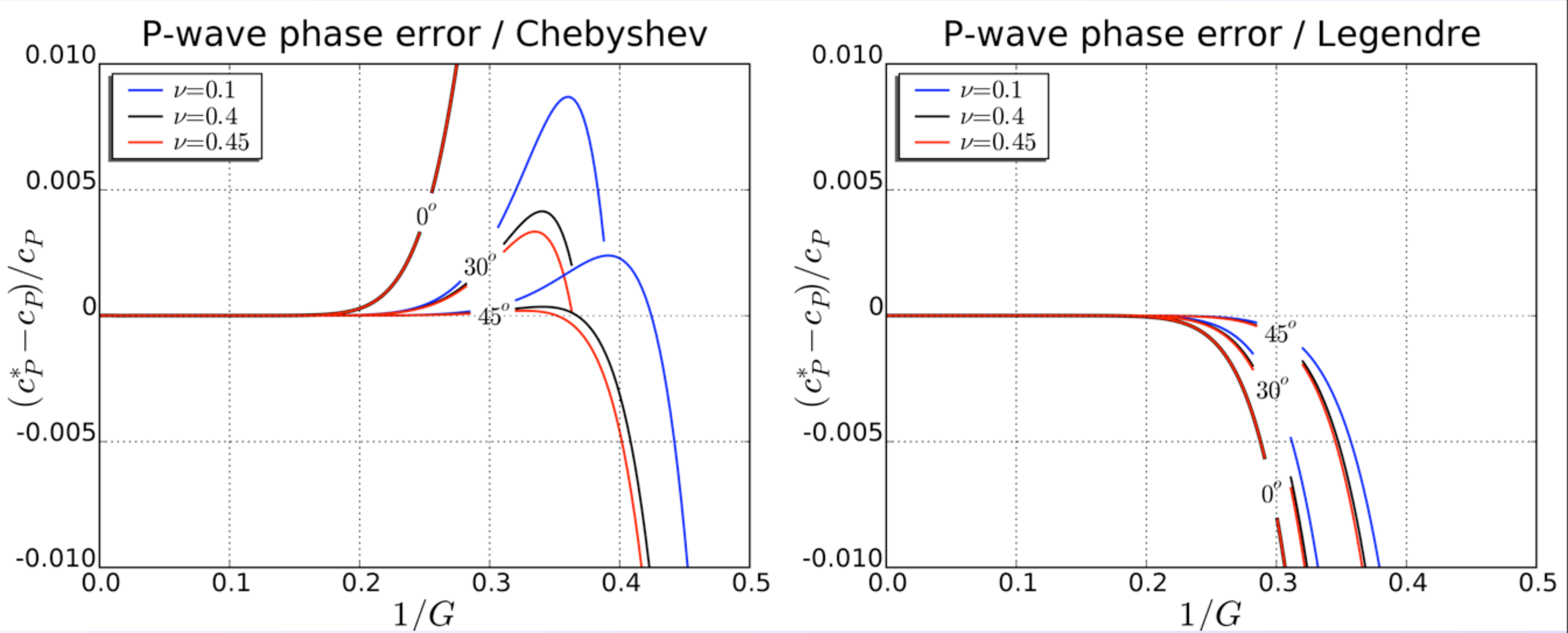


Chebyshev



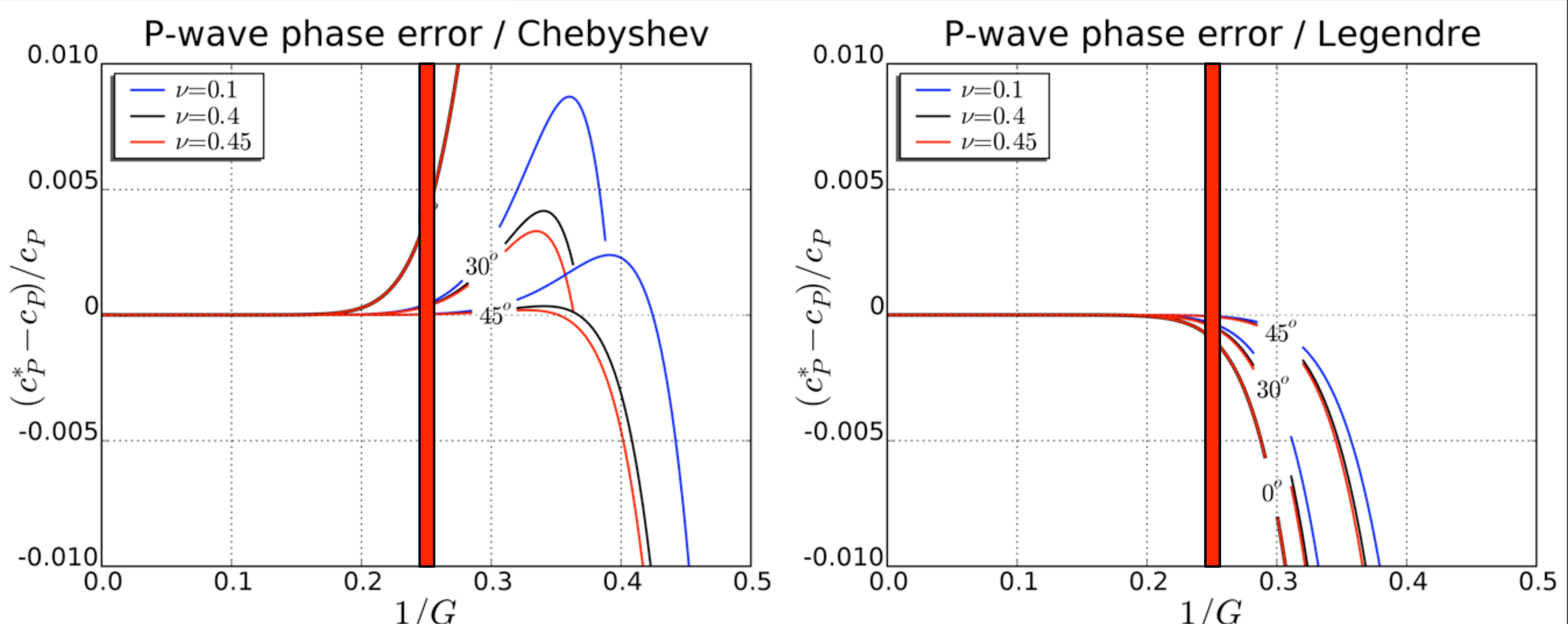
Legendre

Dispersion of P-Waves, 8th-degree



