Simulated and Analog Results in Underwater Acoustics

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Outline

-Introduction to underwater acoustics

-Time reversal in the shallow ocean

-Theory and Simulation in shallow water acoustics

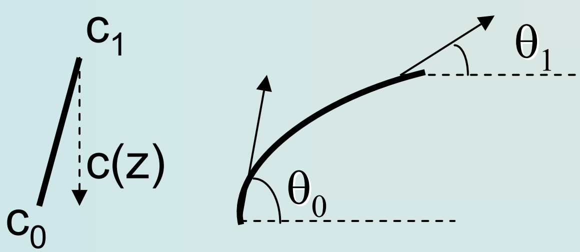
-Exploratory works based on laboratory self-adaptive experiments

Snell's law

Sound likes low speed

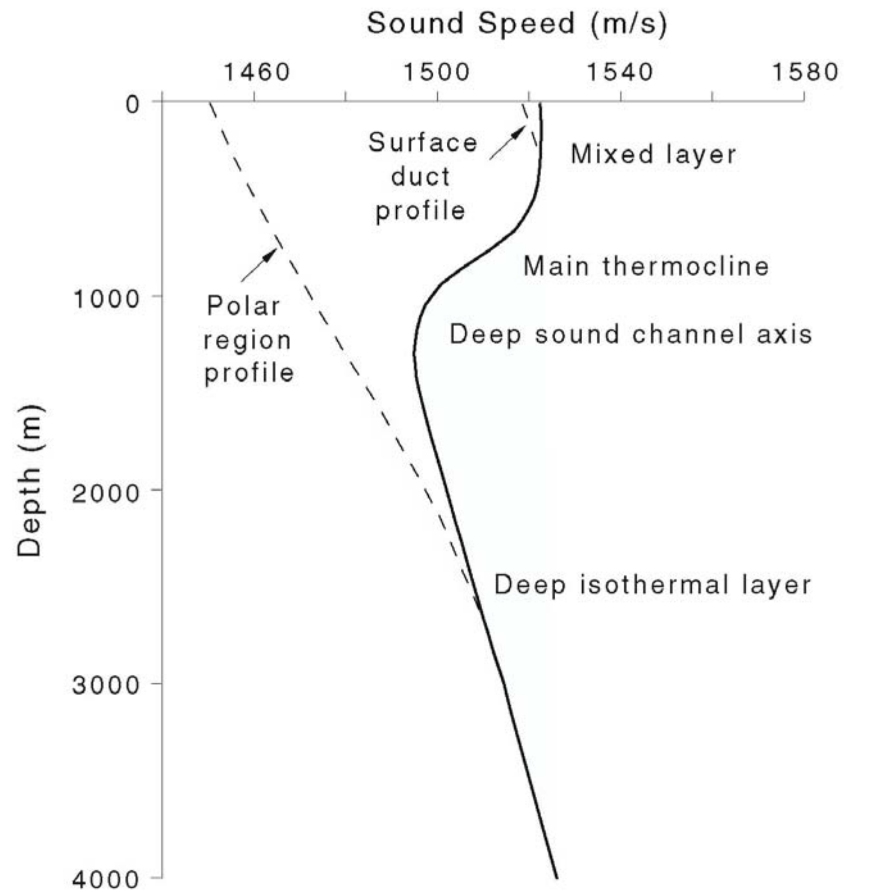
Index of refraction $$n \cos\theta = \frac{\cos\theta}{c} = cte$$

Sound speed   Acoustic ray



$c_0 < c_1$

Generic sound speed structure

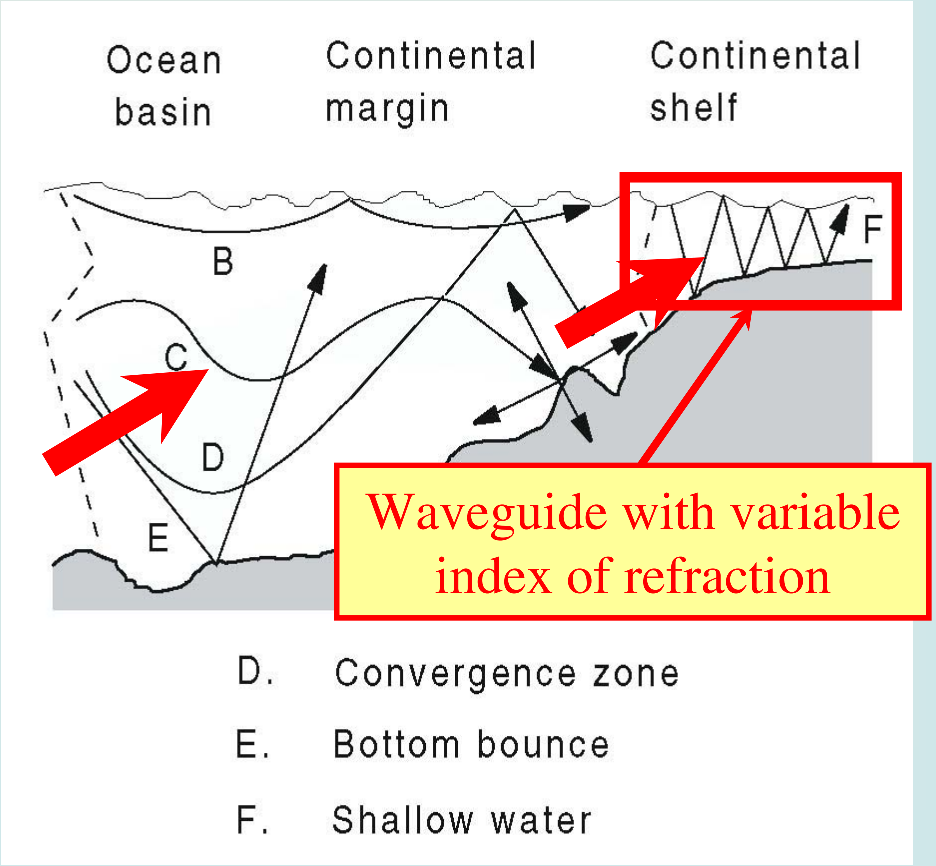


Sound Speed

$$c = 1449.2 + 4.6T - 0.055T^2 + 0.00029T^3 + (1.34 - 0.01T)(S - 35) + 0.016z.$$

The Complex Ocean Environment

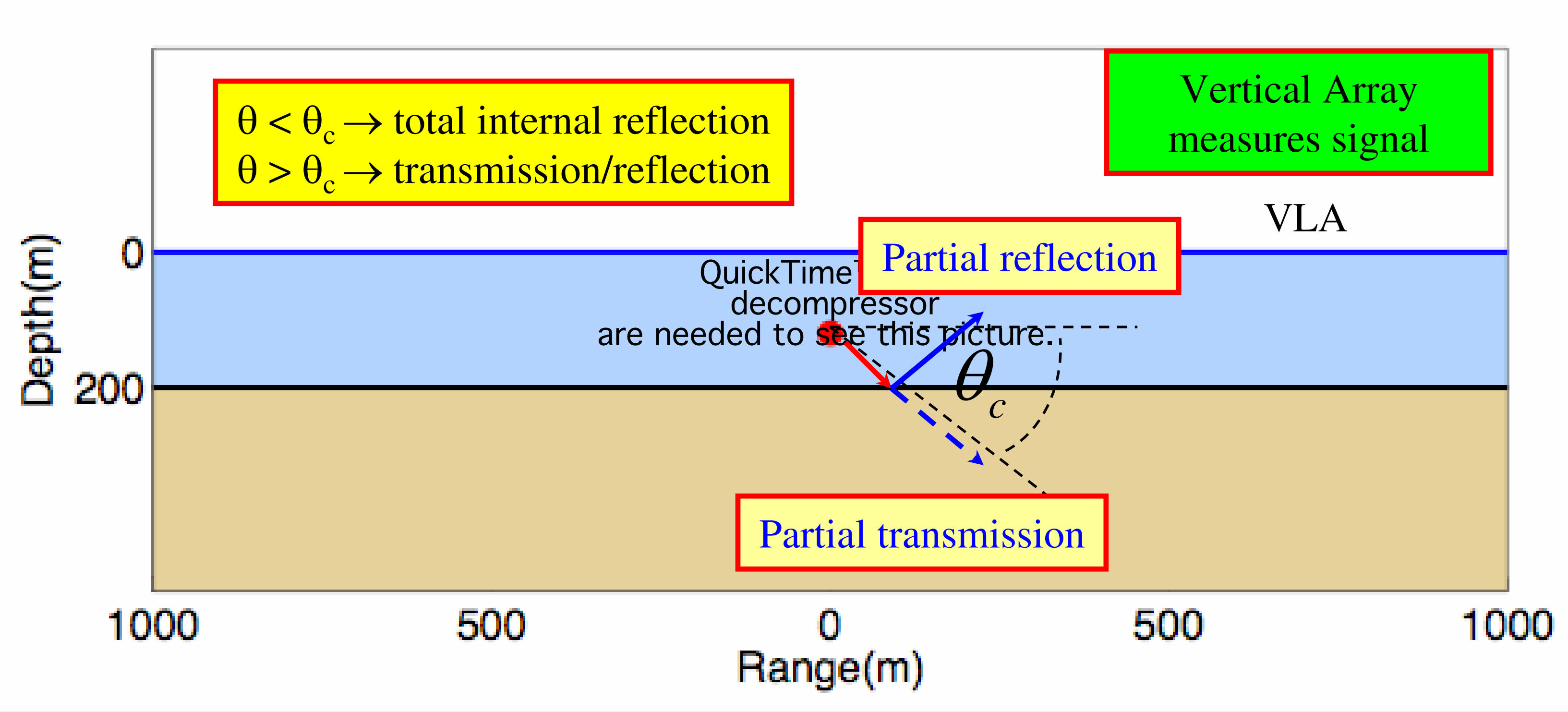
- The ocean is a complex inhomogenous medium
  - Complicated bathymetry
  - $c_{\omega}(z)= c_{\omega}(p(z),T(z),S(z))$ often unknown
  - Layered sediments
  - Dynamic and ever-changing
- Deep ocean propagation dominated by refraction
- Shallow ocean propagation dominated by boundary interactions



Bottom attenuation

- Index of refraction $$n \cos\theta = cte$$
$$\cos\theta_c = \frac{c_w}{c_b}$$

- Acoustic propagation above critical angle (θ_c) refracts into bottom and is attenuated

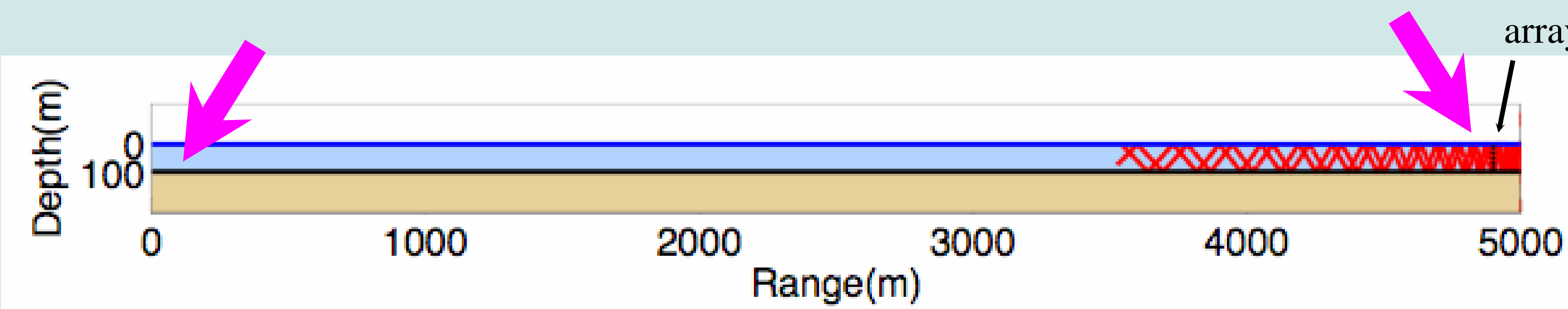


Shallow ocean acoustics challenge

- Boundaries retain information in the waveguide over long ranges
- Can we manipulate acoustic field at long ranges using vertical arrays without knowledge of the complex medium?

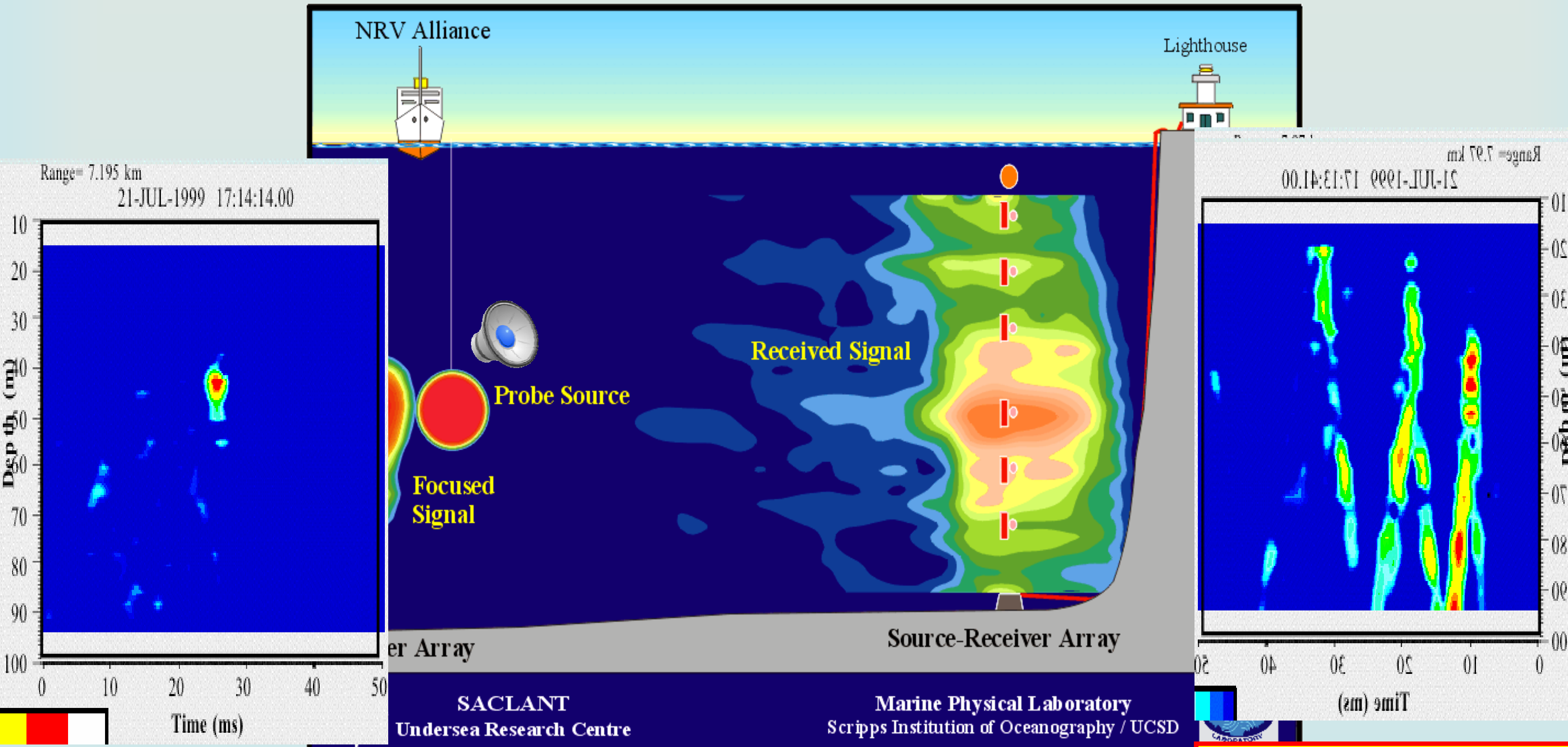
To control acoustic field far from vertical array

Use information embedded in acoustic field at long range



Time reversal in the shallow ocean

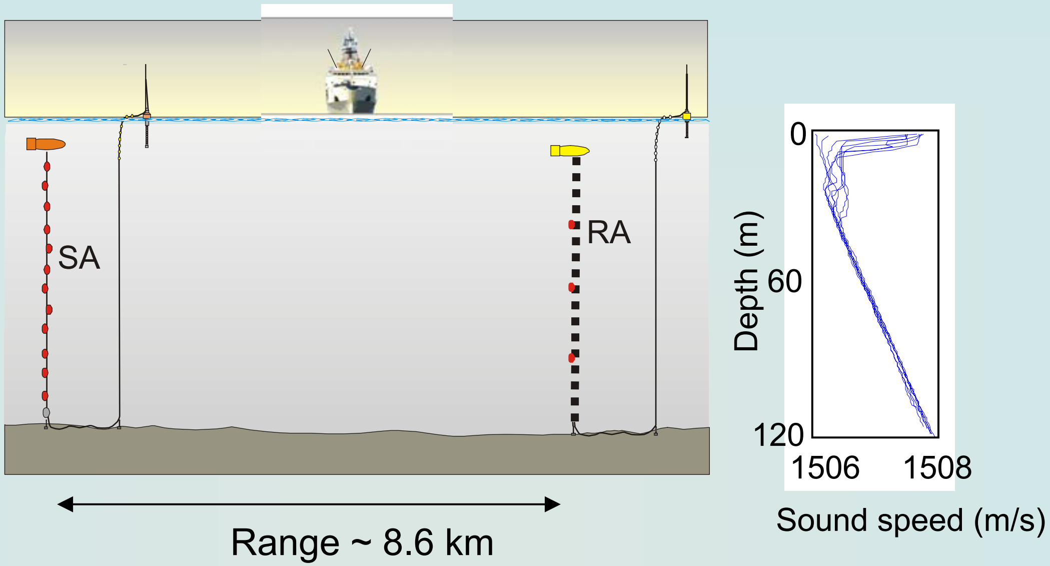
Experimental TR in the Shallow Ocean (1996)

