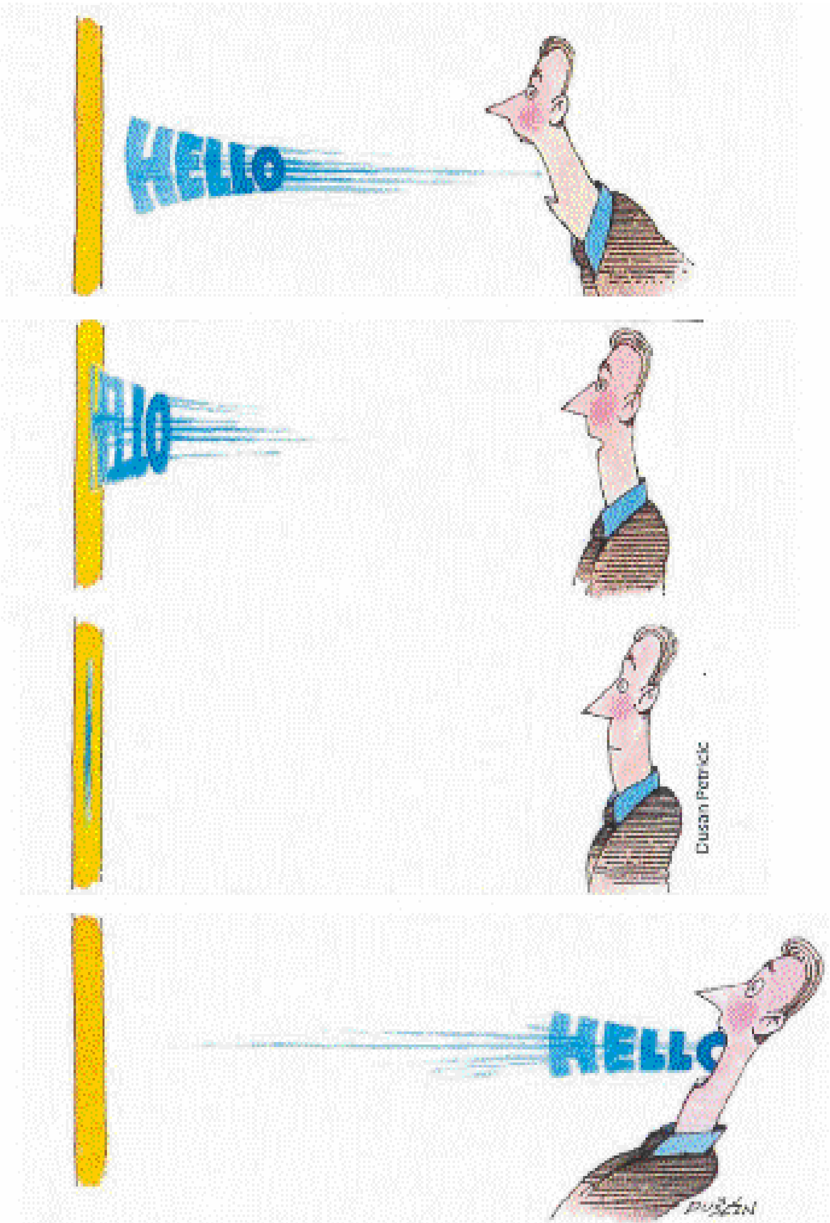
Waves,Time Reversal and Super-resolution



Acoustic propagation in a non dissipative fluid

$\boldsymbol{p}(\vec{\boldsymbol{r}},\boldsymbol{t})$ acoustic pressure field (scalar)

$\rho(\vec{\boldsymbol{r}})$ is the density and $\boldsymbol{c}(\vec{\boldsymbol{r}})$ is the sound velocity in an heterogeneous medium

In linear acoustics
in a domain
without source

→

$$div\left\{\frac{grad(p(\vec{r},t))}{\rho(\vec{r})}\right\}-\frac{1}{\rho(\vec{r})c^2(\vec{r})}\frac{\partial^2 p(\vec{r},t)}{\partial t^2}=0$$