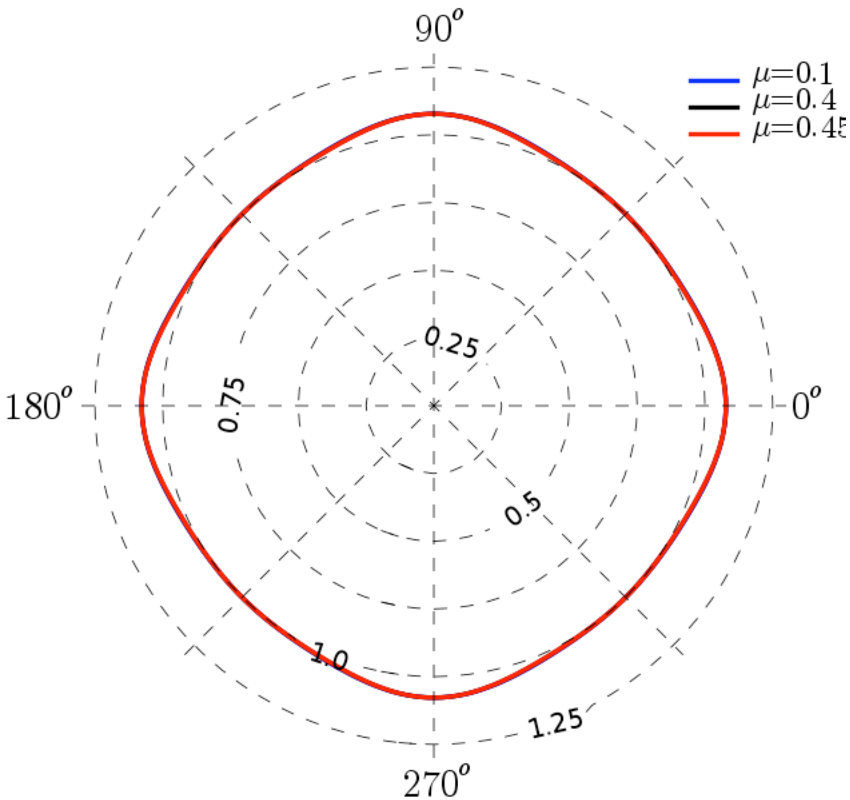
$$c_P^* = \frac{\omega_P^*}{\kappa} \qquad c_P = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$