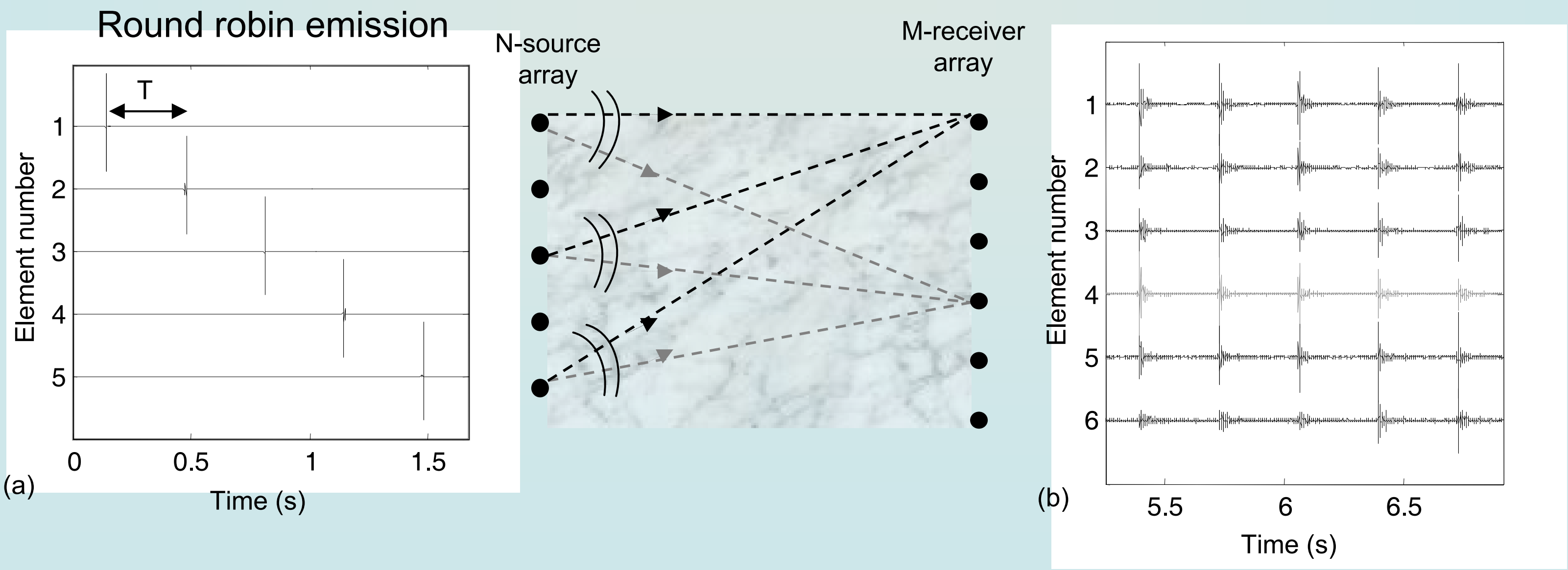


Focused pulse back at source

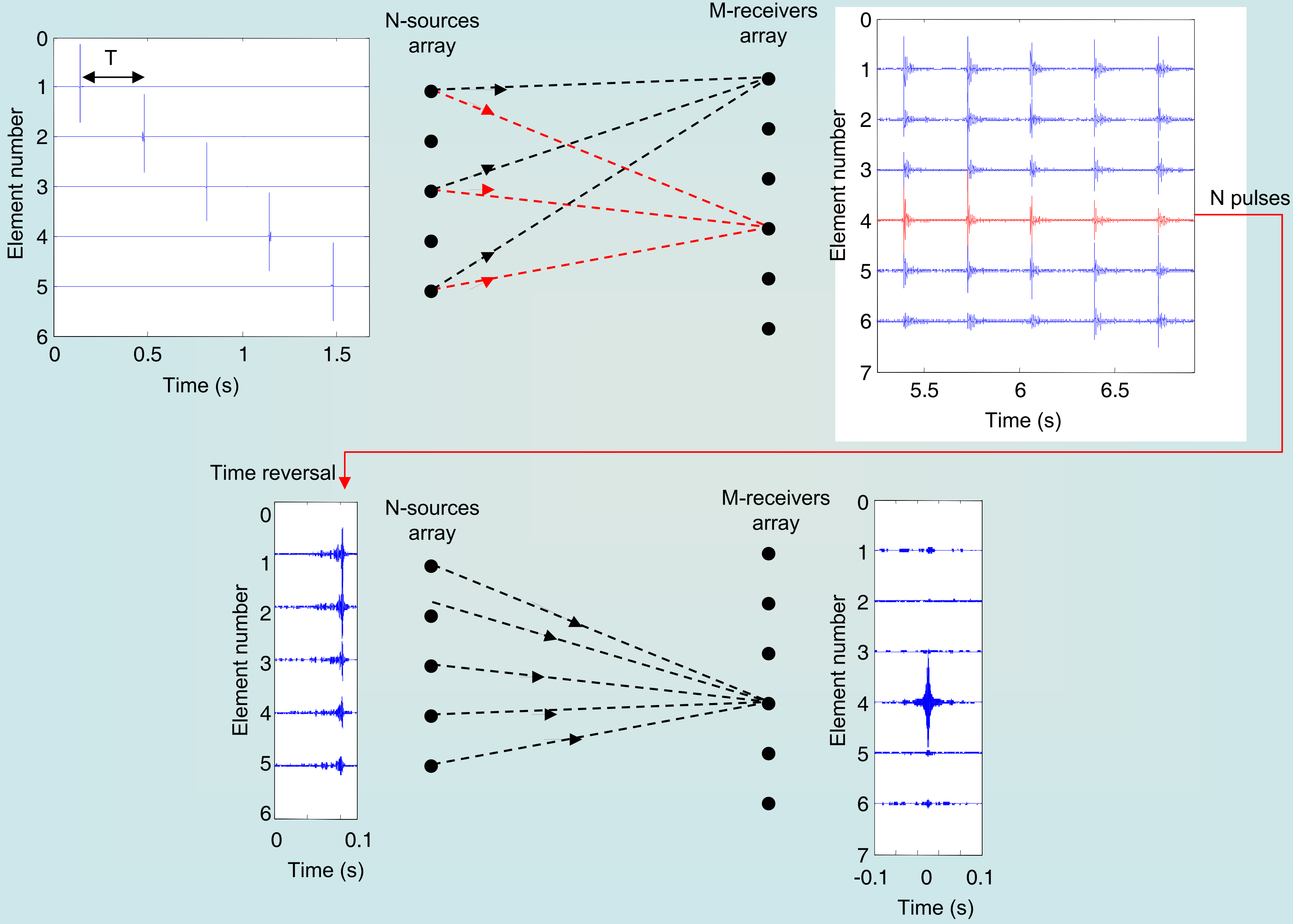
TR = last in first out

Time Reversal Experiment (2003)

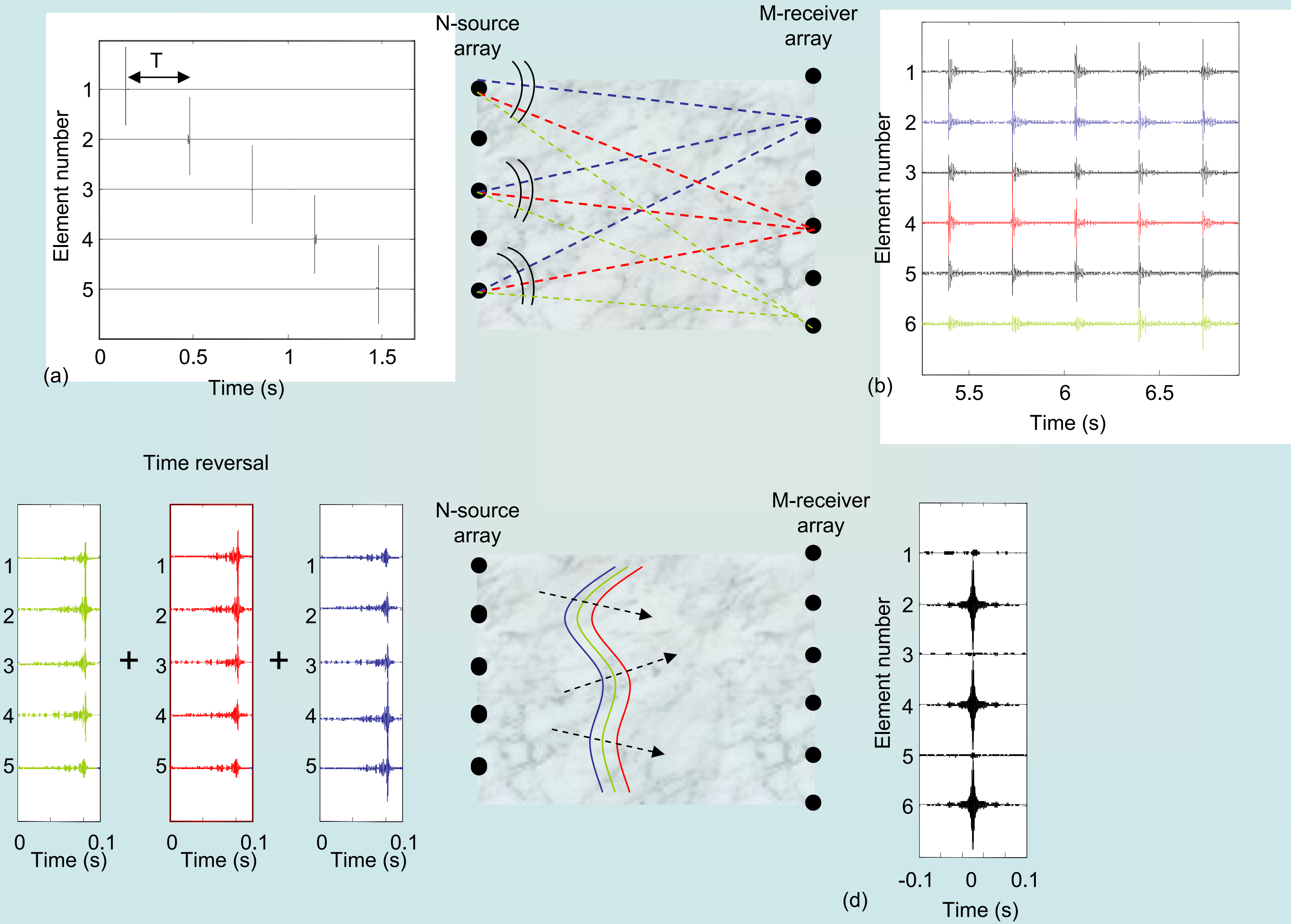




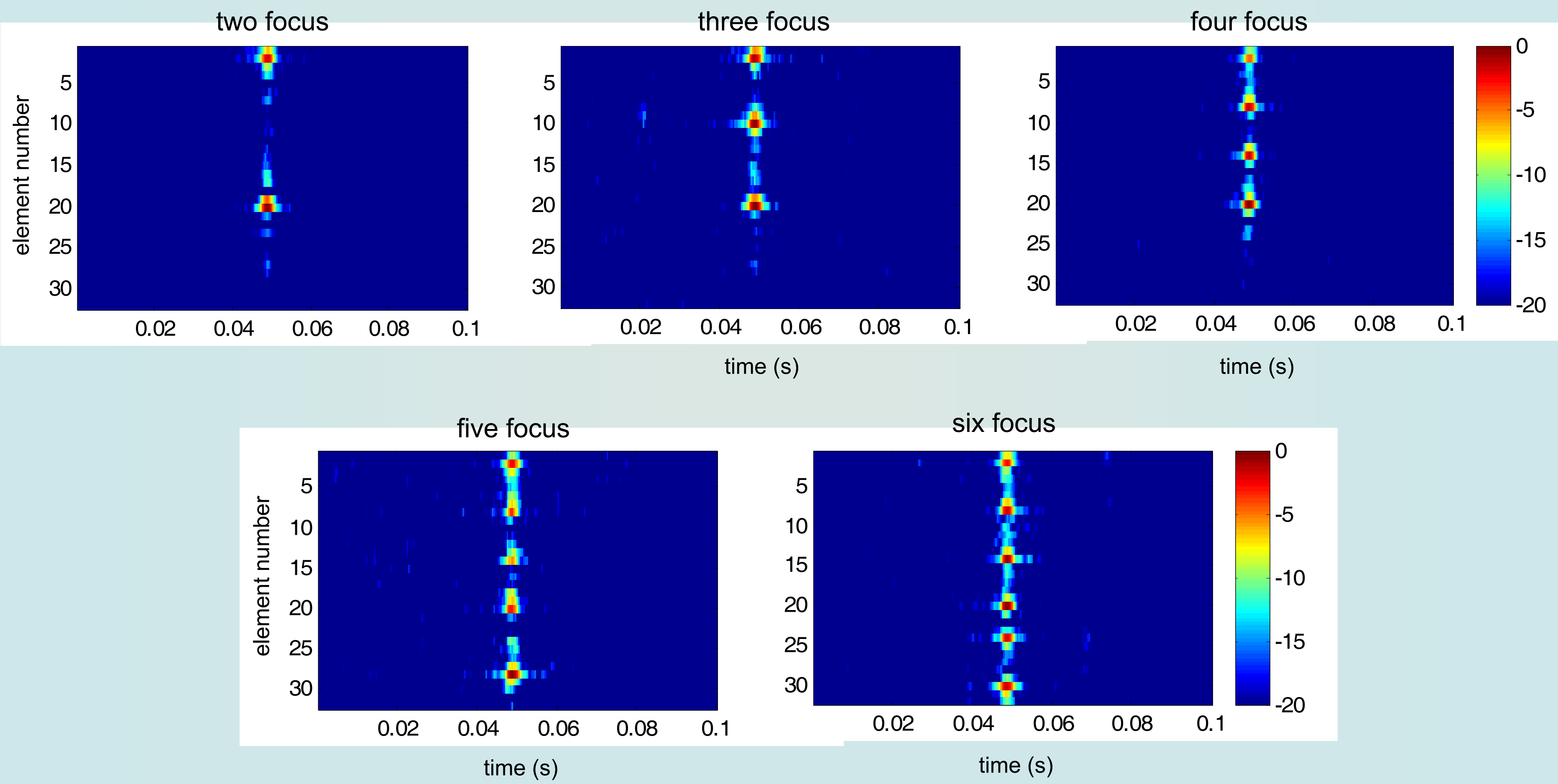
Time Reversal Experiment (2003)