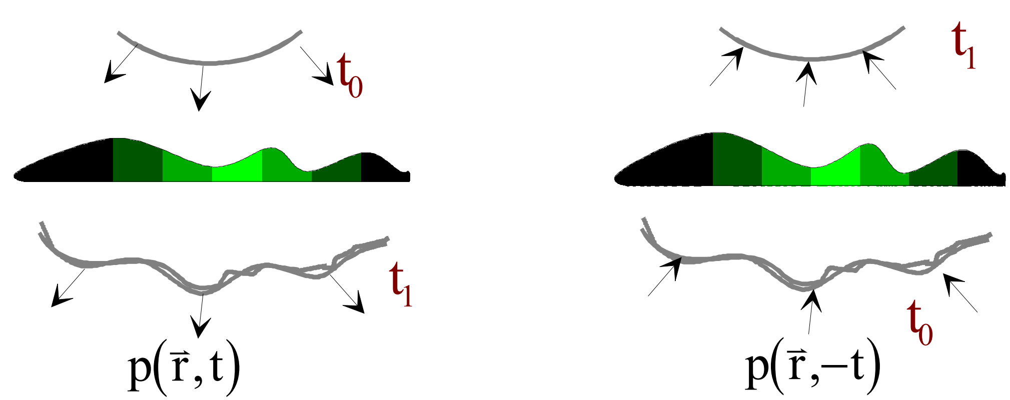
Spatial Reciprocity   Time Reversal Invariance

This equation contains only $\frac{\partial^2 \boldsymbol{p}(\vec{\boldsymbol{r}},\boldsymbol{t})}{\partial \boldsymbol{t}^2}$

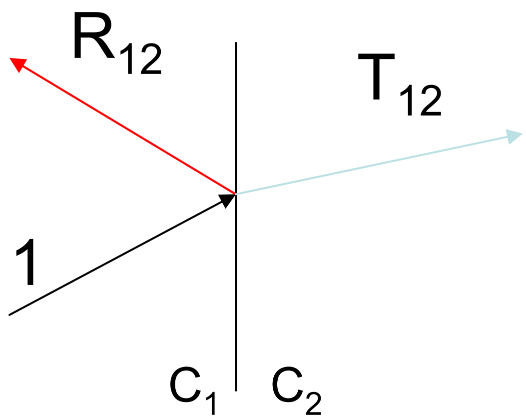
Then if $\boldsymbol{p}(\vec{\boldsymbol{r}},\boldsymbol{t})$ is a solution

$\boldsymbol{p}(\vec{\boldsymbol{r}},-\boldsymbol{t})$ is also a solution

because $\frac{\partial^2 \boldsymbol{p}(\vec{\boldsymbol{r}},\boldsymbol{t})}{\partial \boldsymbol{t}^2}=\frac{\partial^2 \boldsymbol{p}(\vec{\boldsymbol{r}},-\boldsymbol{t})}{\partial \boldsymbol{t}^2}$



Stokes Formula



One incident plane wave : amplitude 1 :