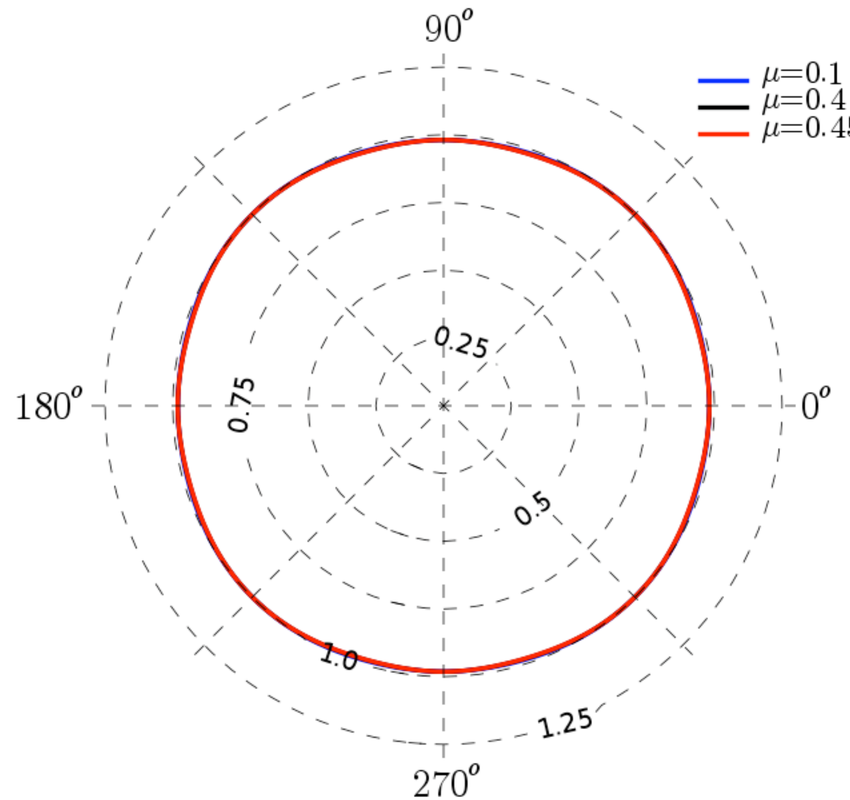
Dispersion of P-Waves, 8^{th}-degree



$$c_P^* = \frac{\omega_P^*}{\kappa} \qquad c_P = \sqrt{\frac{\lambda + 2\mu}{\rho}}$$

