Surface wave propagation in 2D: coupling of proper and improper modes

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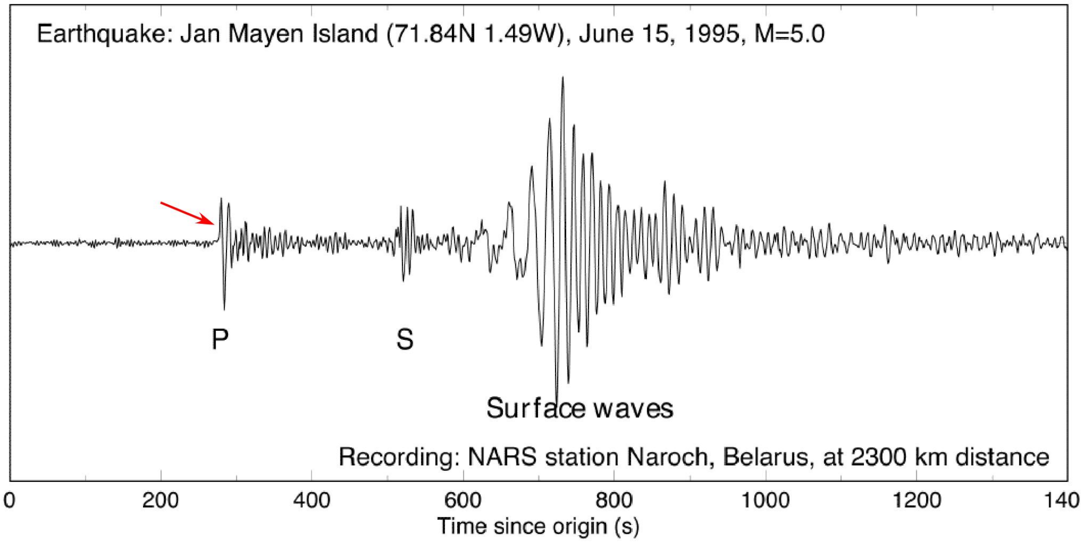
Home Page
Title Page
◀◀ ▶▶
◀ ▶
Page 1 of 15
Go Back
Full Screen
Close
Quit

How to study wave propagation in heterogeneous media?