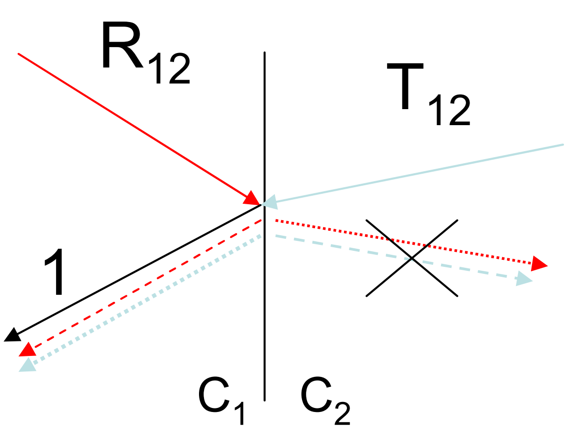
One reflected wave : R_{12}

One transmitted wave : T_{12}

$$T_{12}T_{21}+R_{12}R_{12} =1$$

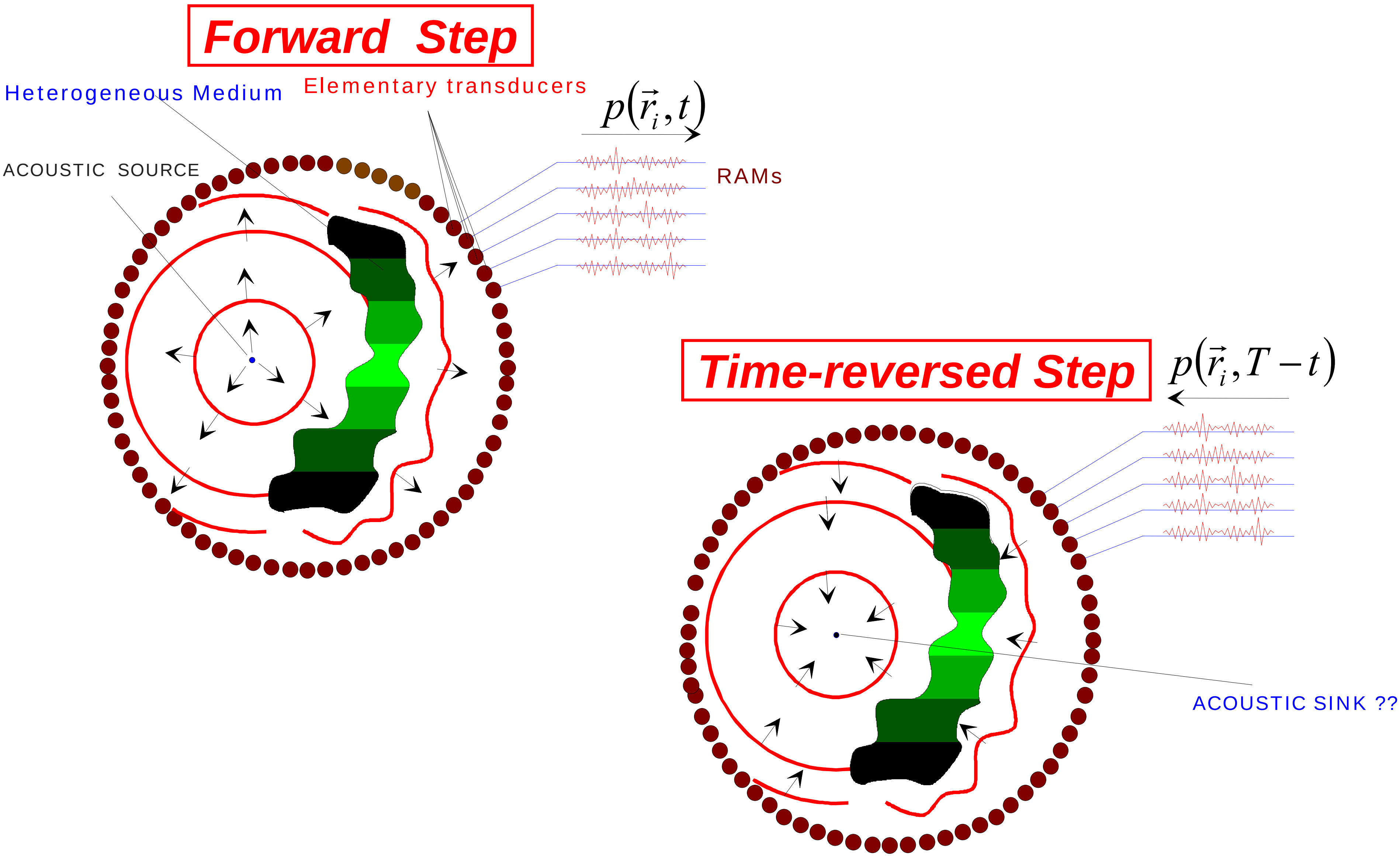
The Time-Reversed Situation

$$T_{12}R_{21}+R_{12}T_{12} =0$$



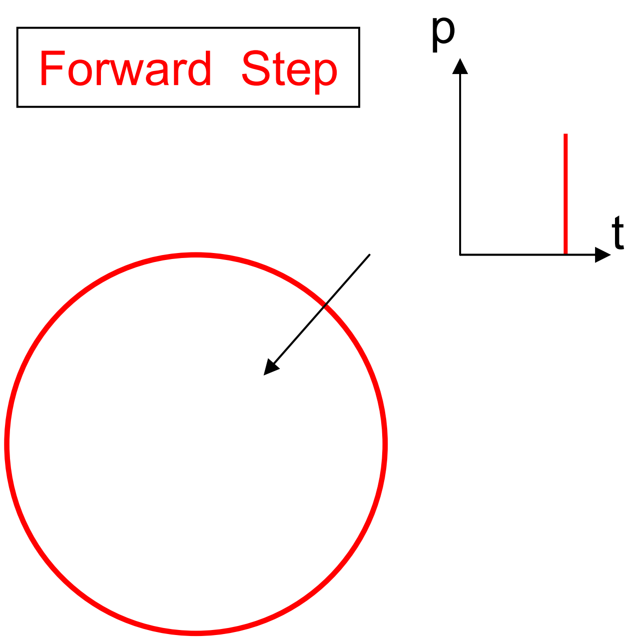
$$R_{12}^2 = T_{12}T_{21}$$
$$R_{12} = -R_{21}$$