Polar plots : $1 + 20(c^*_P - c_P)/c_P$
four grid points per wavelength

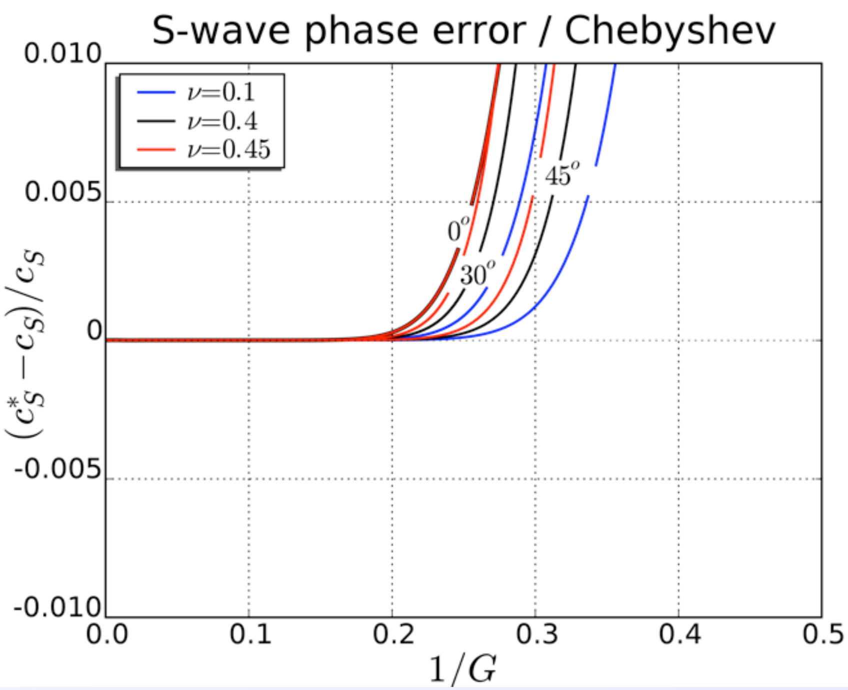


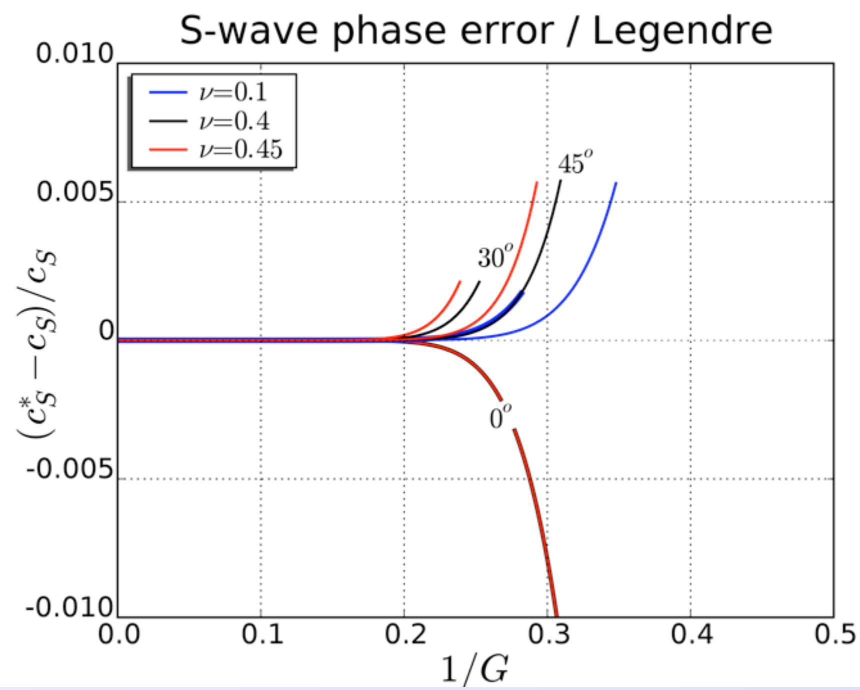


Chebyshev

Legendre

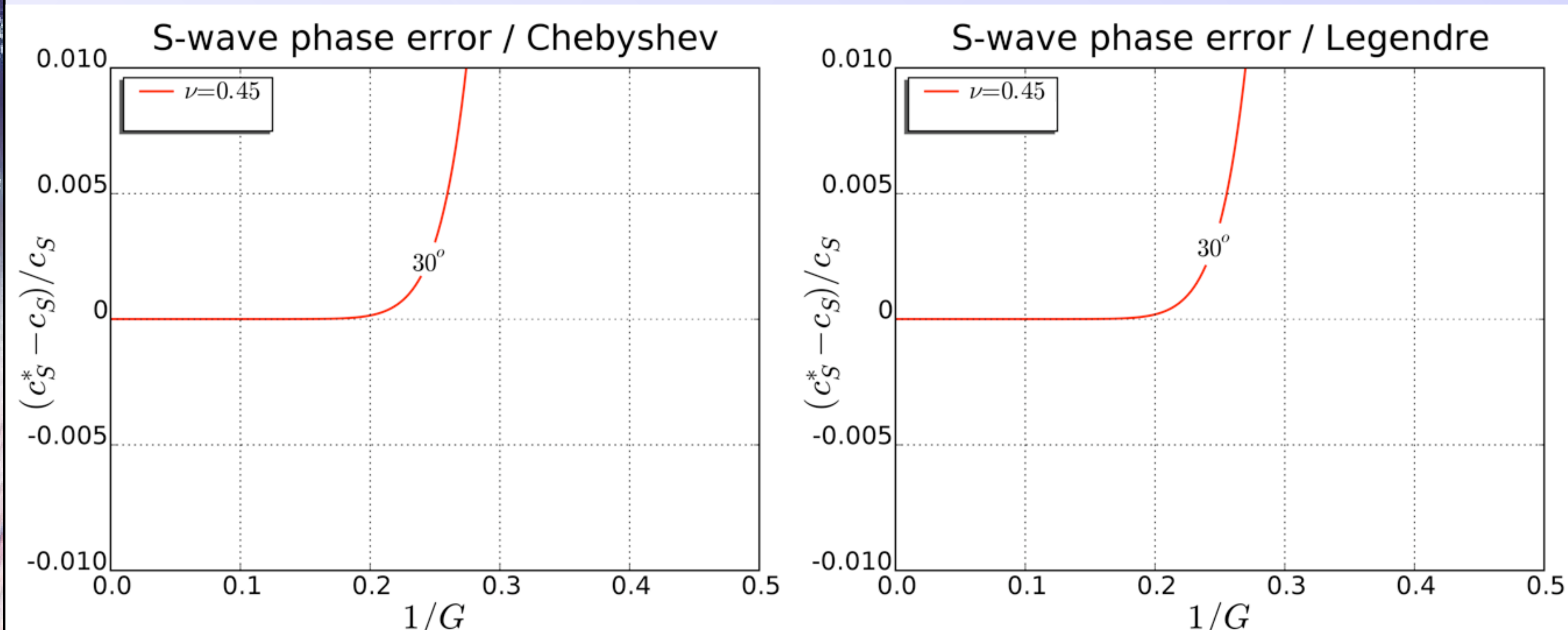
Dispersion / full discretization S-waves