- ↪ exact analytical methods
- ↪ numerical methods
- ↪ semi-analytical methods

→ wavefield expanded on a basis of surface-wave, local normal modes

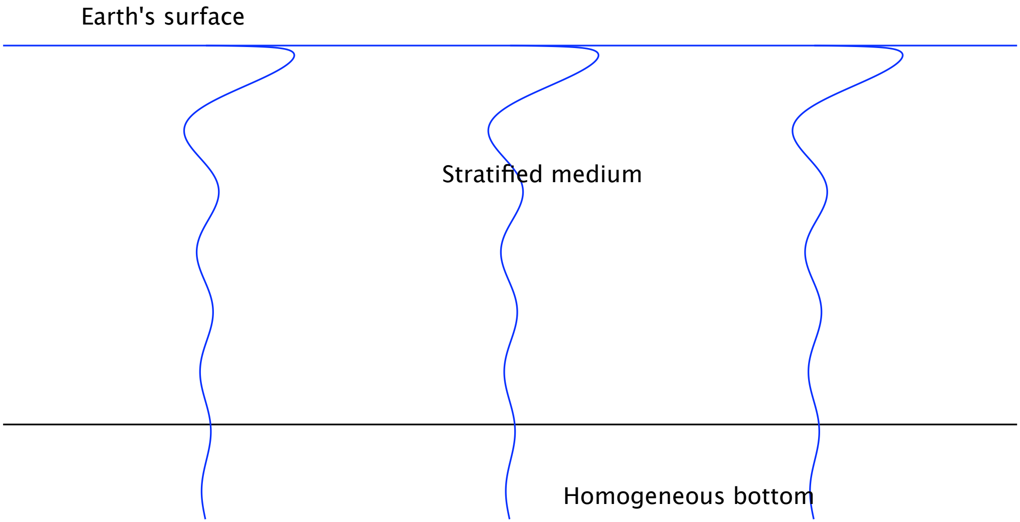


Earthquake: Jan Mayen Island (71.84N 1.49W), June 15, 1995, M=5.0

Recording: NARS station Naroch, Belarus, at 2300 km distance

Surface wave propagation in a laterally homogeneous medium

$\hookrightarrow C_{ijkl}(z),\;\; \rho(z)$



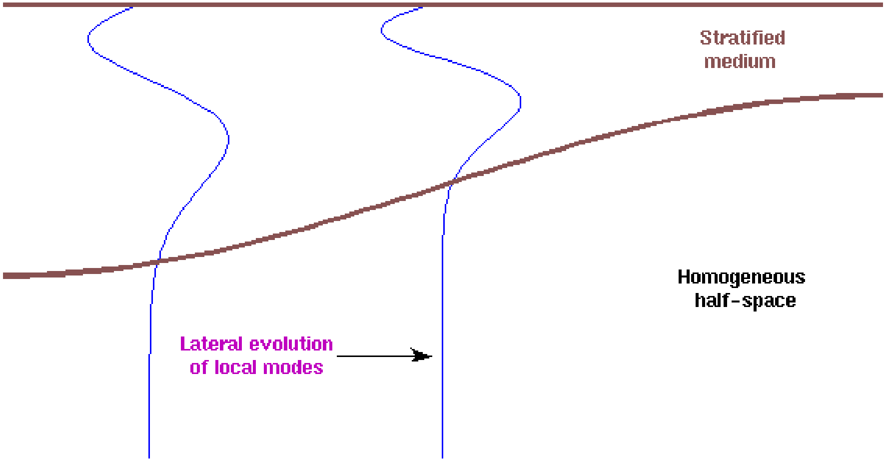
A surface plane wave solution is:

$$\boldsymbol{u}(\boldsymbol{x}^{\boldsymbol{\Sigma}}, z; \omega) = \sum_q c^q(\omega)\boldsymbol{u}^q(z;\omega)\exp(-i\boldsymbol{k}^q(\omega)\boldsymbol{\cdot}\boldsymbol{x}^{\boldsymbol{\Sigma}})$$

Home Page
Title Page
◀◀ ▶▶
◀ ▶
Page 3 of 15
Go Back
Full Screen
Close
Quit

Surface wave propagation in a slowly-varying heterogeneous medium

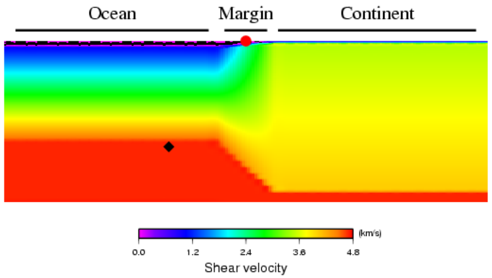
$\hookrightarrow$ $C_{ijkl}(x,z),\ \rho(x,z)$
$\hookrightarrow$ "classical" local modes

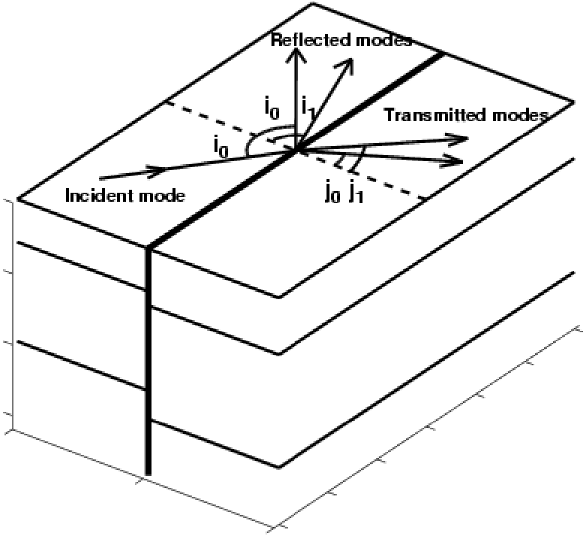


$\hookrightarrow$ 0th-order WKBJ (ray) solution:

$$\boldsymbol{u}(x,z;\omega)=\sum_q \underbrace{c^q(x;\omega)}_{amplitude}\,\underbrace{\boldsymbol{u}^q(x,z;\omega)}_{eigenmodes}\,\underbrace{\exp\left(-i\int^x k^q(\zeta)\mathrm{d}\zeta\right)}_{phase\ term}$$