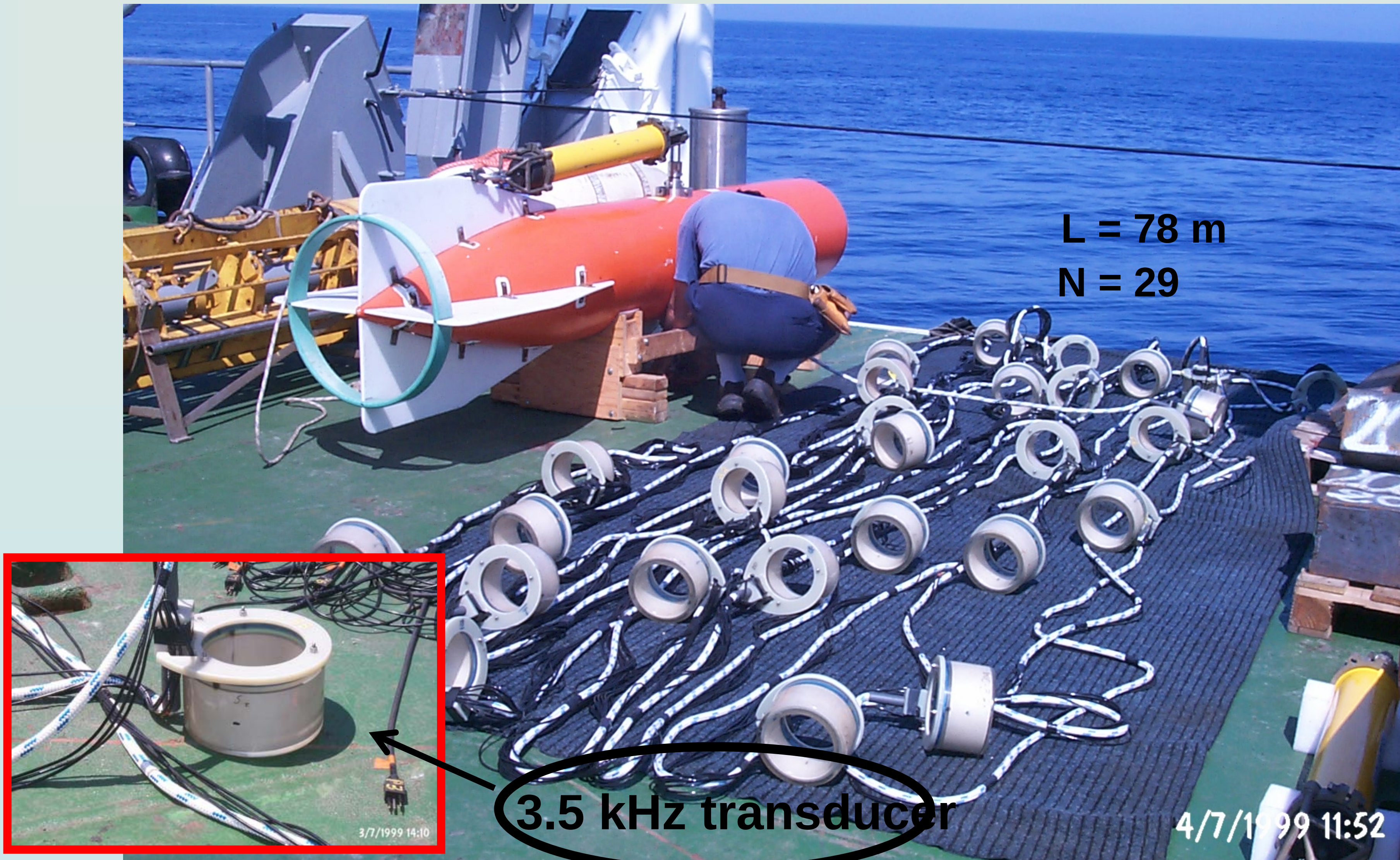
Simultaneous Focus



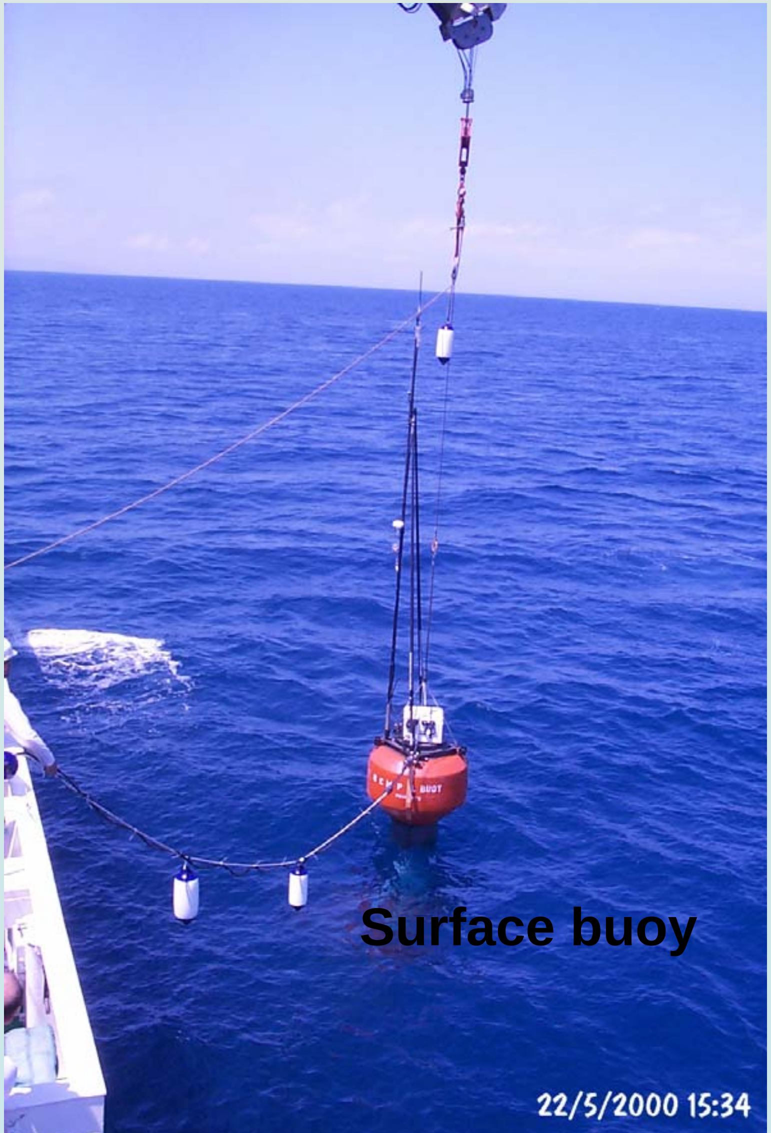
Simultaneous Multiple TR Focus



3.5 kHz SRA

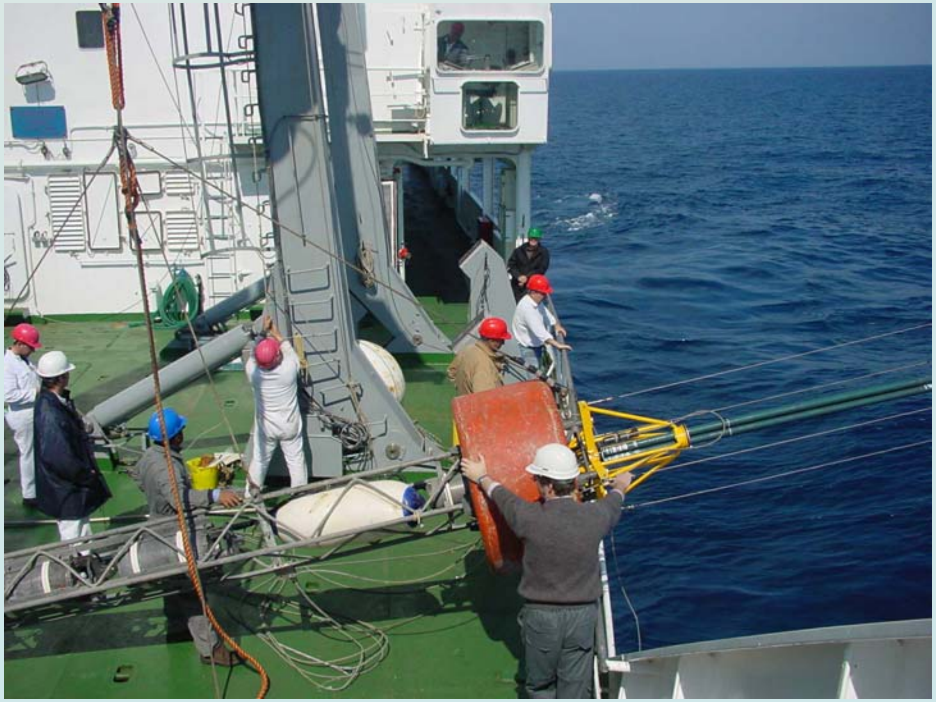


3.5 kHz SRA Deployment



SRA Buoy, PC/LAN, Antennae

Vertical Receive Array (VRA)



Theory and Simulation in Shallow Water Acoustics

Physics of Shallow Ocean Acoustic Propagation

- Physics governed by wave equation:

$$\left(\nabla^2 - \left(\frac{1}{c^2(z)}\right)\frac{\partial^2}{\partial t^2}\right)P(\vec{r},t) = -S(t)\delta^{(3)}\left(\vec{r}-\vec{r}_s\right)$$

- Work in frequency domain and assume azymuthal symmetry (coordinate r,z)

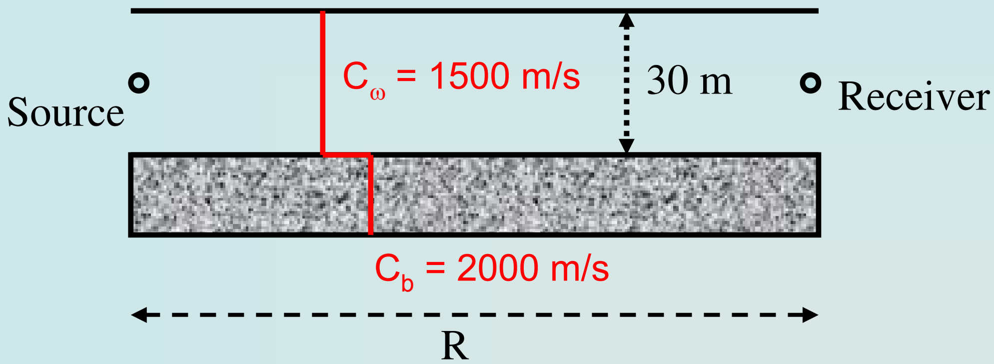
$$\left(\nabla^2 + \frac{\omega^2}{c^2(z)}\right)\tilde{P}(r,z,\omega) = -\tilde{S}(\omega)\left(\frac{\delta\left(r-r_s\right)}{2\pi r}\right)\delta\left(z-z_s\right)$$

- Medium is range-independent ⇨ separation of variables r and z

$$\tilde{P}(r,z) = \int_{-\infty}^{\infty} k\,dk\; g(k,z,z_s)\; H_0^1\left(k(r-r_s)\right)$$

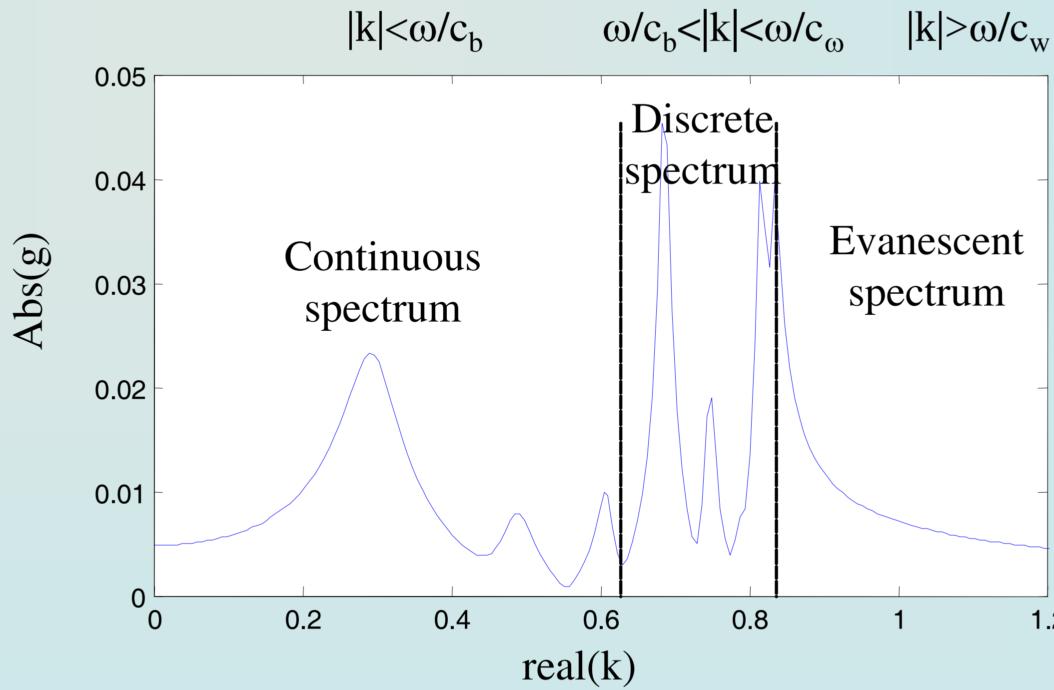
$$\frac{d^2 g}{dz^2} + \left(\frac{\omega^2}{c^2(z)} - k^2\right)g = -\frac{1}{2\pi}\delta(z-z_s)$$

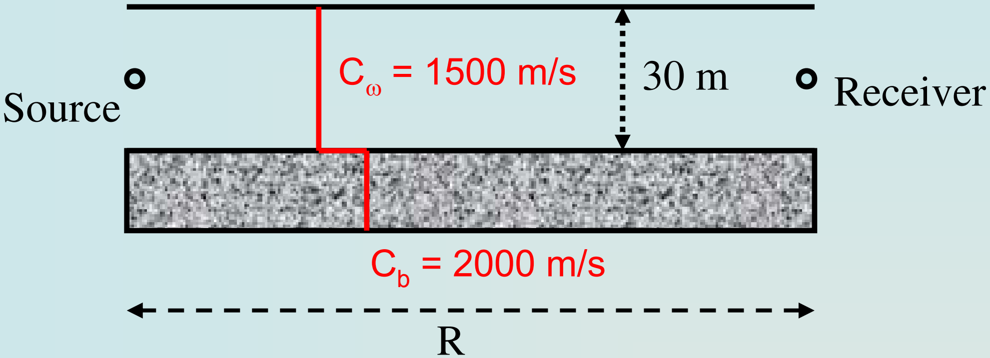
can be solved numerically….



F=15 Hz

R=900 m or 9 λ





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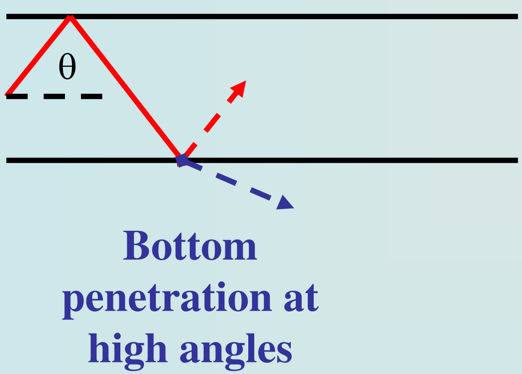

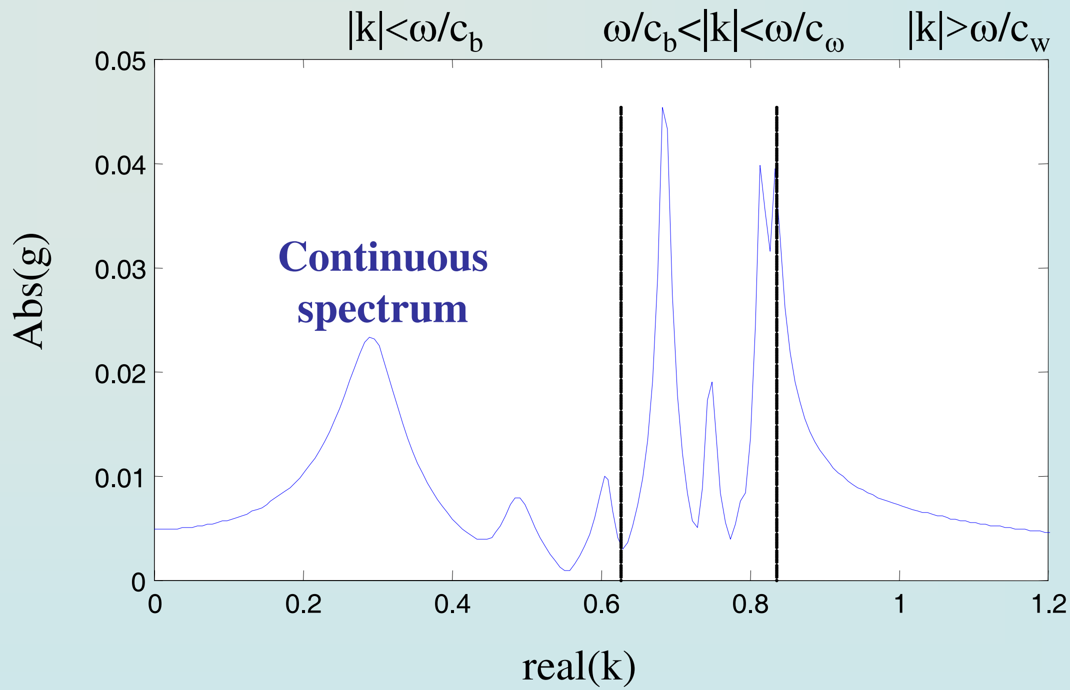

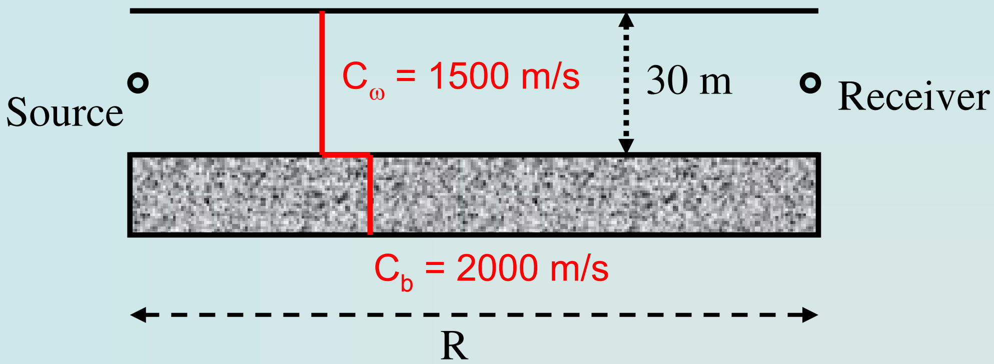

F=15 Hz

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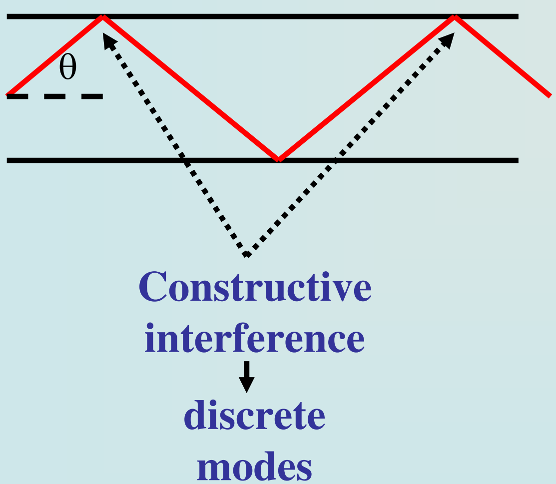