$$CFL = \frac{c_P \Delta t}{h}$$

Dispersion / full discretization S-waves



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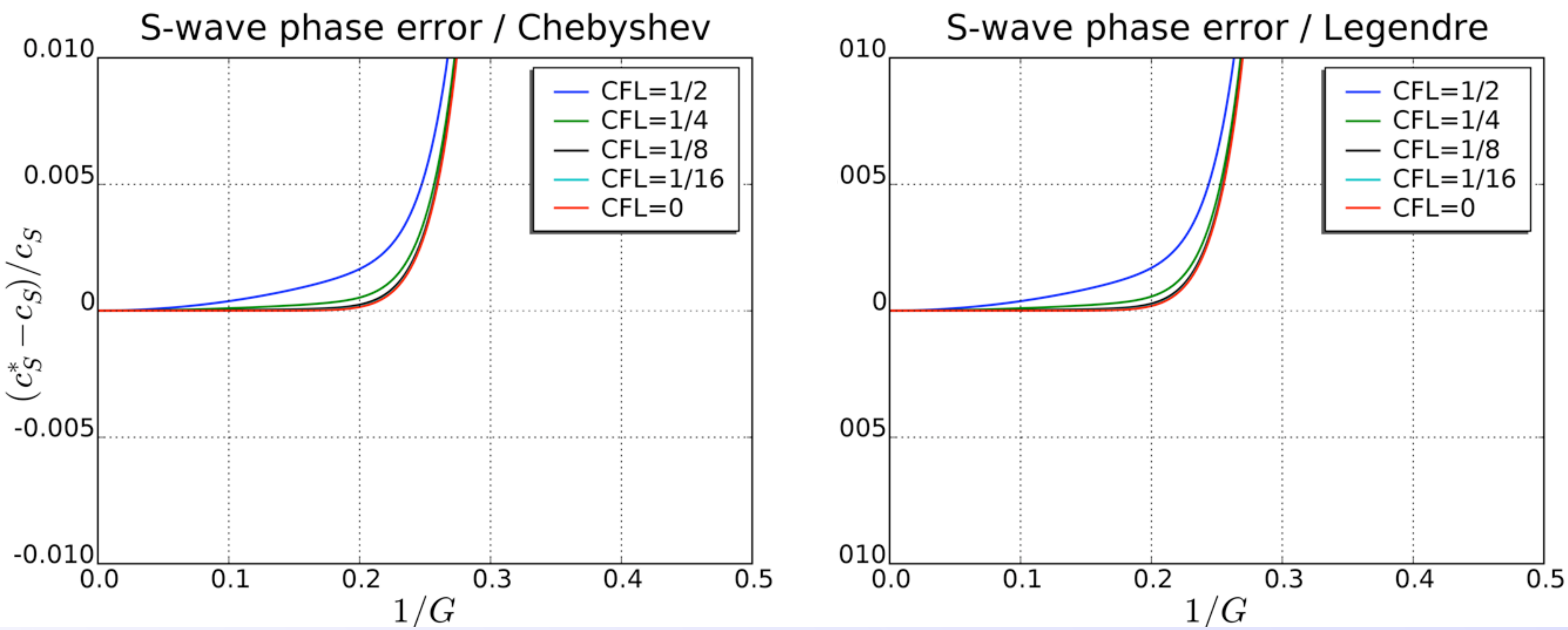
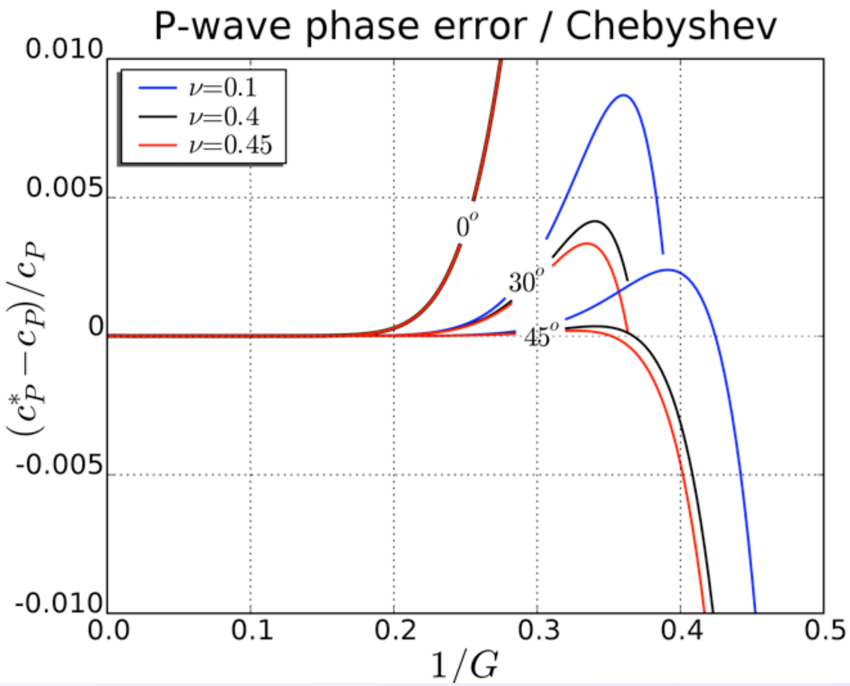
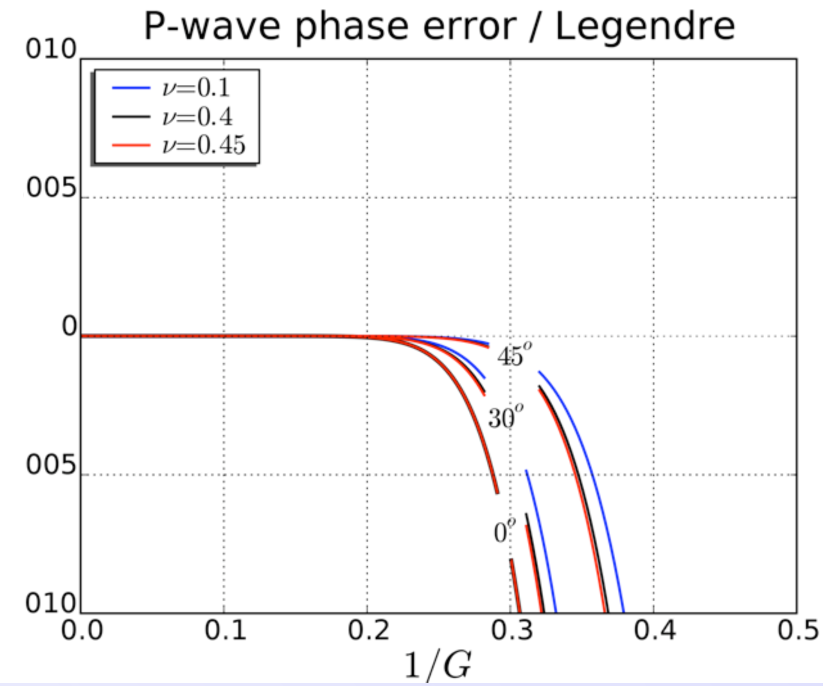
Dispersion / full discretization
S-waves