Mode coupling





↪ WKBJ-iterative solution:

$$\boldsymbol{u}(x,z;\omega)=\sum_q \underbrace{c^q(x;\omega)}_{excitation}\,\underbrace{\boldsymbol{u}^q(x,z;\omega)}_{eigenmodes}\,\underbrace{\exp\left(-i\int^x k^q(\zeta)\mathrm{d}\zeta\right)}_{phase\ term}$$

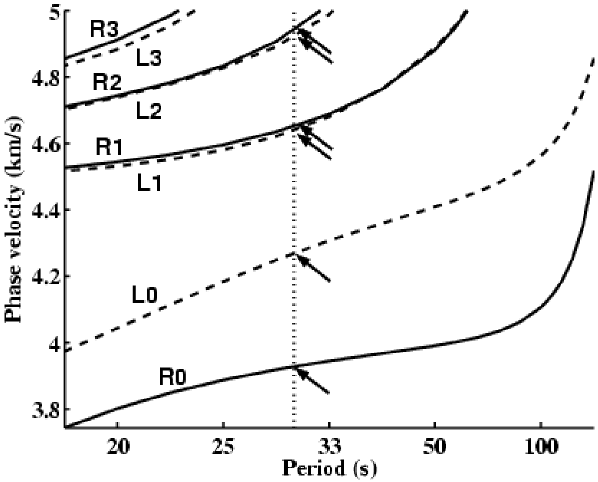
Home Page
Title Page
◀◀ ▶▶
◀ ▶

Go Back
Full Screen
Close
Quit

Mode coupling - evolution equation

↪ evolution equations for amplitude coefficients:

$$\frac{dc^q(x)}{dx} = \underbrace{B_r^q\left(\frac{1}{k^r - k^q}, \boldsymbol{u}^q, \boldsymbol{u}^r, \frac{d(\text{elasticity})}{dx}\right)}_{\textit{Coupling matrix: energy exchanges}} c^r(x)$$

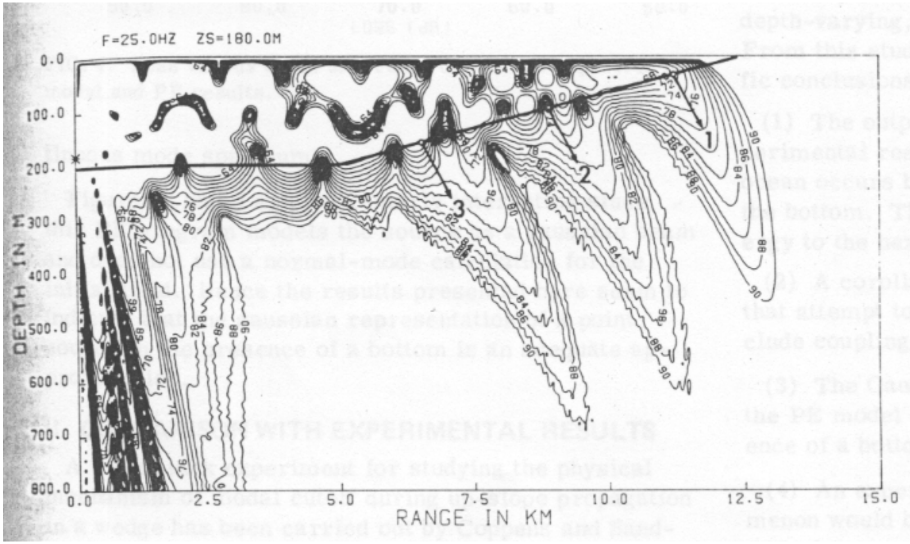


↪ $B_r^{q\dagger} = -B_q^r$: conservation of energy

↪ system of 1st order ODEs to solve

Limitation

↪ parabolic equation simulations (Kuperman et al. 1982)