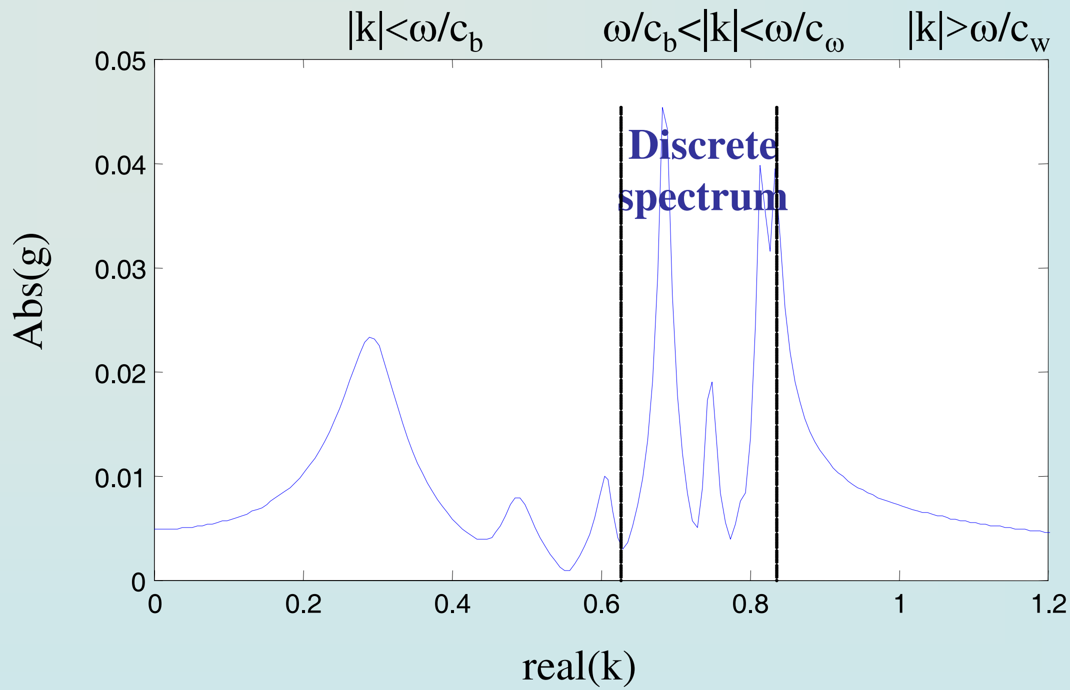

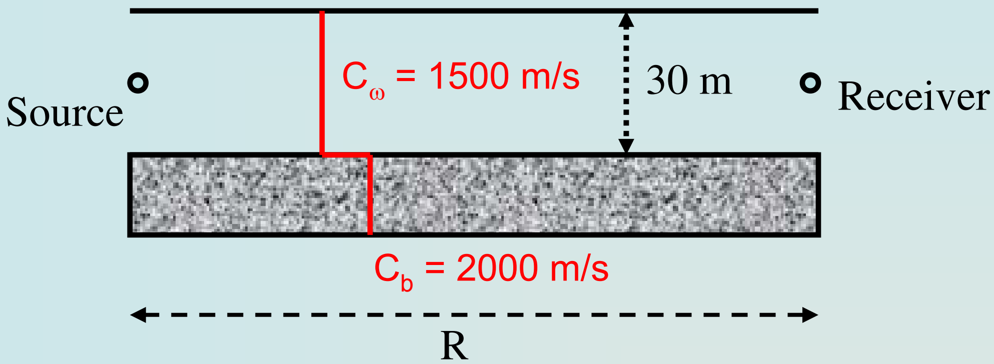

F=15 Hz

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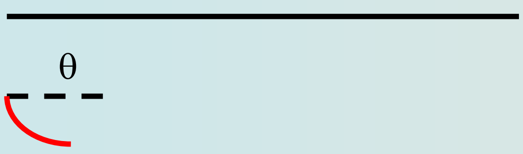
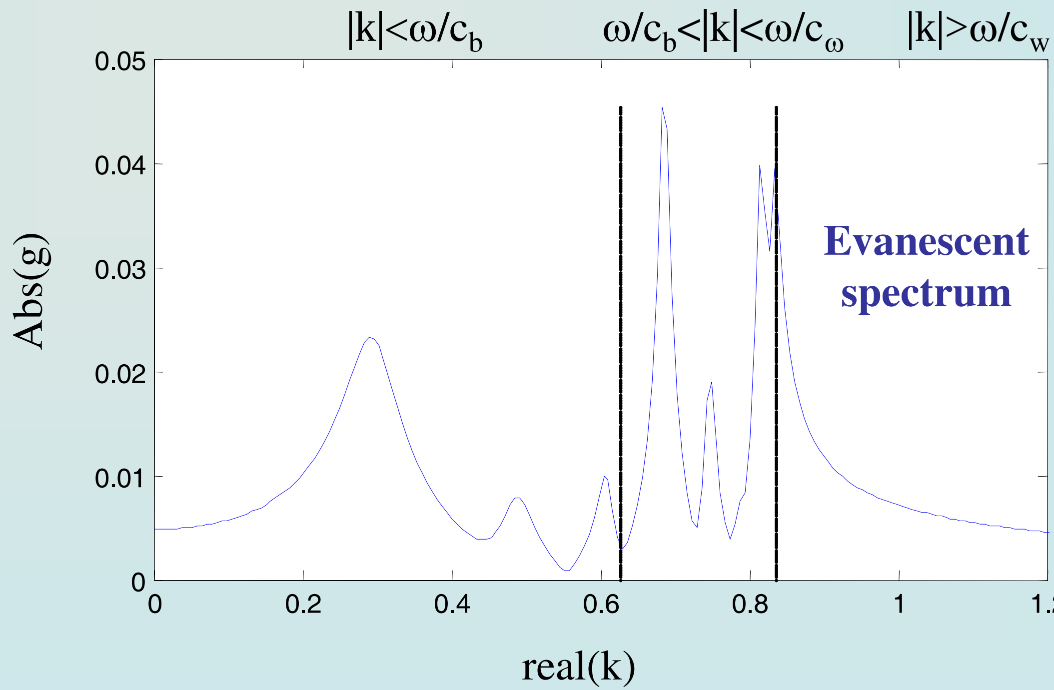
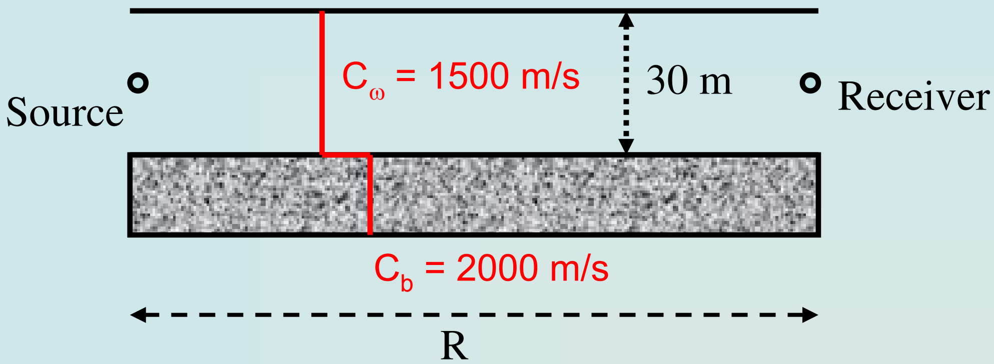


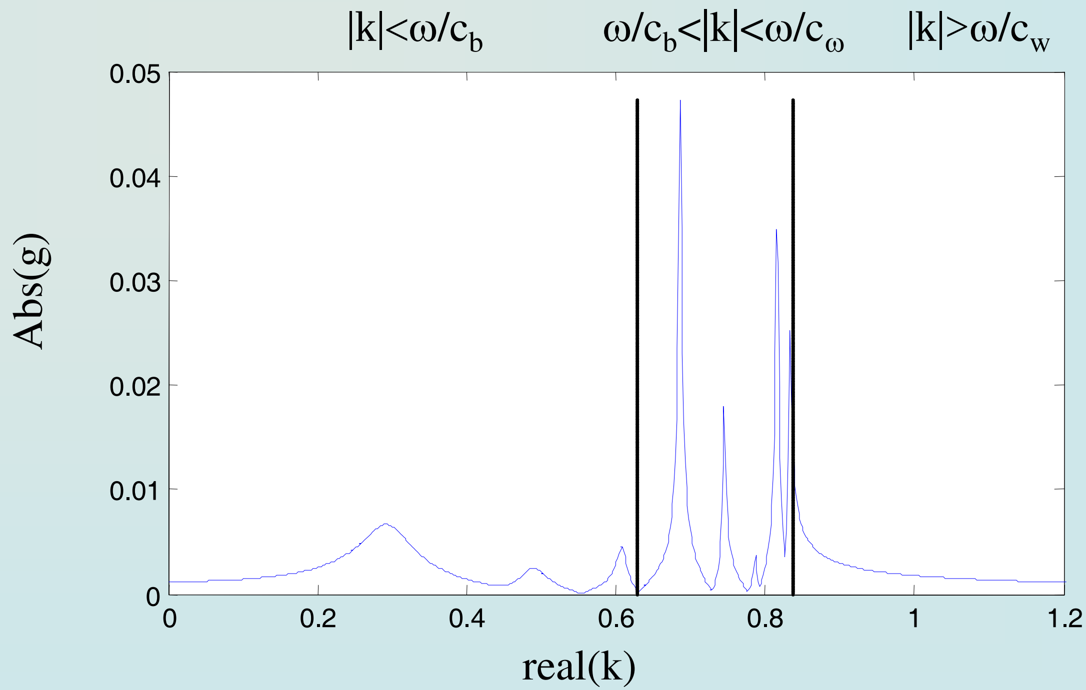
No propagation possible at $c<c_\omega$

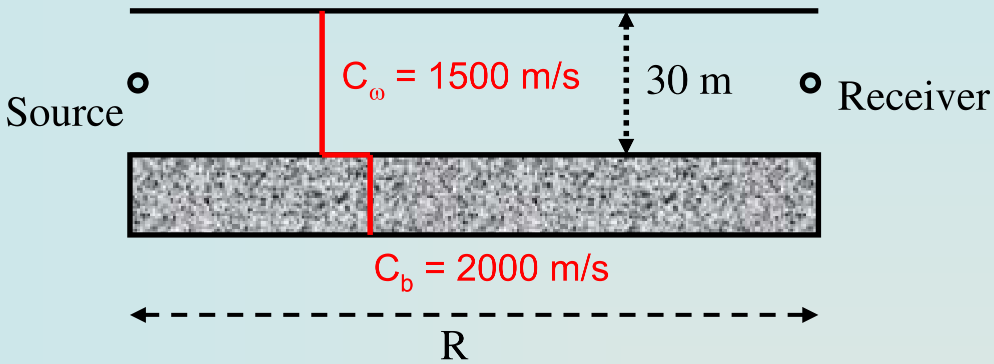




F=15 Hz

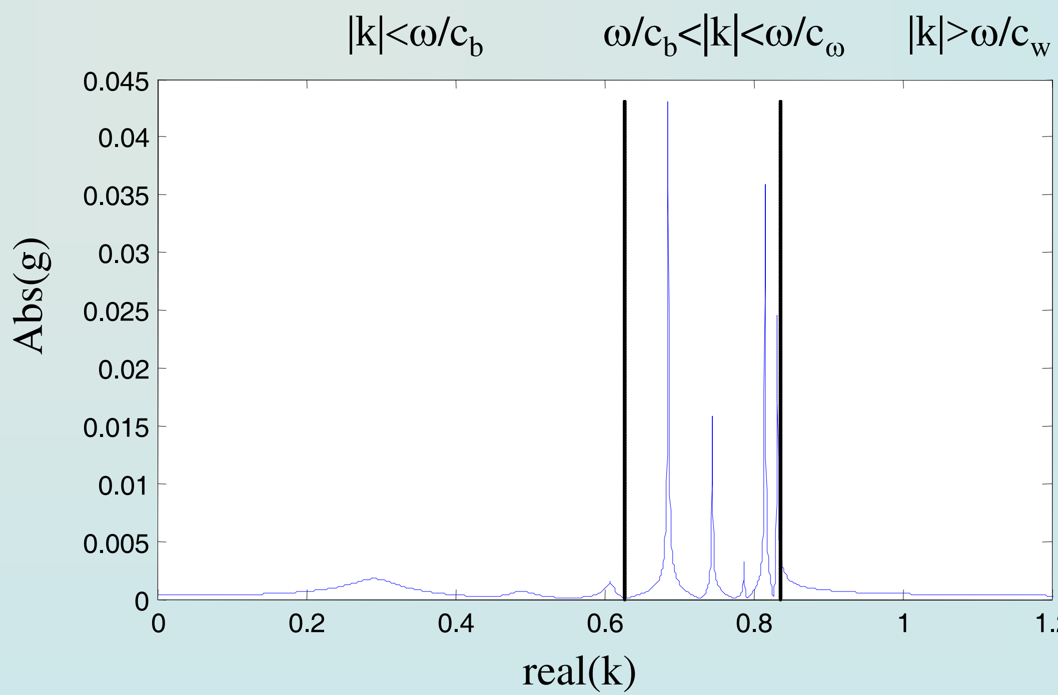
R=3600 m or 36 λ

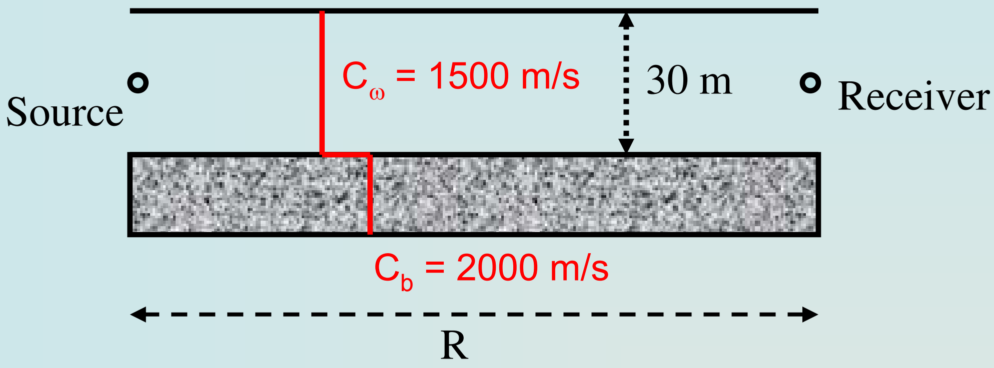




F=15 Hz

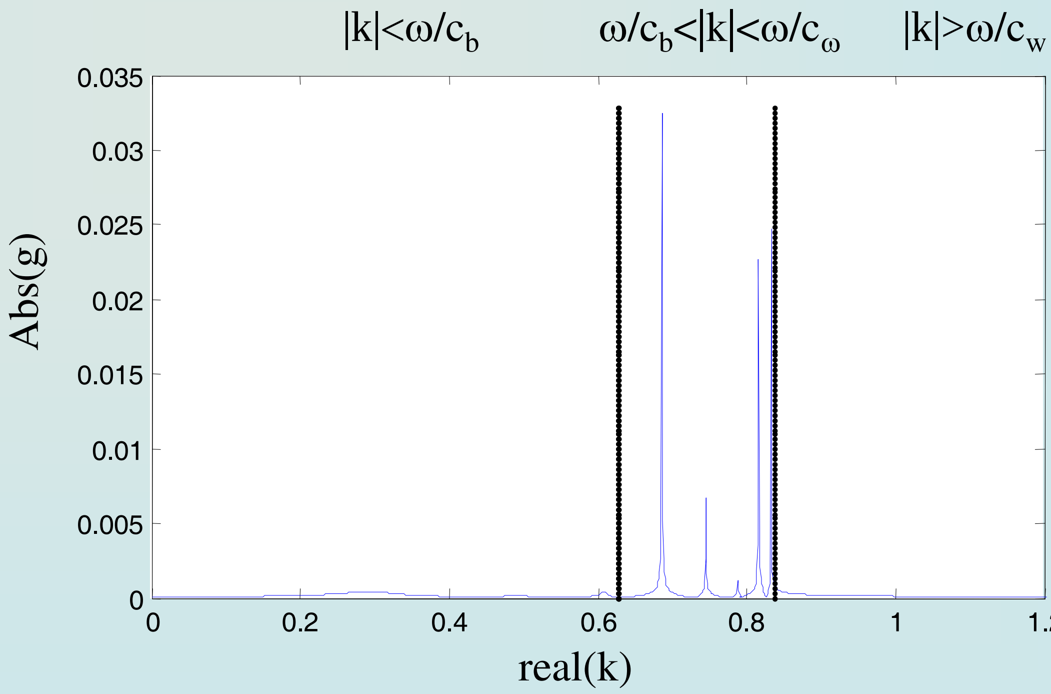
R=14200 m or 142 λ





F=15 Hz

R=56800 m or 568 λ



Physics of Shallow Ocean Acoustic Propagation

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- Work in frequency domain and assume azymuthal symmetry (coordinate r,z)

$$\left(\nabla^2 + \frac{\omega^2}{c^2(z)}\right)\tilde{P}(r,z,\omega) = -\tilde{S}(\omega)\left(\frac{\delta(r-r_s)}{2\pi r}\right)\delta(z-z_s)$$

- Medium is range-independent ⇨ separation of variables r and z

$$\tilde{P}(r,z) = \int_{-\infty}^{\infty} k\,dk\; g(k,z,z_s)\, H_0^1(k(r-r_s))$$

$$\frac{d^2 g}{dz^2} + \left(\frac{\omega^2}{c^2(z)} - k^2\right)g = -\frac{1}{2\pi}\delta(z-z_s)$$

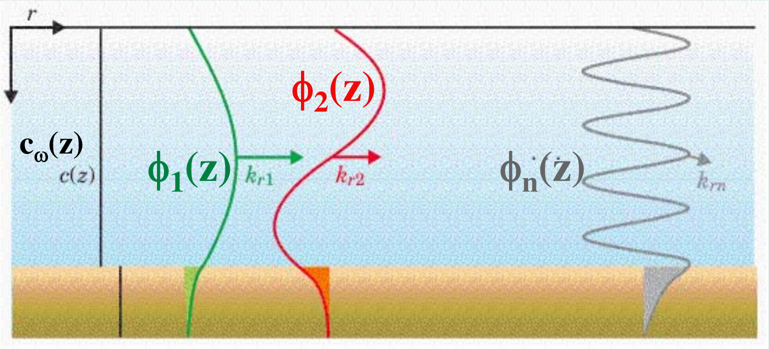
- Waveguide propagation + long range ⇨ modal decomposition

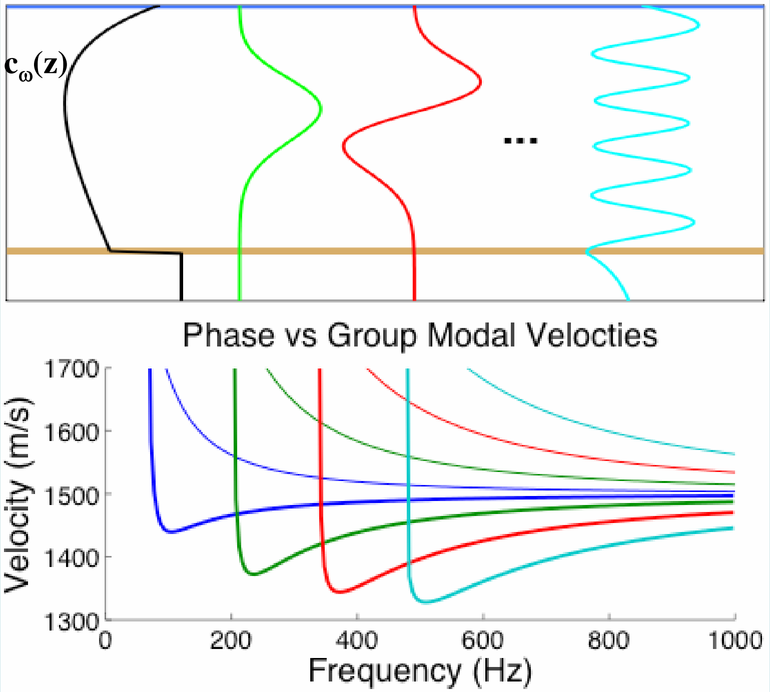
$$g(k,z) = \sum_{n=1}^{N} a_n(k)\,\Phi_n(z)$$

$$\frac{d^2\Phi_n}{dz^2} + \left(\frac{\omega^2}{c^2(z)} - k_n^2\right)\Phi_n(z) = 0$$

Summary of mode properties

- Modes arise from constructive interference in waveguide
- Eigen-value solutions to depth equation
- Mode shapes and wavenumbers depend on $c(z) = [c_\omega(z)\ c_b(z)]$
- Orthogonal over half space below water surface
- Travel at different velocities so that they disperse with time/range
- Attenuate with range due to bottom interactions
- Invariant in range