$$CFL = \frac{c_P \Delta t}{h}$$

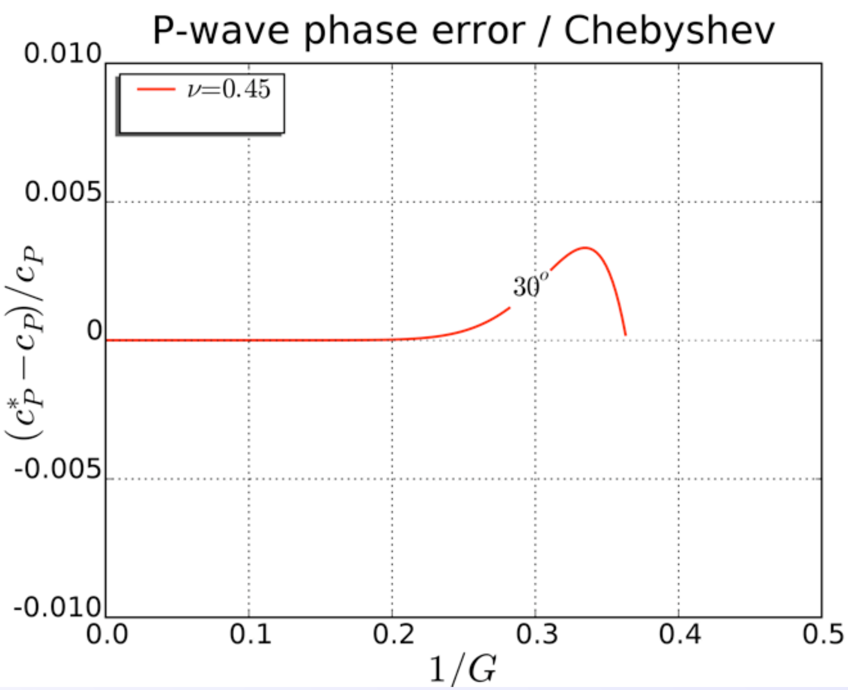
Dispersion / full discretization
P-waves