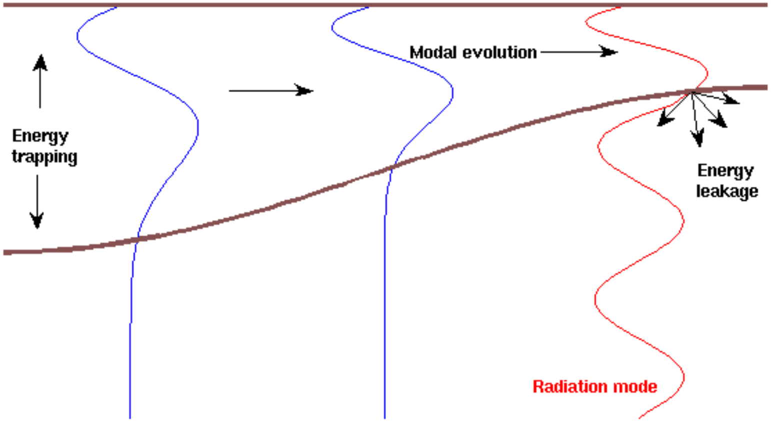


Home Page
Title Page
◀◀ ▶▶
◀ ▶

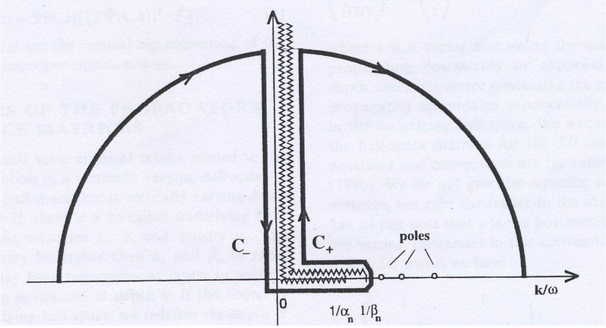
Go Back
Full Screen
Close
Quit

Local modes-3: radiation modes

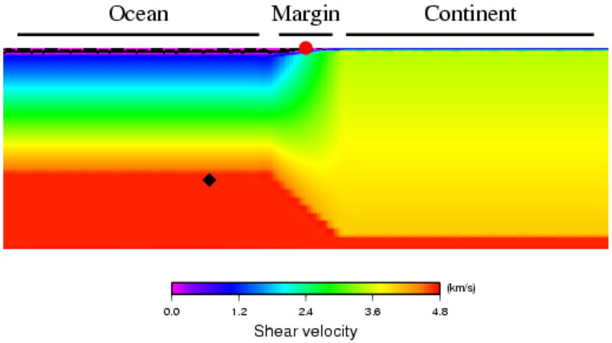
↪ energy not always trapped

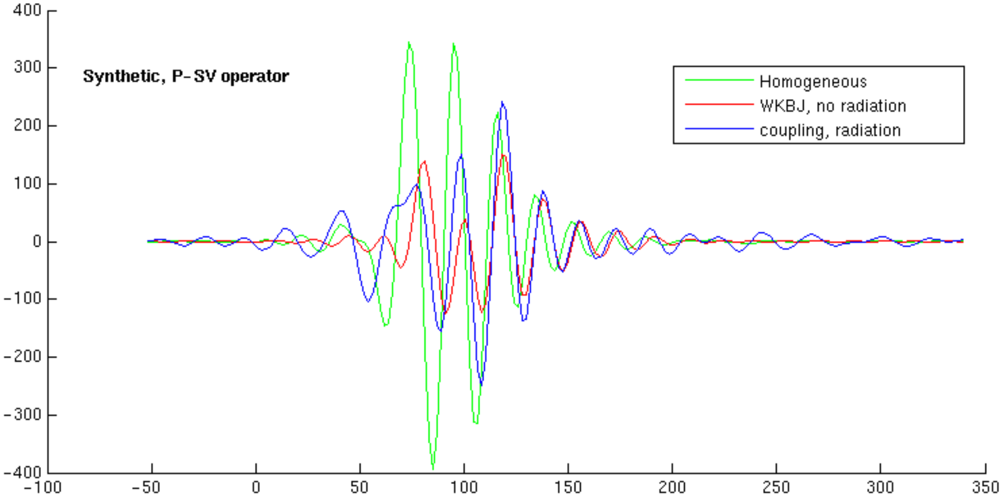


↪ cut-off wavenumber



Importance of the radiation modes





Home Page
Title Page
◀◀ ▶▶
◀ ▶
Page 9 of 15
Go Back
Full Screen
Close
Quit

Radiation modes at the cut-off wavenumber

- radiation modes of the SH-operator, homogeneous bottom:

$$\begin{pmatrix} u \\ \sigma \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{\pi\omega\rho\beta^2\eta}} \cos\left(\omega\eta z + \theta\right) \\ -\sqrt{\frac{\omega\rho\beta^2\eta}{\pi}} \sin\left(\omega\eta z + \theta\right) \end{pmatrix} ,$$

$$\theta = \frac{\pi}{2} - \arctan\left(\frac{P_{22}}{P_{21}} \cdot \omega\rho\beta^2\eta\right) + n\pi \; , n \in N$$

- near the cut-off: $\theta \overset{\eta\to 0^+}{\longrightarrow} \pi/2$, and $u \longrightarrow 0$

$\hookrightarrow$ importance of mode coupling

Love fundamental mode coupling near the cut-off-1

- T ≈ 85s
- simplified profile of the Møre margin:

Incident (forward) Love wave

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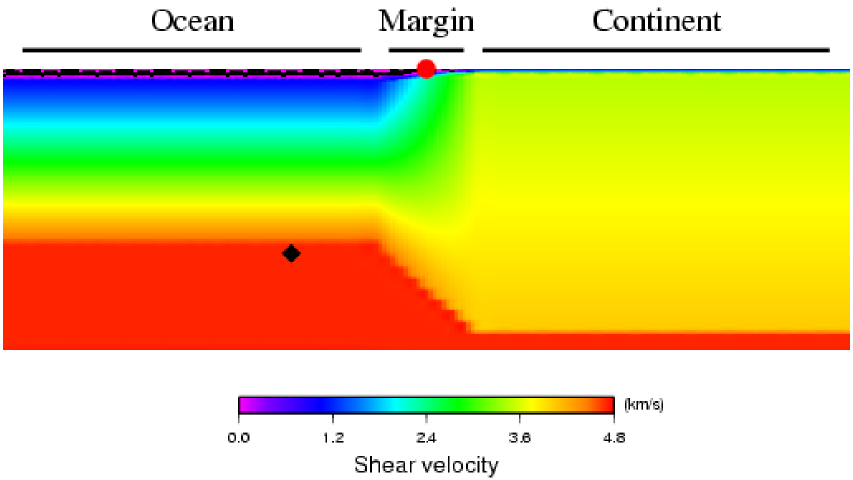

Radiated field??

↓

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Reflected Love wave??

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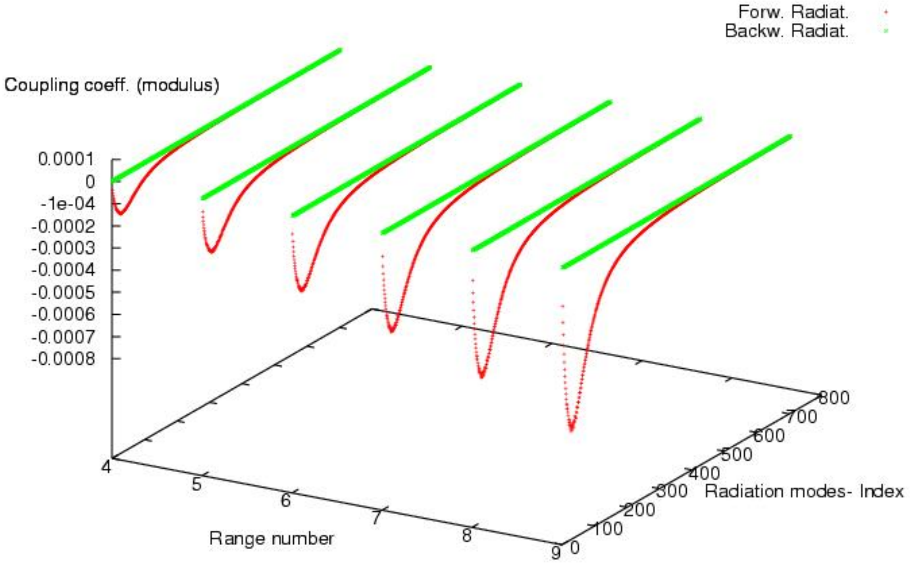
Home Page
Title Page
◀◀ ▶▶
◀ ▶
Page 11 of 15
Go Back
Full Screen
Close
Quit