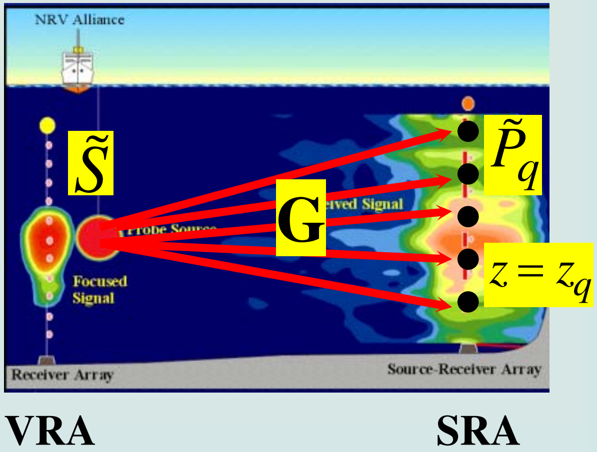




Modal propagation

- Pressure field : Green's function convolved by source spectrum

$$\tilde{P}_q(z_s, r, \omega) = G(z_q, z_s, r - r_s, \omega)\, \tilde{S}(r_s, z_s, \omega)$$



- Acoustic field propagates as depth dependent modes

$$G(z_q, z_s, r - r_s, \omega) = \left(\frac{i}{\sqrt{8\pi}\rho(z_s)} \sum_{n=1}^{N} \phi_n(z_q, \omega)\phi_n(z_s, \omega) \frac{e^{-i(k_n(\omega) - i\alpha_n(\omega))(r - r_s)}}{\sqrt{k_n(\omega) r}} \right)$$

depth dependence

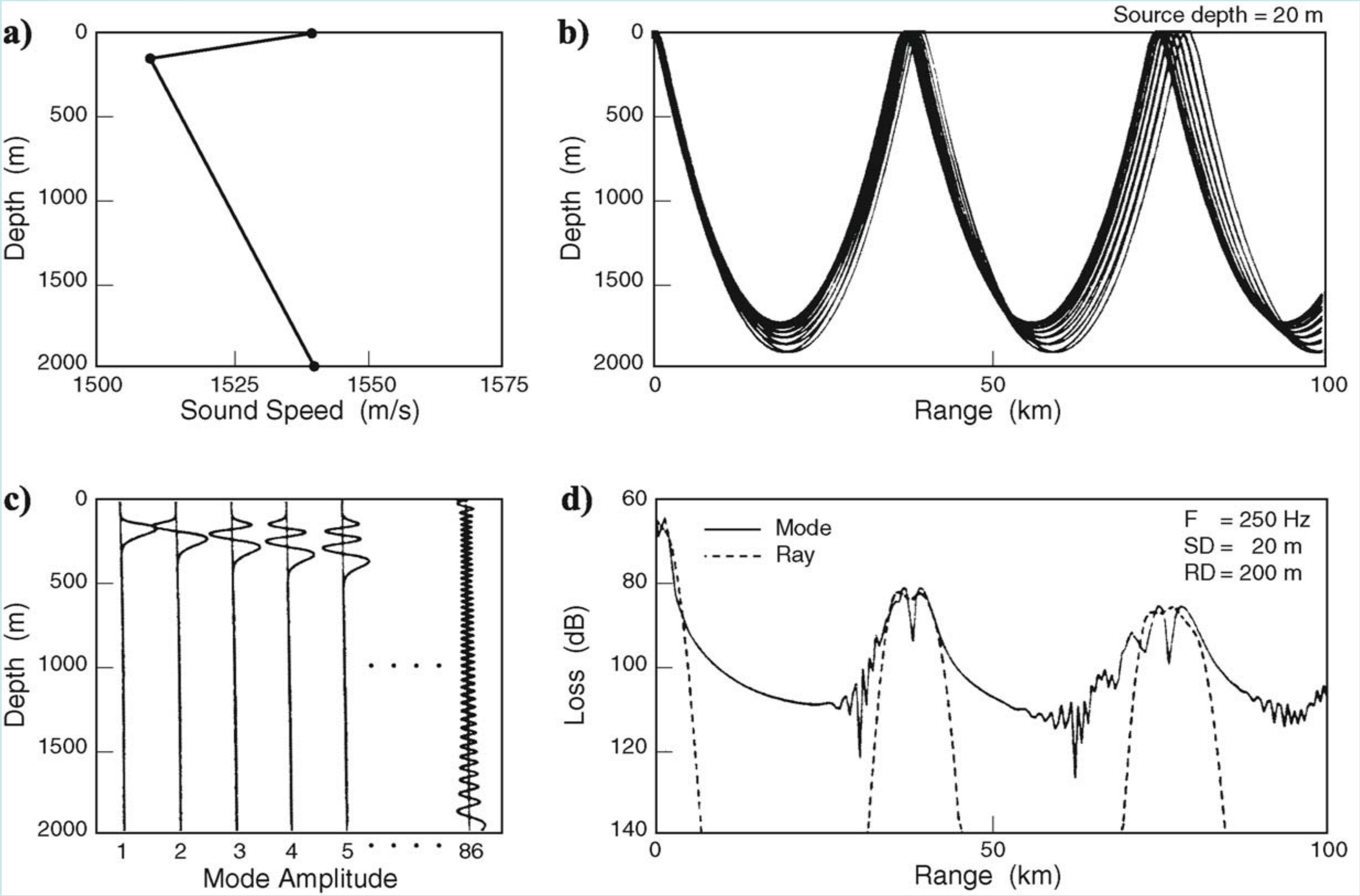
phase

modal attenuation

modal excitation

cylindrical spreading

Model consistency : modes and rays



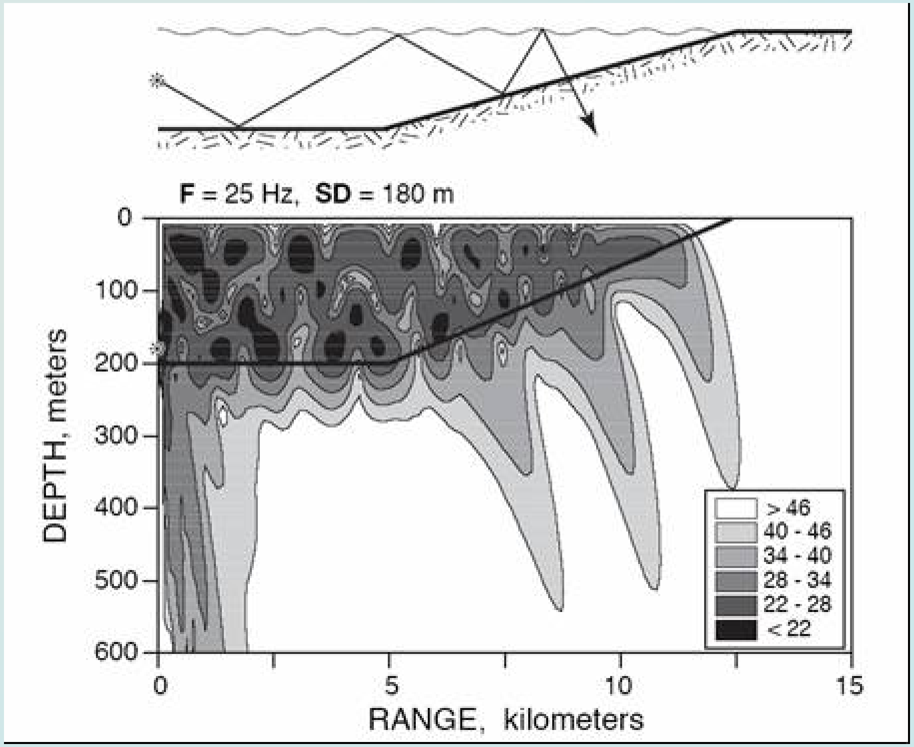
The most advanced simulation: Parabolic Equation

$$\frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} = \left(\frac{\partial}{\partial x} - \frac{1}{c}\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial x} + \frac{1}{c}\frac{\partial}{\partial t}\right)$$

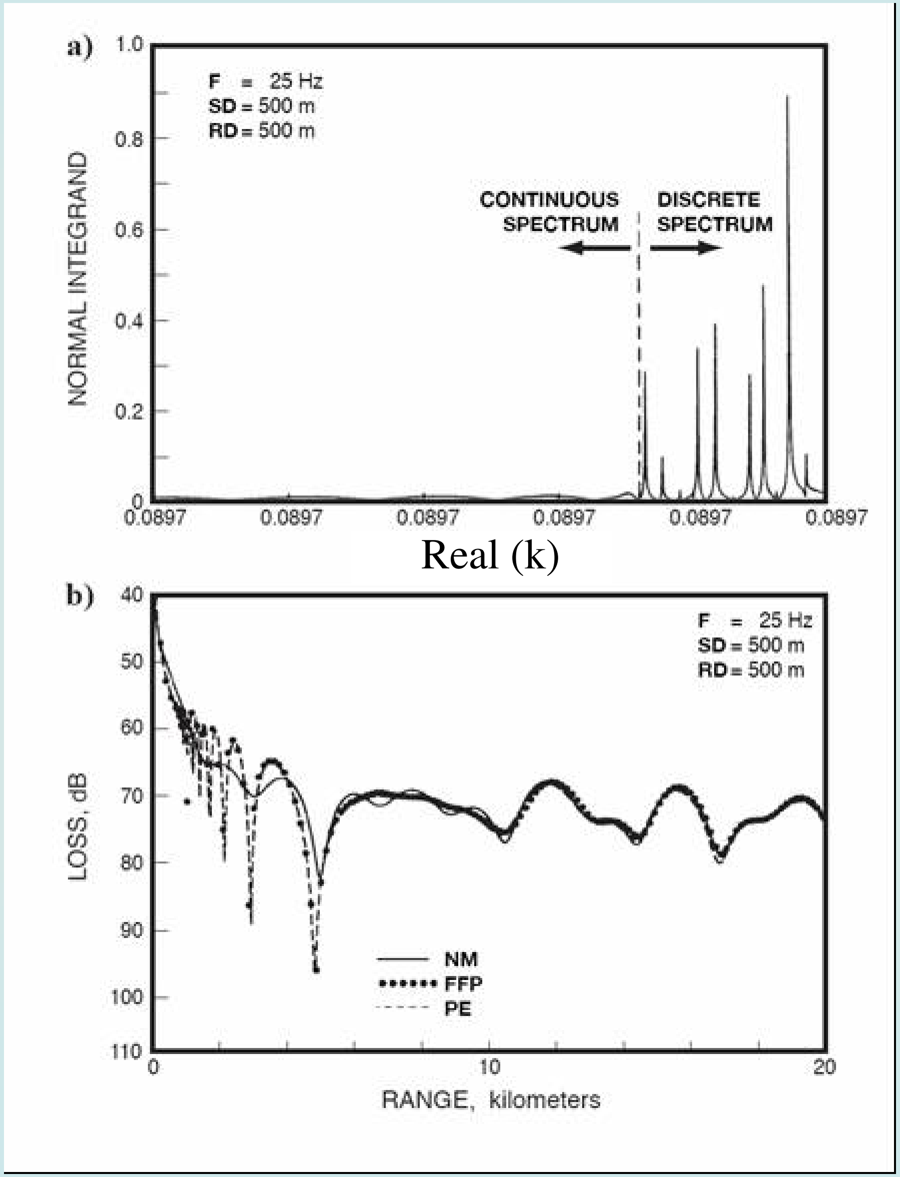
\+ paraxial approx.

⇨ small angle PE (Tappert, 1977)

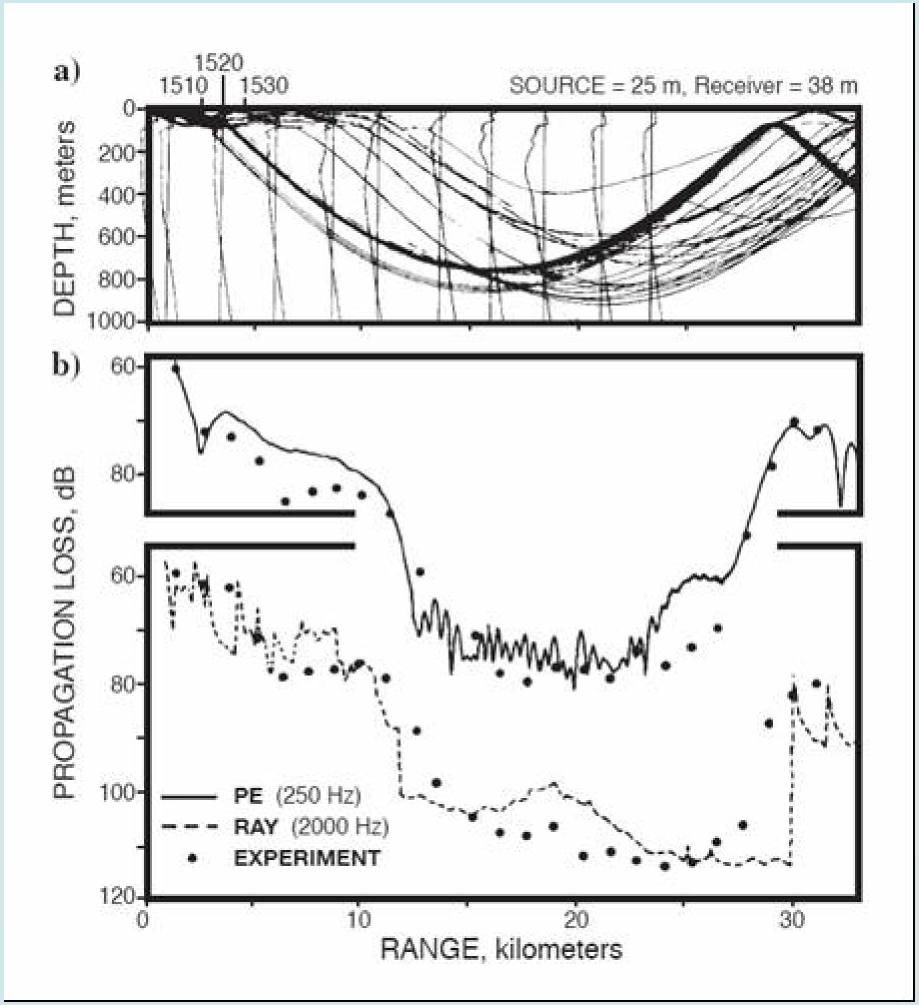
⇨ high-angle PE + energy conservation (Collins, 1991)



Simulation results



Comparison with data



What simulation to use?

Wave equation

Freq~ 10 Hz -10,000 Hz

L~0.1 km-10,000 km

H~10 m – 5000 m

Range indep.

Spectral code

- Shallow water
- $H/\lambda<100$
- Short range
- $L/\lambda<100$
- Freq~10 Hz -10 kHz

Normal mode

- Shallow water
- $H/\lambda<100$
- Long range
- $L/\lambda>>10$ & $L/H>>10$
- Freq~10 Hz – 10 KHz

Range dep.

Ray

- Deep water
- Very long range
- $L/\lambda>>10000$
- High Freq
- Large bandwidth

PE

- Shallow/Deep water
- All range
- $L/\lambda>10$
- All Freq

Finite Difference

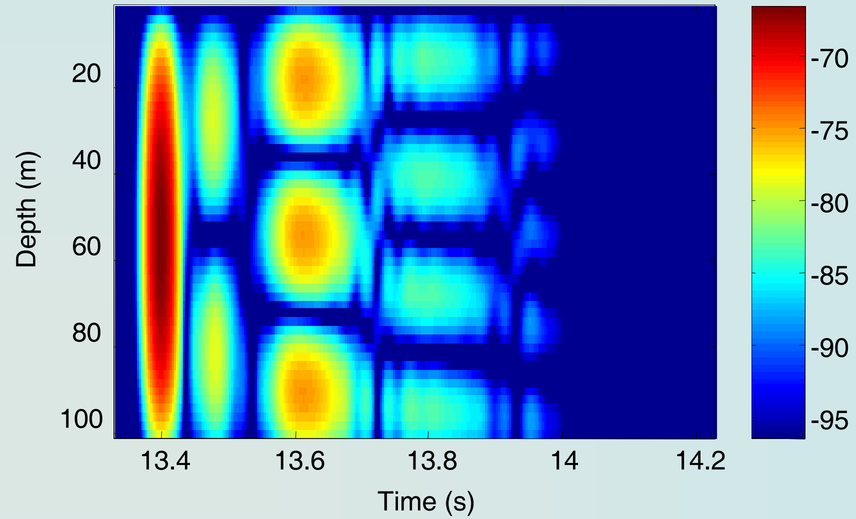
- Shallow/Deep water
- Very short range
- $L/\lambda<100$
- All Freq

⇨ **Gain in efficiency : coupling between simulations**

Shallow water vs Deep water propagation

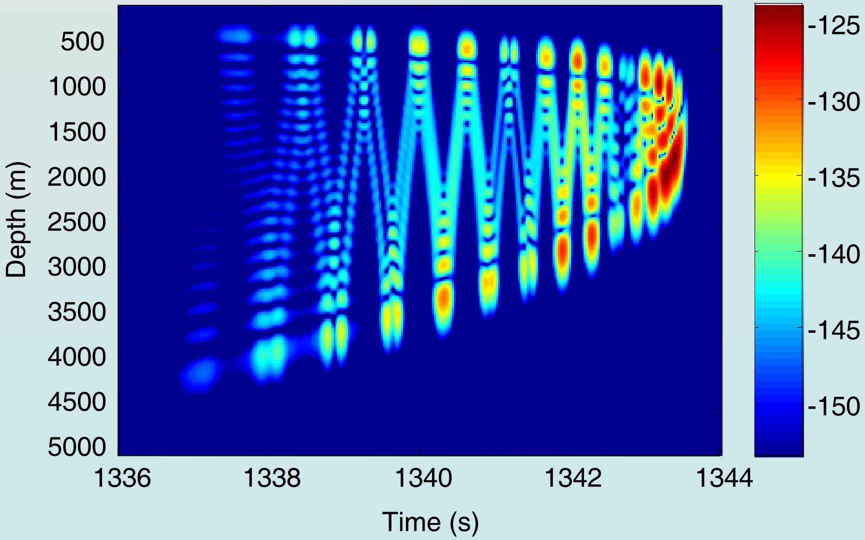
Acoustic propagation dominated by reflection