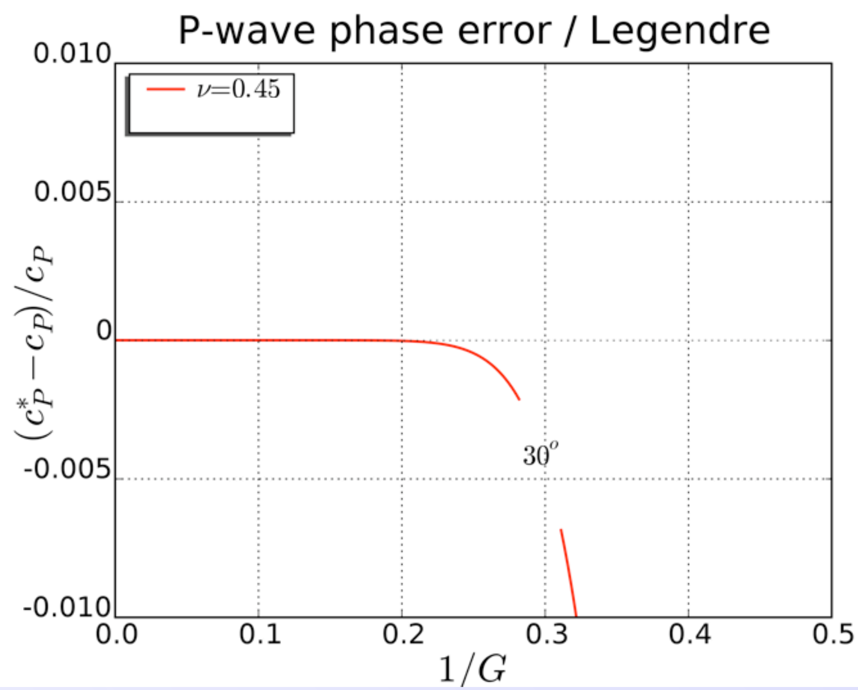




$$CFL = \frac{c_P \Delta t}{h}$$

Dispersion / full discretization P-waves





$$CFL = \frac{c_P \Delta t}{h}$$

Dispersion / full discretization
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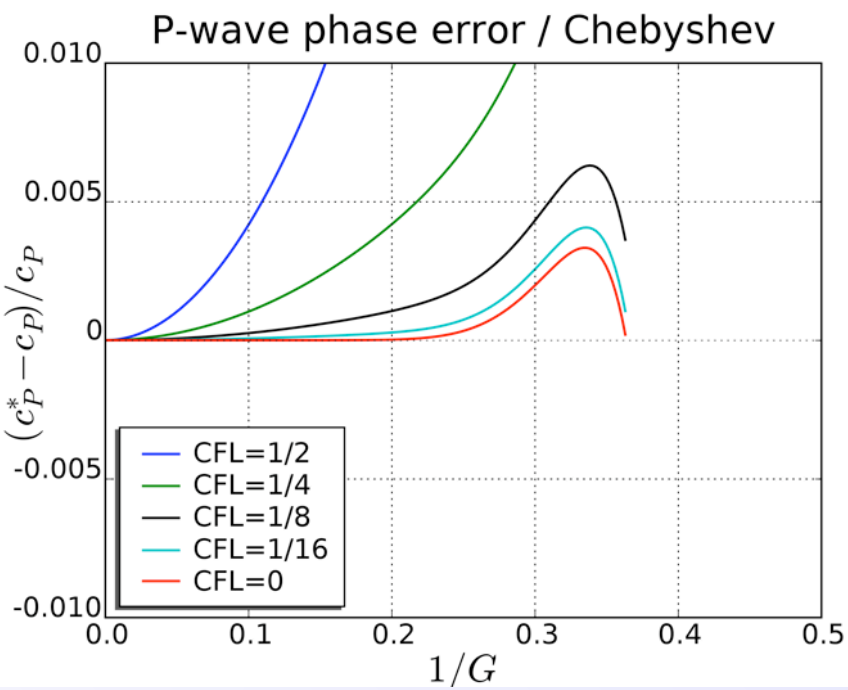
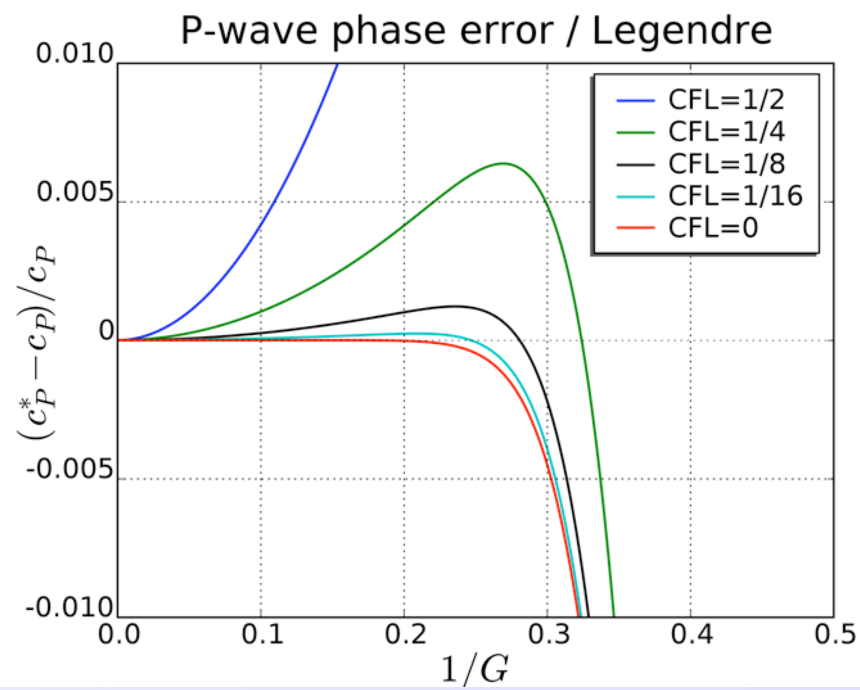
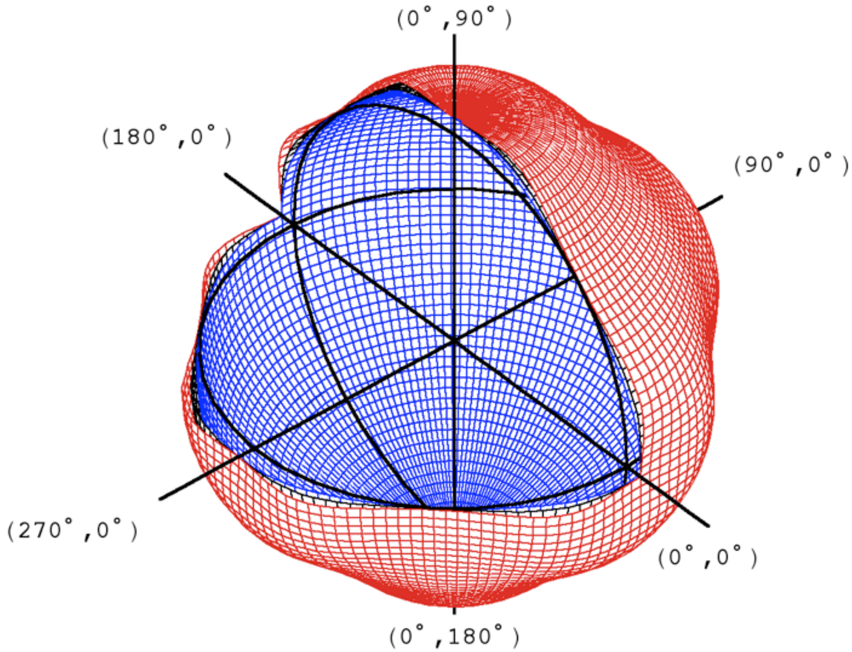