Time-Reversal Cavity

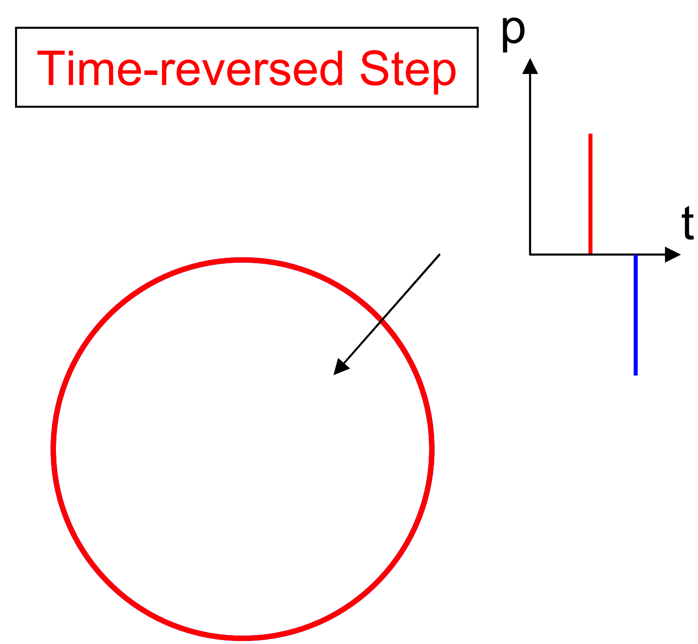


Origin of Diffraction Limits in Wave Physics

Pulsed mode – the homogeneous medium

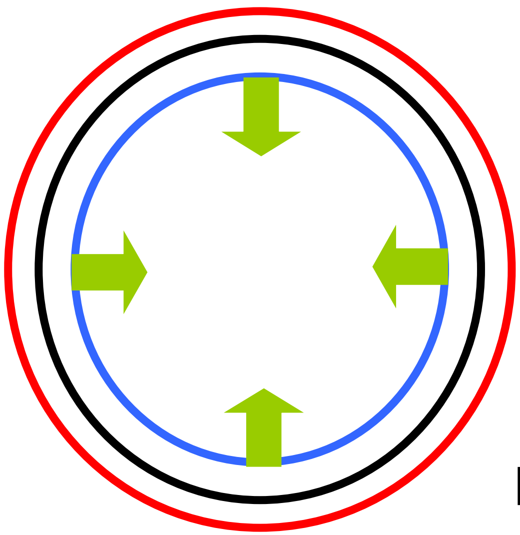


$$G^{0}_{ret}(R,t)$$



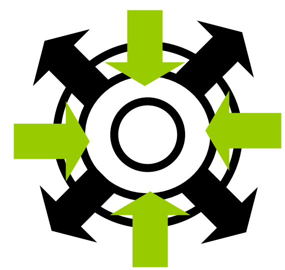
$$G^{0}_{adv}(R\ ,t)-G^{0}_{ret}(R,t)$$

The time-reversed step for monochromatic waves : origin of the diffraction limits

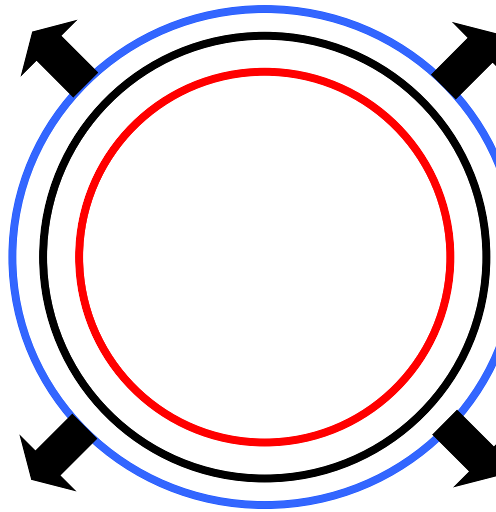


Converging only

$$\mathrm{Im}\left\{\hat{G}^{0}_{ret}(R,\omega)\right\}$$



Both converging and diverging waves interfere



Diverging only

$$\hat{G}^{0}_{adv}(R,\omega)=\frac{\exp\{j(-kR-\omega t)\}}{R}$$

with a singularity

$$\frac{\sin\{kR\}}{R}\exp(-j\omega t)$$

without singularity

Diffraction limit (λ/2)