Shallow water
- Water depth=100 m
- Range=10 km
- Freq=75-125 Hz

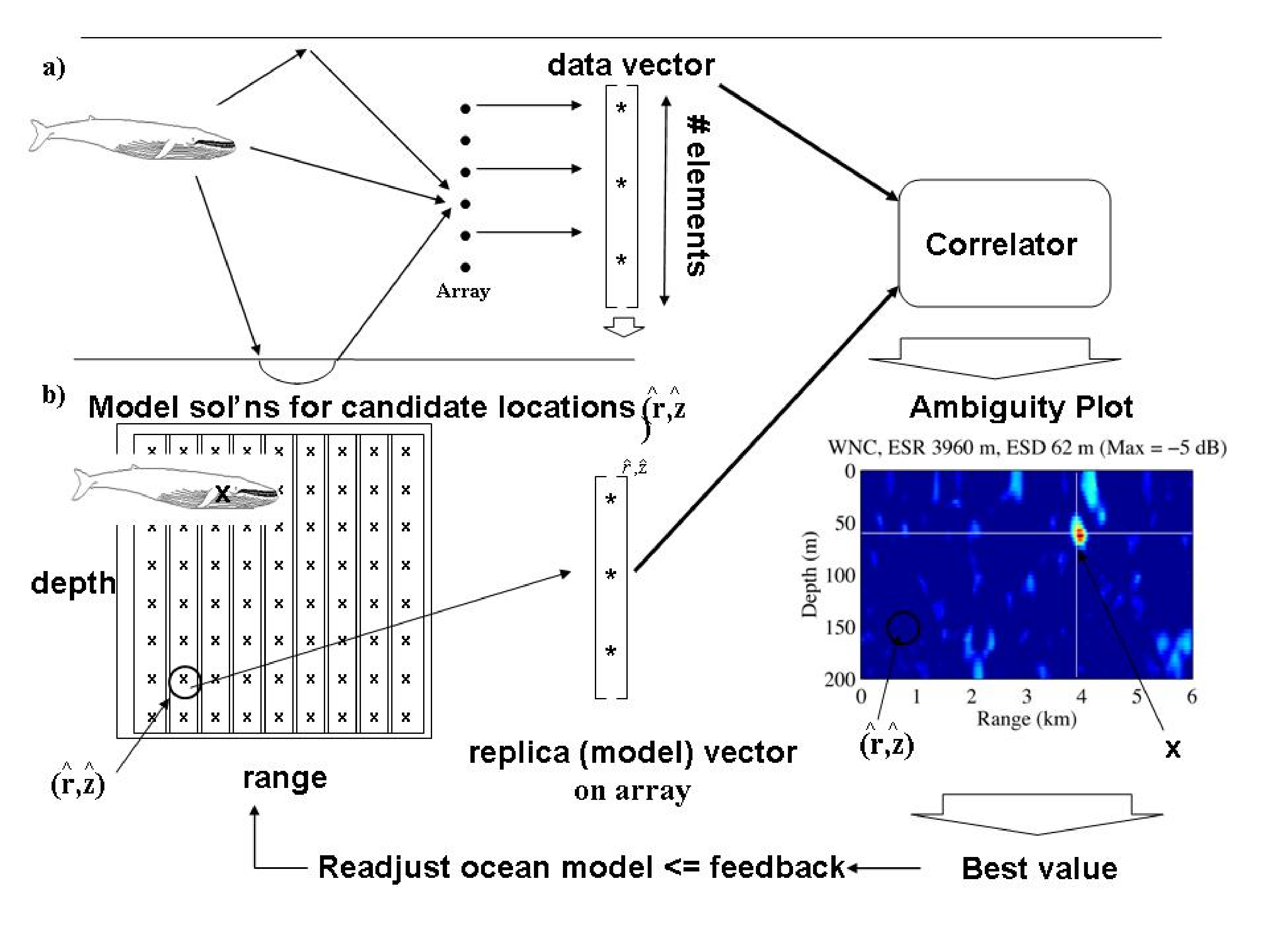


Acoustic propagation dominated by refraction

Deep water
- Water depth=5000 m
- Range=2000 km
- Freq=20-40 Hz

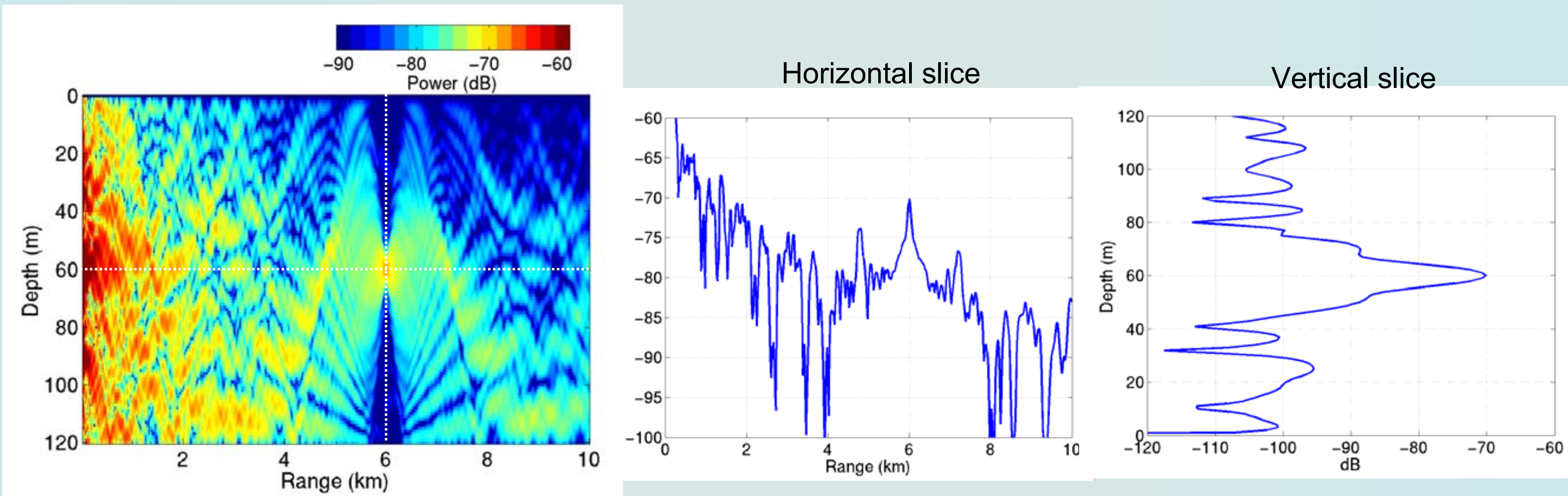


Matched-field processing (MFP)



Time Reversal vs Matched-Field Processing

Focused field (1 kHz, 6 km, 60 m)



Time-reversal (TR) vs Matched-field processing (MFP)

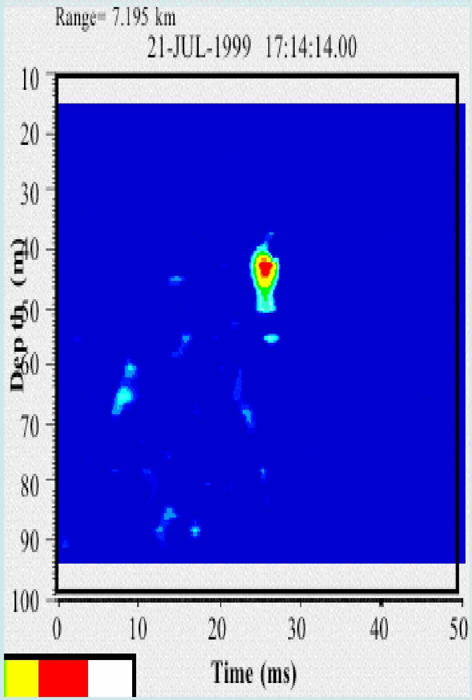
MFP: $$\left|W^{+}(r)D(r_s)\right|^2 = W^{+}(r)K(r_s)W(r)$$

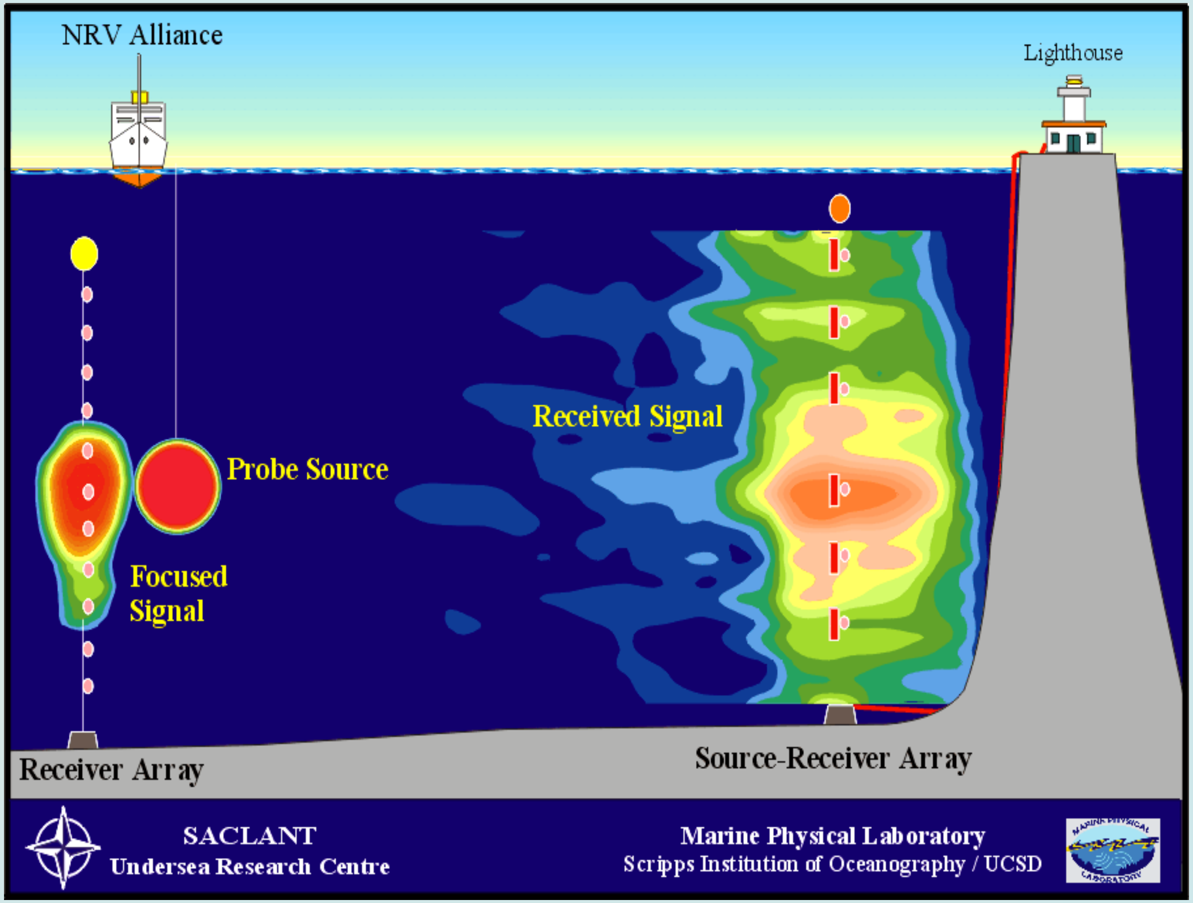
TR: $$G^{+}(r_{trm}; r_s)G(r;\ r_{trm})$$

W : Normalized computer generated replica fields

G : Ocean generated replica field (Green's function)

TR versus MFP ?





Focused pulse back at source

! MFP does not work at 3 kHZ for L >2 km !

Exploratory works based on laboratory self-adaptive experiments

Why laboratory experiments ?

- Signal processing techniques for detection, communications, and tomography require an accurate model of the environment
  - Complicated by multi-path arrivals $\rightarrow$ too many parameters
- Fluctuations due to wind, surface waves, and source motions complicate signal processing
- Characterizing the ocean environment is expensive, impractical, and often impossible

Outline

Mode extraction over a single partial-spanning vertical array

- Use a single partial-spanning vertical array and range sampled field to extract modes without modeling
- Use modes to depth shift Time-Reversal focus

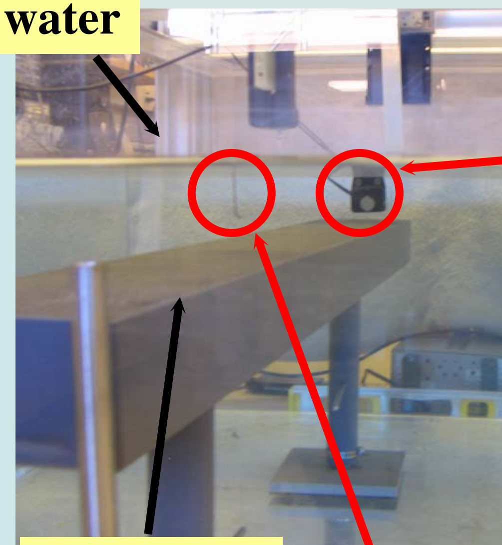
Huygens principle applied to Time-Reversal imaging

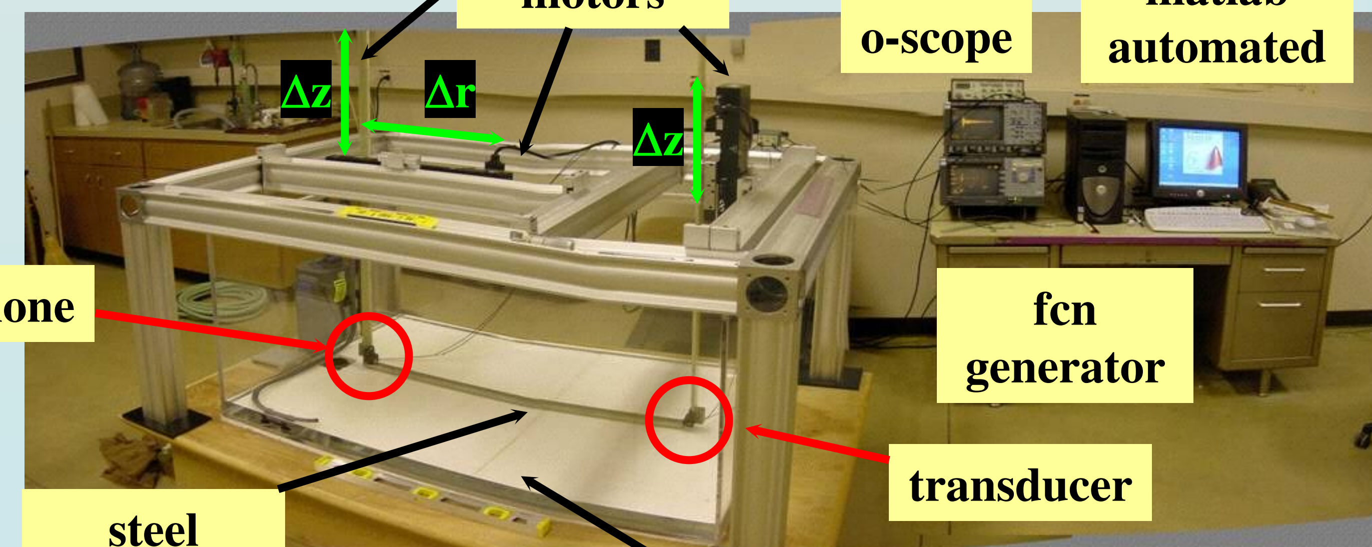
- Use the pressure response between 2 vertical arrays to produce Time-Reversal foci at arbitrary locations without knowledge of the environment

Ultrasonic experiment

- We test our theories in a scaled down waveguide at ultrasonic frequencies

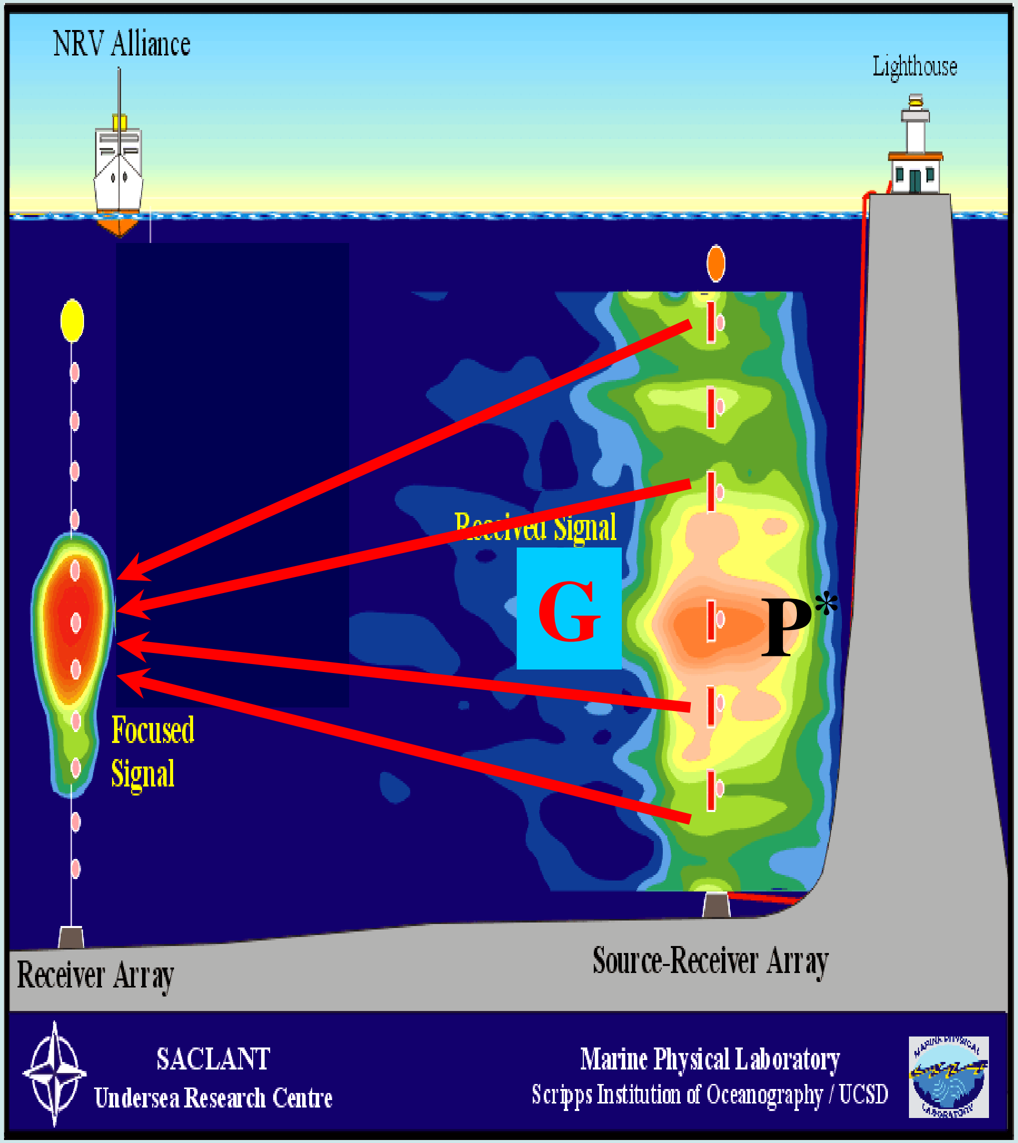
$$\left(\frac{\lambda}{D},\frac{D}{R}\right)_{\exp} \approx \left(\frac{\lambda}{D},\frac{D}{R}\right)_{ocean}$$