Love fundamental mode coupling near the cut-off-2

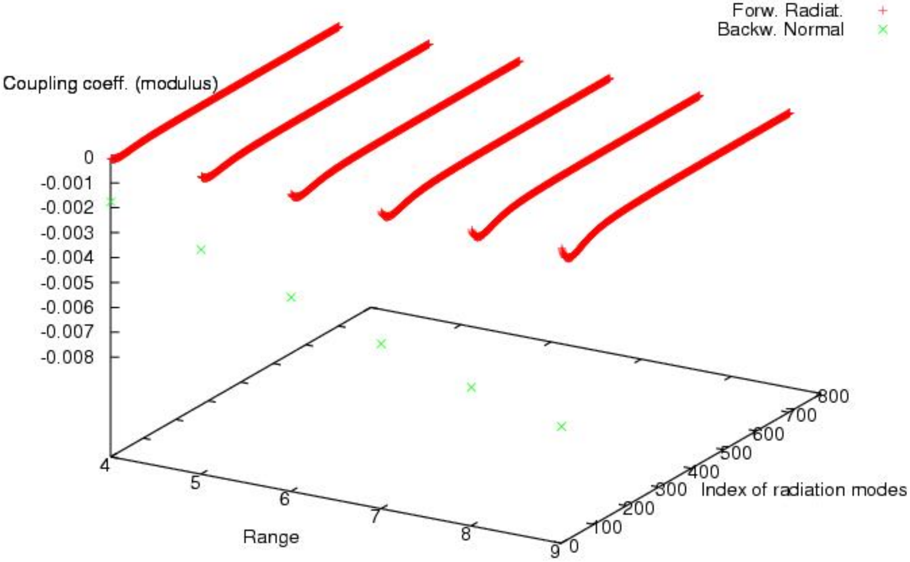
↪ measure of the coupling coefficients with radiation modes



- maximum leakage: ≈ 32°
- aperture: ± 10°

Love fundamental mode coupling near the cut-off-3

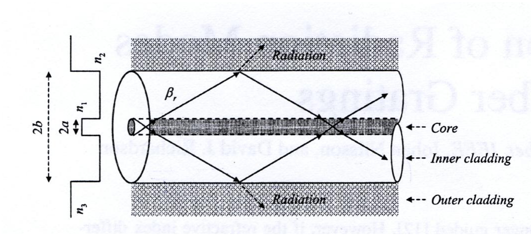
↪ measure of the coupling coefficients with backward (reflected) mode

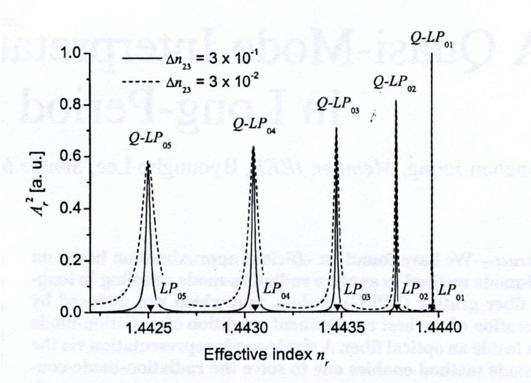


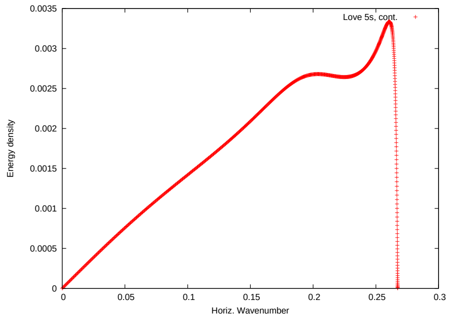
↪ importance of considering leakage (42%)<u>and</u> reflection (58%)