$$-\hat{G}^{0}_{ret}=-\frac{\exp\{j(kR-\omega t)\}}{R}$$

with a singularity

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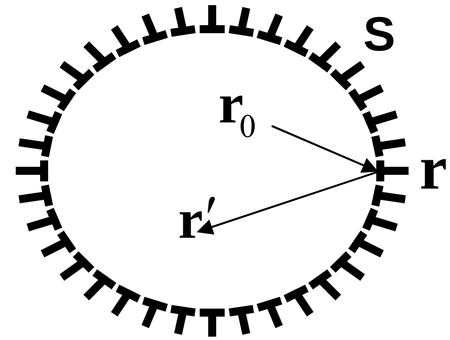
In a Non Dissipative Heterogeneous Medium

STEP 1

$$\left.\begin{aligned}&(L_r + L_t)\,G_{ret}(\mathbf{r},\mathbf{r}_0;t) = -\delta(\mathbf{r}-\mathbf{r}_0)\,\delta(t)\\ &L_r = \vec{\nabla}.\left(\frac{1}{\rho(\mathbf{r})}\vec{\nabla}\right),\; L_t = -\kappa(\mathbf{r})\,\partial_{tt}\end{aligned}\right\}$$



STEP 2 **Generating on the surface S monopole and dipole sources**

$$\varphi_s(\mathbf{r},t),\,\partial_n\varphi_s(\mathbf{r},t) \quad \textbf{with} \quad \varphi_s(\mathbf{r},t) = G_{ret}(\mathbf{r},\mathbf{r}_0;T-t)$$

Representation theorem

$$\varphi_{tr}(\mathbf{r}',t') = \int_{-\infty}^{+\infty} dt \iint_S \left[G(\mathbf{r}',\mathbf{r};t'-t)\,\partial_n\varphi_s(\mathbf{r},t) - \varphi_s(\mathbf{r},t)\,\partial_n G(\mathbf{r}',\mathbf{r};t'-t)\right]\frac{d^2\mathbf{r}}{\rho(\mathbf{r})}$$

monopole **dipole**

Taking into account that $L_t = L_{T-t}$ **, we obtain**

$$\varphi_{tr}(\mathbf{r}',t') = G_{ret}(\mathbf{r}',\mathbf{r}_0;T-t') - G_{ret}(\mathbf{r}',\mathbf{r}_0;t'-T)$$

A convolution approach

$$\iint_S \left[G(\mathbf{r}',\mathbf{r};t) \otimes \partial_n G(\mathbf{r},\mathbf{r_0};T-t) - G(\mathbf{r},\mathbf{r_0};T-t) \otimes \partial_n G(\mathbf{r}',\mathbf{r};t) \right] \frac{d^2\mathbf{r}}{\rho(\mathbf{r})} =$$

$$G(\mathbf{r}',\mathbf{r}_0;T-t) - G(\mathbf{r}',\mathbf{r}_0;t-T)$$

Paraxial Approximation

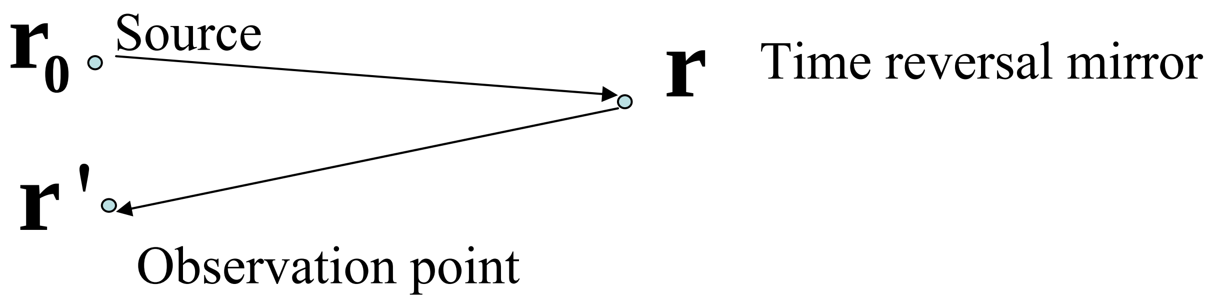
$$2\frac{\partial}{c\partial t} \iint_S G(\mathbf{r}',\mathbf{r};t) \otimes G(\mathbf{r},\mathbf{r_0};T-t) \frac{d^2\mathbf{r}}{\rho(\mathbf{r})} \simeq G(\mathbf{r}',\mathbf{r}_0;T-t) - G(\mathbf{r}',\mathbf{r}_0;t-T)$$