Focal depth shifting of a time reversal mirror in a range-independent waveguide

TR in the shallow ocean



Pressure field on array

$$P_q(z',r,\omega) = \underline{G(z_q,z',r,\omega)}\tilde{S}(\omega)$$

Modal depth-dependence

$$P_q(z',r,\omega) = \sum_{m=1}^{N} \underline{\phi_m(z_q,\omega)\phi_m(z',\omega)}\,\psi_m(r,\omega)$$

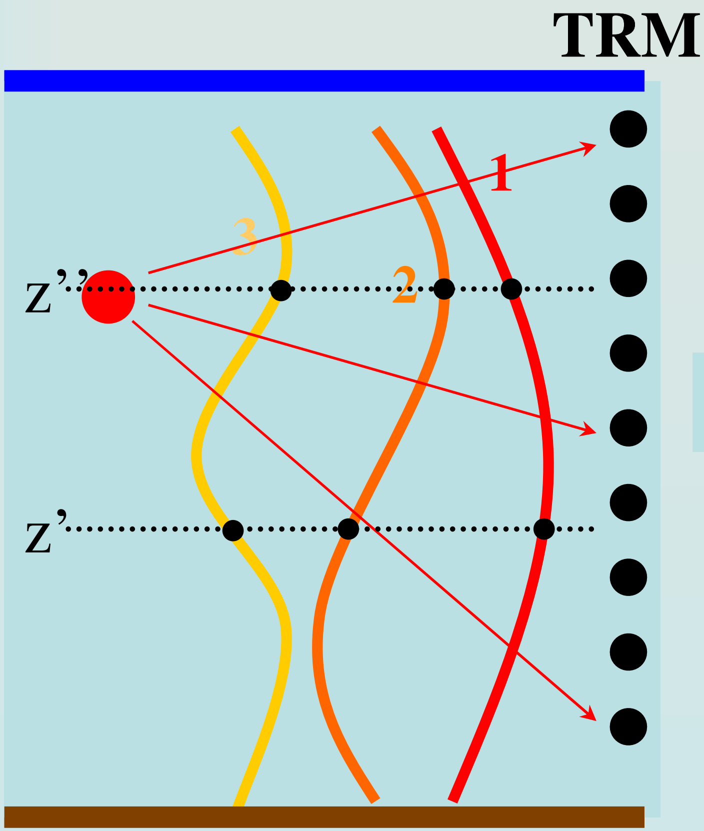
Range-dependent phase

$$\psi_m(r,\omega) \equiv \left(\frac{i}{\sqrt{8\pi}\rho(z_s)}\right)\frac{\boxed{e^{-i(k_m(\omega))r}}}{\sqrt{k_n(\omega)r}}\tilde{S}(\omega)$$

Time-reversal focus

$$\sum_{q=1}^{N_q} \underline{G(z_q,z,r,\omega)}P_q^*(z',r,\omega) \approx \boxed{\delta(z-z')}$$

Modal remodulation depth shifting



depth-dependent modal excitations

$$P_q(z',r,\omega) = \sum_{m=1}^{N} \phi_m(z_q,\omega)\underbrace{\phi_m(z',\omega)}\,\psi_m(r,\omega)$$

$P_q(z' \rightarrow z'')$

Modes must be known

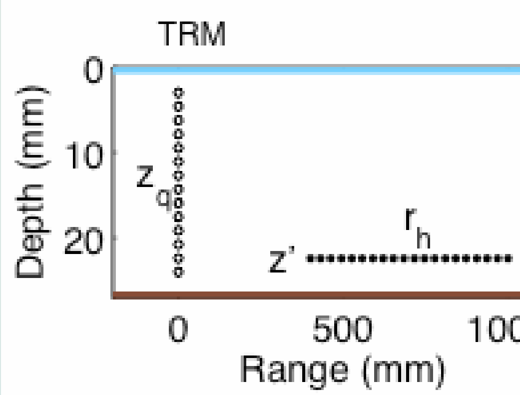
$$P_q(z' \rightarrow z'',r,\omega) =$$
$$\sum_{m=1}^{N} \phi_m(z_q,\omega)\underbrace{\left(\frac{\phi_m(z'',\omega)}{\phi_m(z',\omega)}\right)}\phi_m(z',\omega)\,\psi_m(r,\omega)$$

Modes must be uncoupled

modal remodulation
z'→ z''

Frequency-wavenumber (f-k) structure

- Measure pressure fields from broad-band sources at many ranges

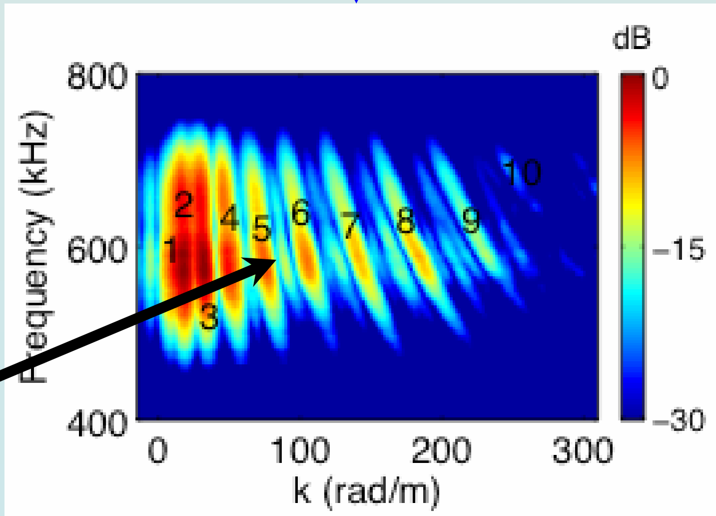


$$F_c = 600\,kHz$$
$$\Delta F = 400\,kHz$$
$$\lambda_c = 2.5\,mm$$

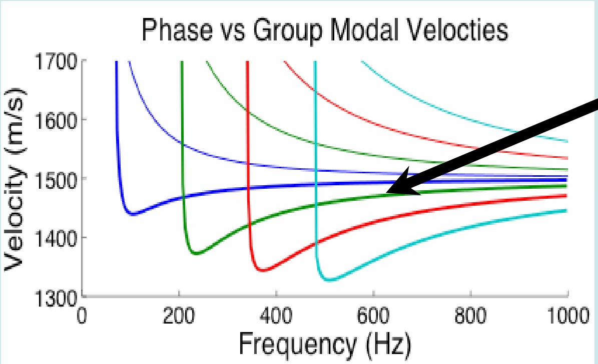
Fourier Transform

$$\tilde{P}_q(z',k,\omega) = \frac{1}{2\pi}\iint drdt\, P_q(z',r,t)e^{-i(kr+\omega t)}$$

- To construct f-k structure of the waveguide due to source at z'

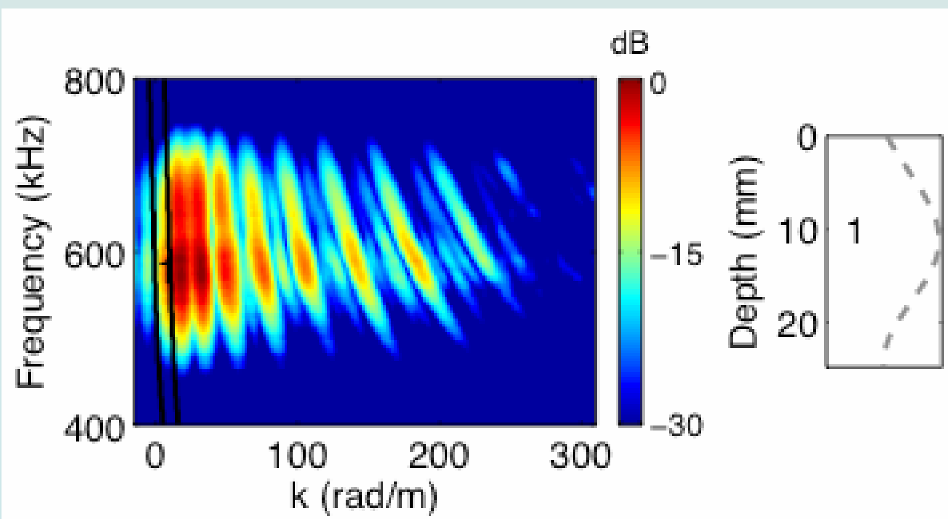


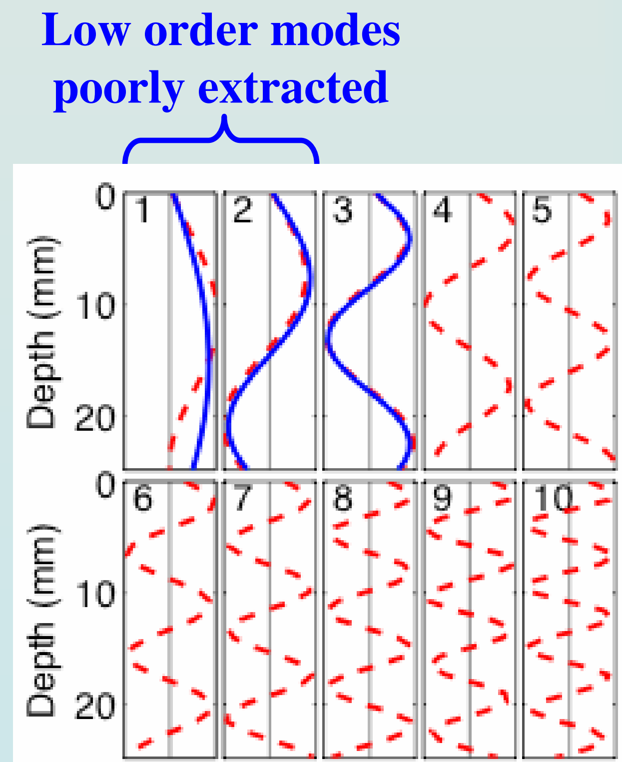
Dispersion due to different modal velocities



Mode extraction

Modes are isolated (masked) and extracted individually using SVD

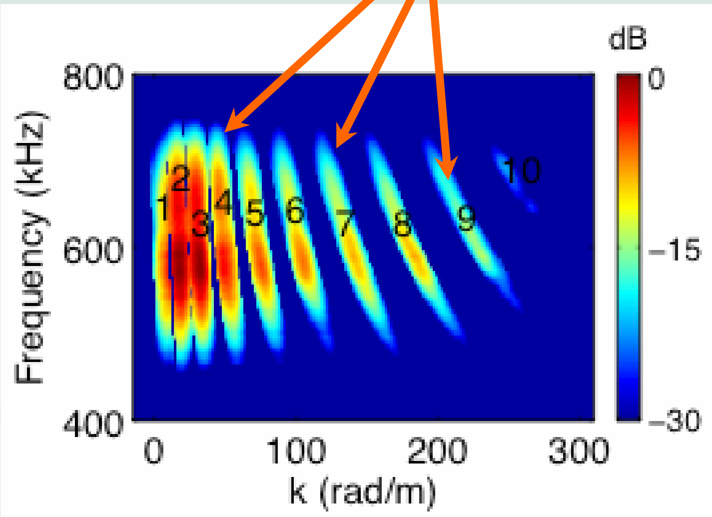




Extracted modes

Depth shifted TR foci

$$\left(\frac{\phi_m(z'')}{\phi_m(z')}\right)$$



- Use extracted modes to remodulate mask regions of f-k structure

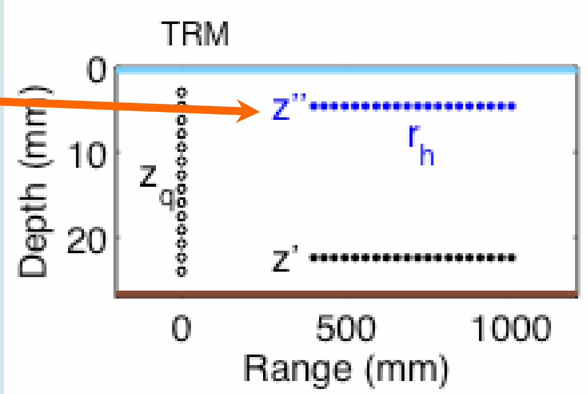
↓

Inverse Fourier Transform

$$P_q(z',r,t)=\frac{1}{2\pi}\iint d\omega dk\,\tilde{P}_q(z',k,\omega)e^{i(kr+\omega t)}$$

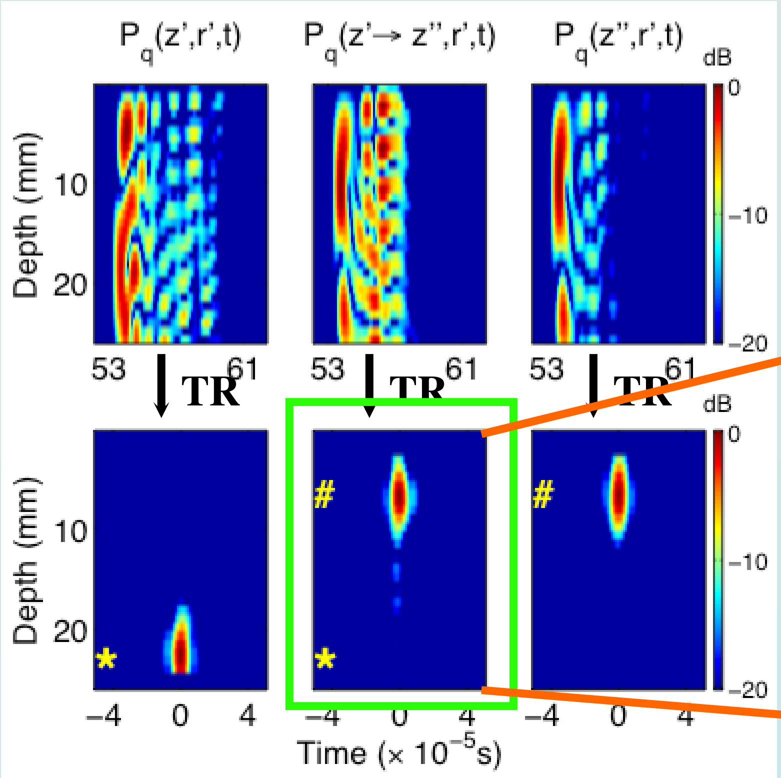
↓

- Can produce depth shifted TR foci at all original source ranges



Experimental DS results at many depths

Depth shifted TR focus at a single range vs TR focus control

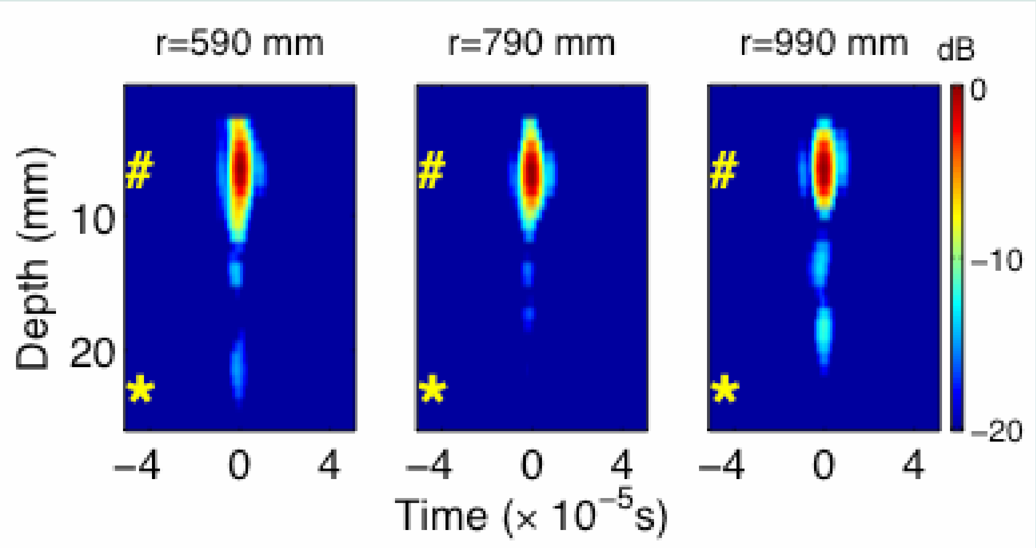


Depth shifted TR focus at many depths

QuickTime™ and a
decompressor
are needed to see this picture.