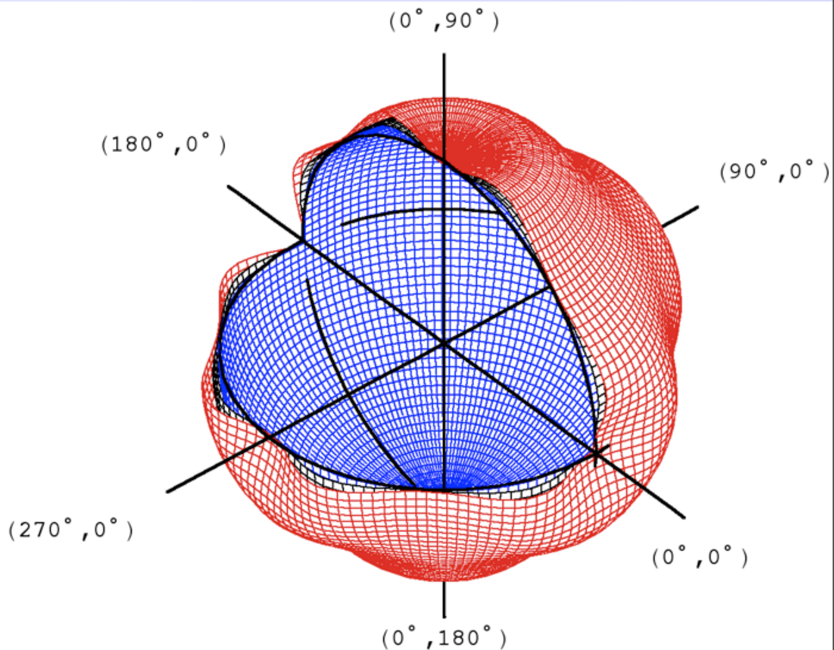


$$CFL = \frac{c_P \Delta t}{h}$$

3D plots : $1 + 20(c^*_S - c_S)/c_S$
spherical coordinates
four grid points per wavelength



Chebyshev



Legendre

3D plots : $1 + 20(c^*_P - c_P)/c_P$
spherical coordinates
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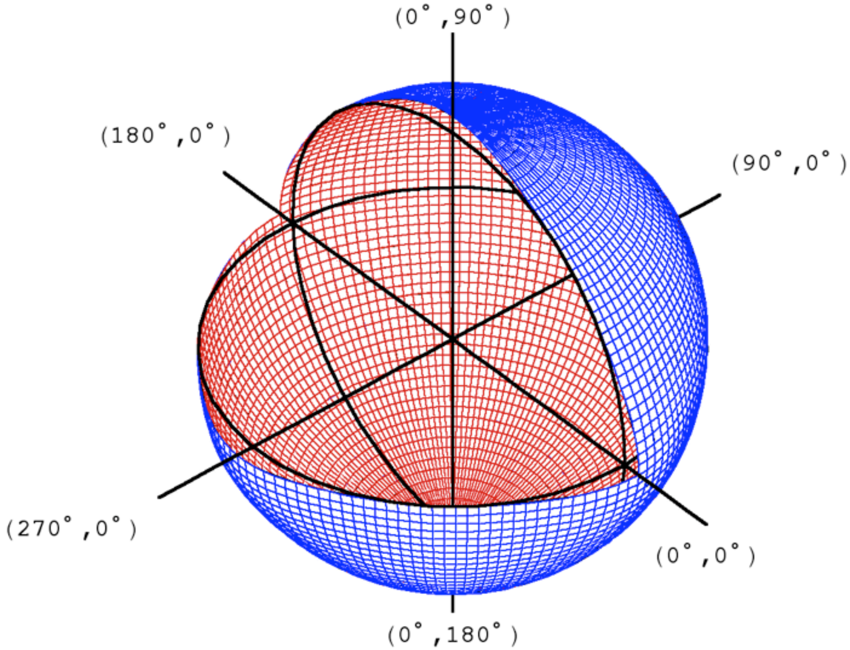

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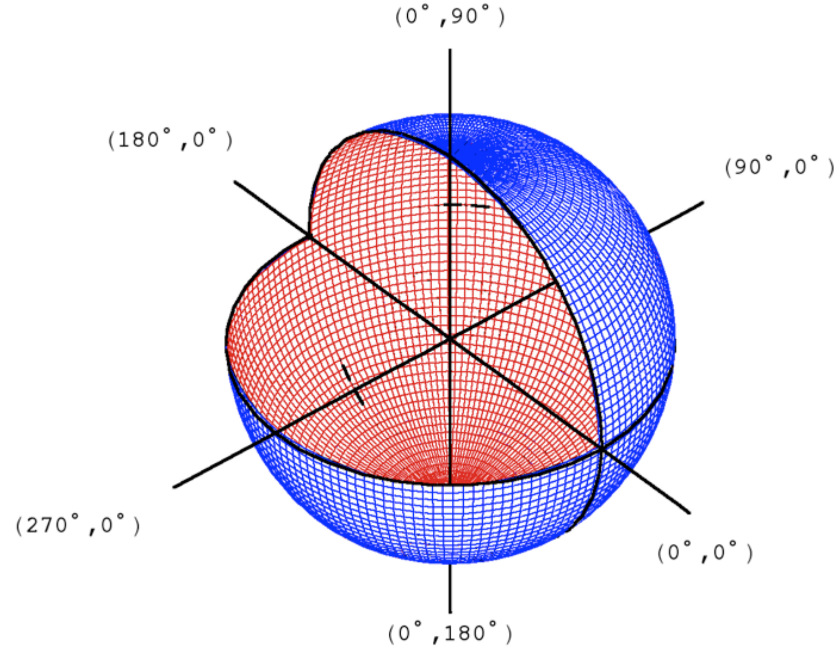

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Seismic wave Propagation and Imaging in Complex media: a European network

The End!