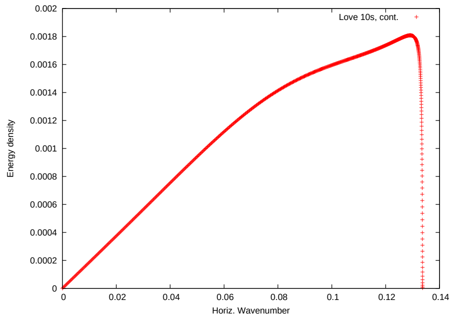
Difficulties with the continuum

- discretization problem
- WKBJ theory for a mode within a continuum??
- degeneracy



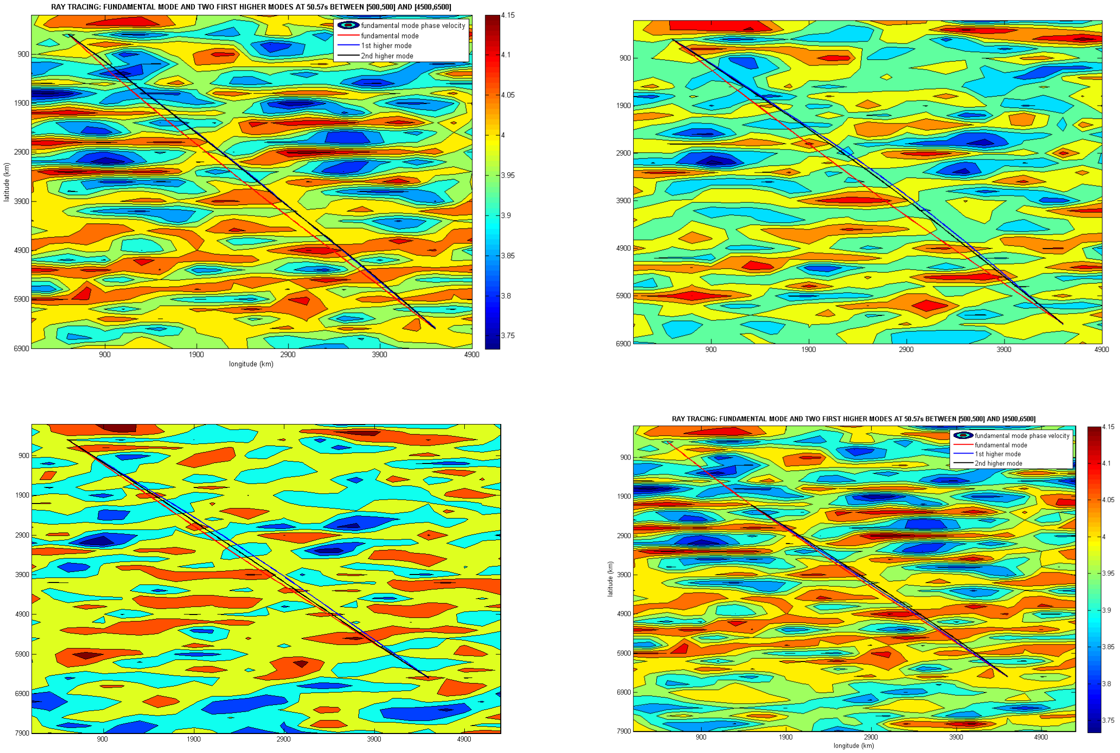






Coupling local modes in 3D

- no plane wave decoupling for a given wavenumber/y (Tromp 1994). Coupling along a ray trajectory.



(Bérénice Froment)

- apply the parabolic approximation to the obtained system of PDEs (Abawi, Kuperman & Collins, 1997)

Home Page
Title Page
◀◀ ▶▶
◀ ▶
Page 15 of 15
Go Back
Full Screen
Close
Quit