Experimental DS at many ranges

Depth shifted TR foci are produced at any depth **at all initial source ranges**



Conclusions

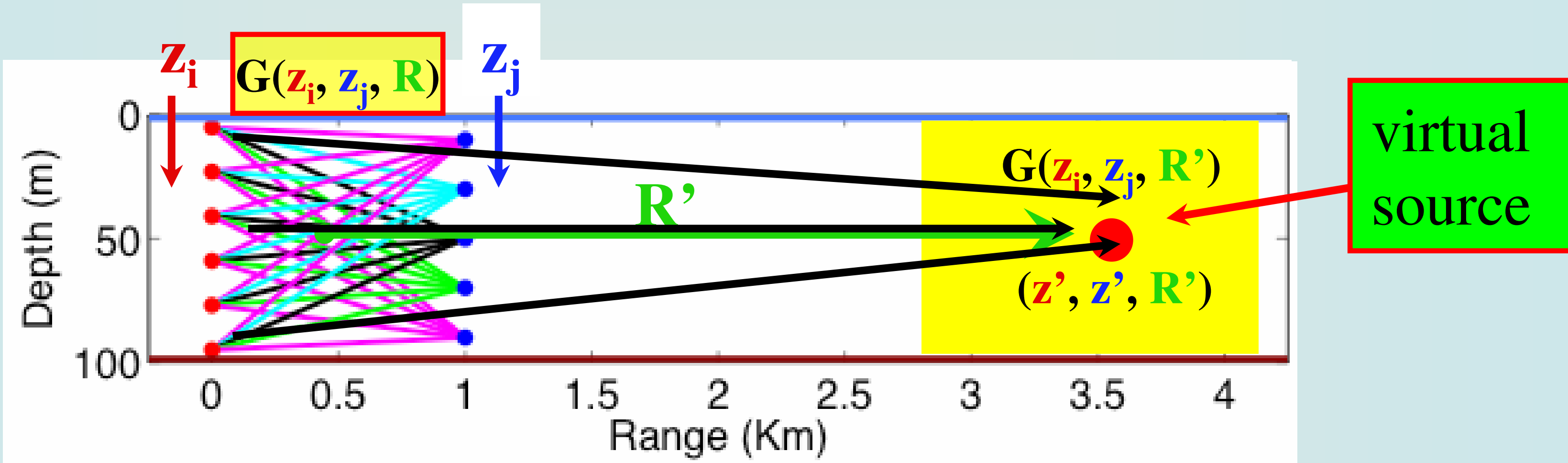
Self-adaptive ocean acoustics

Extract information from the data and apply manipulations to the same data set without modeling or inversions

- Time reversal
  - No information other than detection, requires a source
- Dispersion based mode extraction
  - **Works on partial-spanning arrays**
  - Requires moving source
- Extracted modes can be applied to TR
  - **Focal spot manipulation**
  - **Communications and sonar applications**

Huygens principle applied to TR imaging

Problem definition



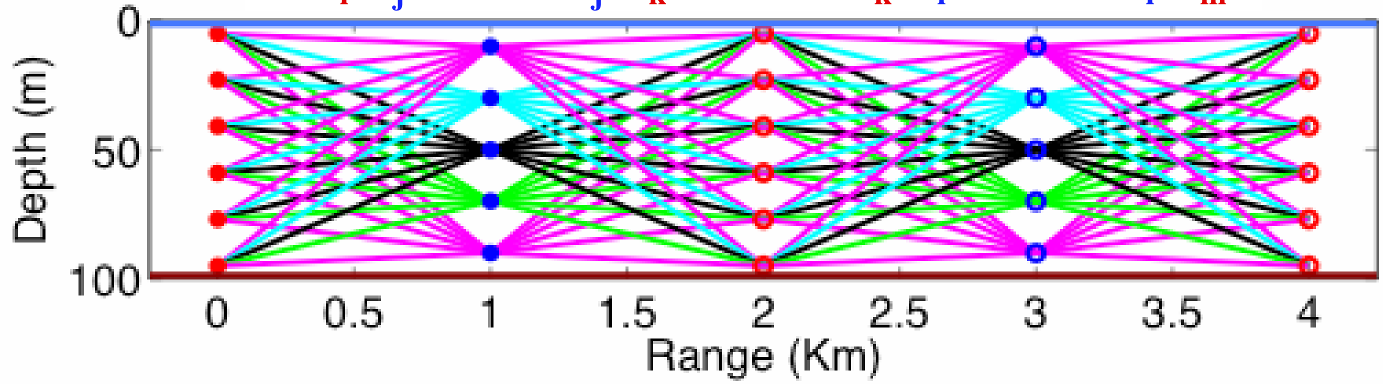
- Shallow ocean **range independent**, **stationary** waveguide
- A pair of vertical line arrays z_i, z_j, separated by distance **R**
- Use $G(z_i, z_j, R) \Rightarrow G(z_i, z_j, R')$
- Imaging at (z', z', R')

Time-reversal requires a signal from the source location

Huygens Principle: Estimating Down Range Greens Function

Huygens Huygens Huygens

$G(z_i, z_j) \otimes G(z_j, z_k) \otimes G(z_k, z_l) \otimes G(z_l, z_m)$

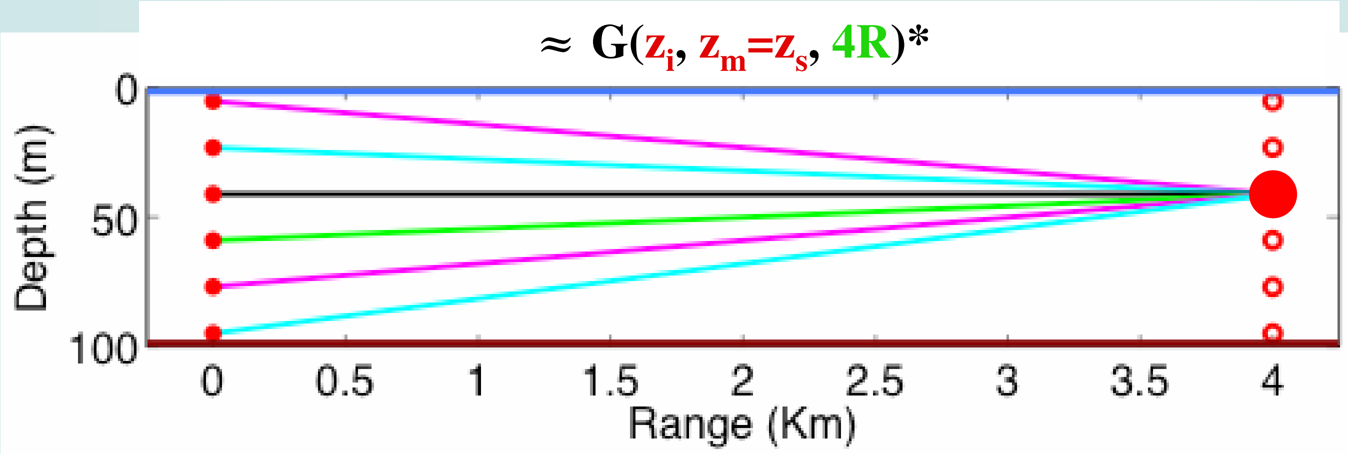


Given $G(z_i, z_j, R)$: Use **Huygens principle** to propagate to **4R**:

$$G(z_i, z_j, R) \otimes G(z_j, z_k, R) \otimes G(z_k, z_l, R) \otimes G(z_l, z_m, R) \approx G(z_i, z_m, 4R)$$

3R)

Down Range Imaging



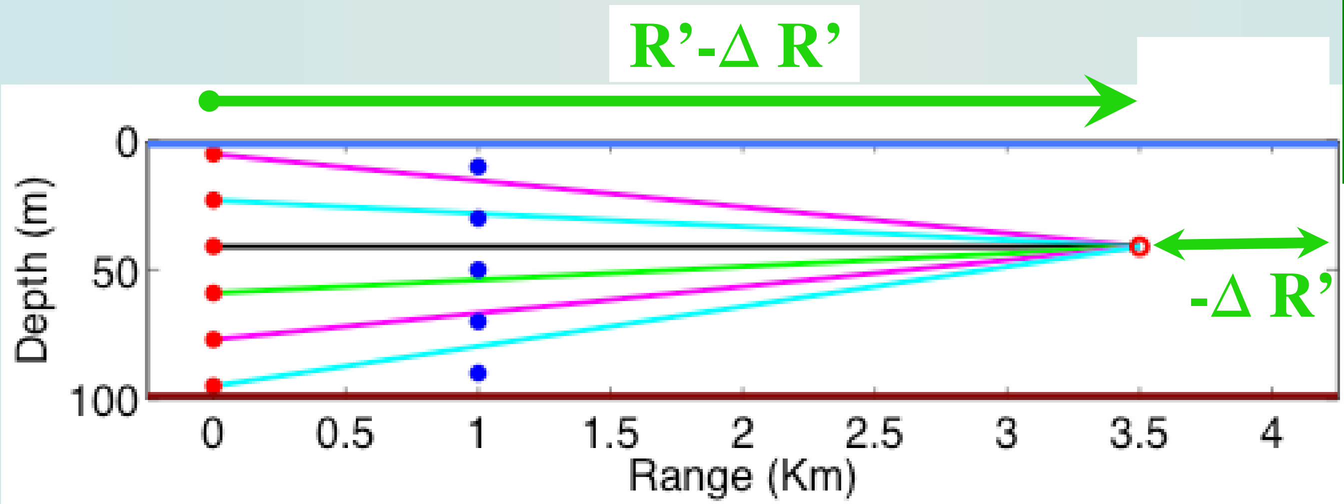
- Choose image depth
- Approximate $\mathbf{G} \approx G(z_i, z_m = z_s, 4R)$
- Perform imaging: $G(z_i, z_m = z_s, 4R) \Rightarrow G(z_i, z_m = z_s, 4R)^*$

Increasing the Range Target Window: Frequency Based Range Shiftng

$$f \Rightarrow f \pm \delta f$$

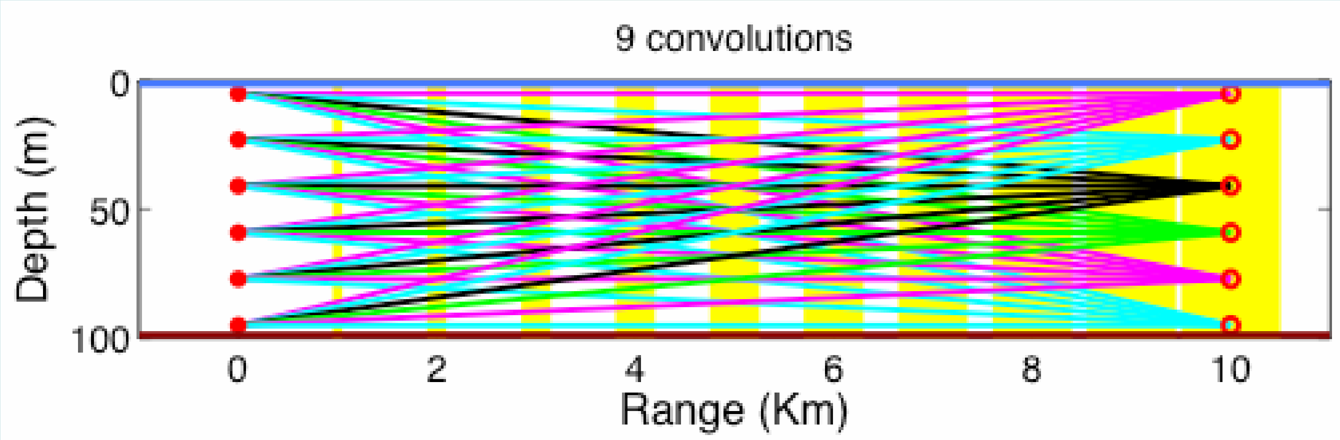
$$G(z_i\,,\, z_i\!=\!z_s,\, R') \quad \Rightarrow \quad G(z_i\,,\, z_i\!=\!z_s,\, R' \pm \Delta R')$$

$$\left|\frac{\Delta R'}{R'}\right|_{max} \approx 5\%$$



H. C. Song et al., J. Acoust. Soc. Am. ***103*** *(6), June 1998*

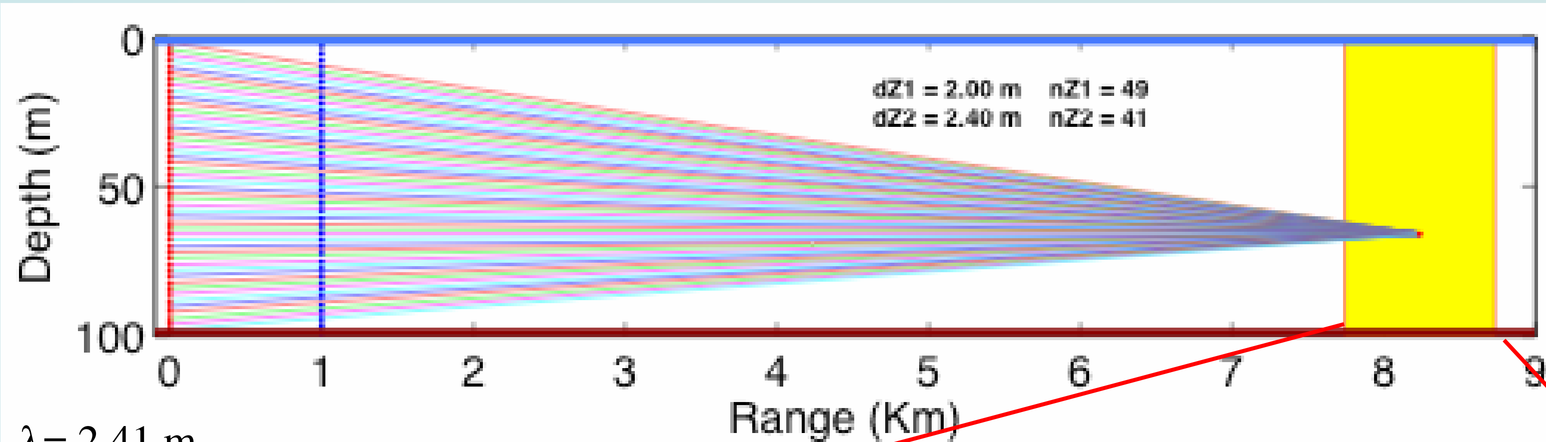
Down Range Imaging: Combining Huygens Principle with Range Shifting



By combining Greens function iteration with range shifting it is possible to cover all ranges and depths down range

Simulated Imaging Results

Imaging using iterated signal **with** frequency dependent range shifting

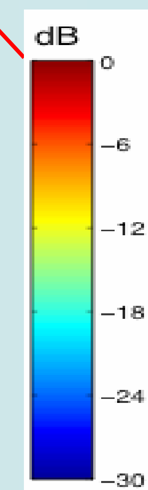


λ= 2.41 m
dZ_1 = 2.00 m
dZ_2 = 2.40 m
R = 1km
R' = 8.237 km

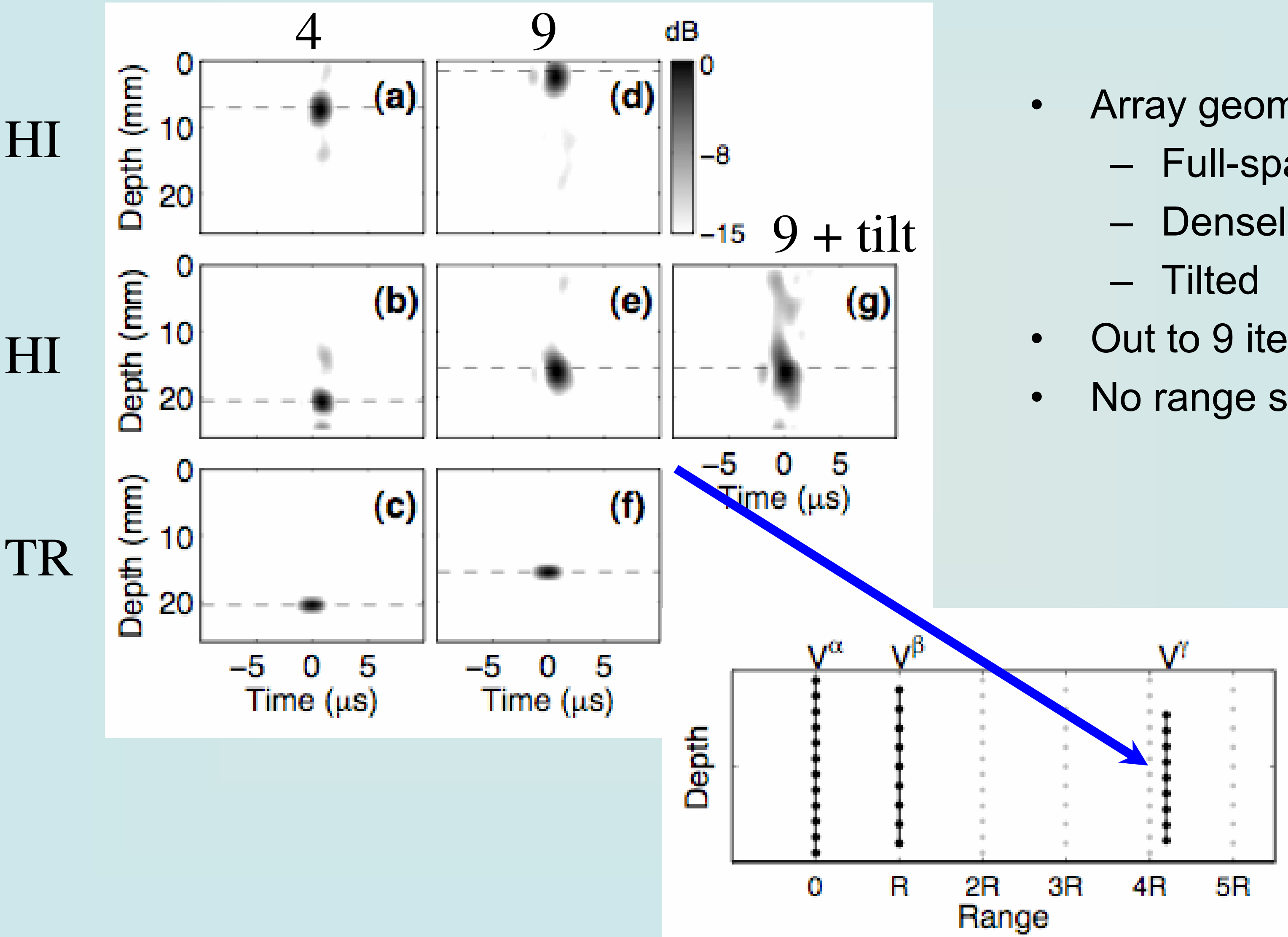
Depth (m)
0
50
100

QuickTime™ and a
decompressor
are needed to see this picture.

-0.4 -0.2 0 0.2 0.4
Time(s)



Experimental Imaging Results



- Array geometry
  - Full-spanning
  - Densely sampling
  - Tilted
- Out to 9 iterations
- No range shift