in monochromatic regime

$$\iint_S \left[\widehat{G}(\mathbf{r}',\mathbf{r};\omega) \partial_n \widehat{G}^*(\mathbf{r},\mathbf{r_0};\omega) - \widehat{G}^*(\mathbf{r},\mathbf{r_0};\omega) \partial_n \widehat{G}(\mathbf{r}',\mathbf{r};\omega) \right] \frac{d^2\mathbf{r}}{\rho(\mathbf{r})} =$$

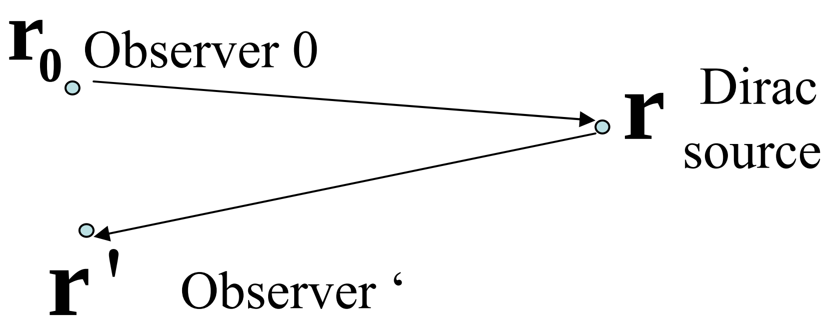
$$2j \,\mathrm{Im}\, \widehat{G}(\mathbf{r}',\mathbf{r}_0;\omega) \simeq \frac{2j\omega}{c} \iint_S \widehat{G}(\mathbf{r}',\mathbf{r};\omega) \widehat{G}^*(\mathbf{r},\mathbf{r_0};\omega) \frac{d^2\mathbf{r}}{\rho(\mathbf{r})}$$

A one channel time- reversal mirror as an estimate of a spatial correlator

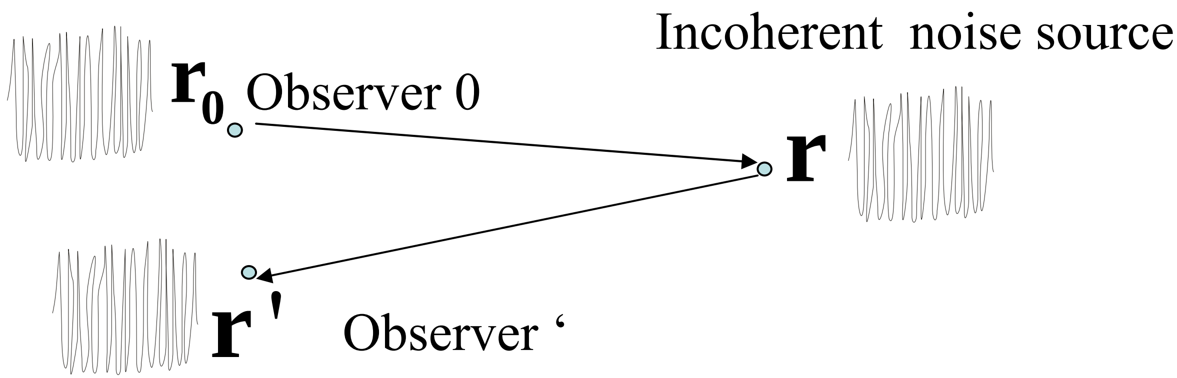


$$\varphi_{tr}\left(\mathbf{r}',t'\right) \prec \; G\left(\mathbf{r}',\mathbf{r};t\right) \otimes G\left(\mathbf{r},\mathbf{r}_{\mathbf{0}};T-t\right)$$

Time reversal field observed at point $\mathbf{r}'$ coming from a source at $\mathbf{r}_{\mathbf{0}}$



$$Corr\left(\mathbf{r}_{\mathbf{0}},\mathbf{r}',t\right) \prec G\left(\mathbf{r}',\mathbf{r};t\right) \otimes G\left(\mathbf{r}_{\mathbf{0}},\mathbf{r};-t\right)$$



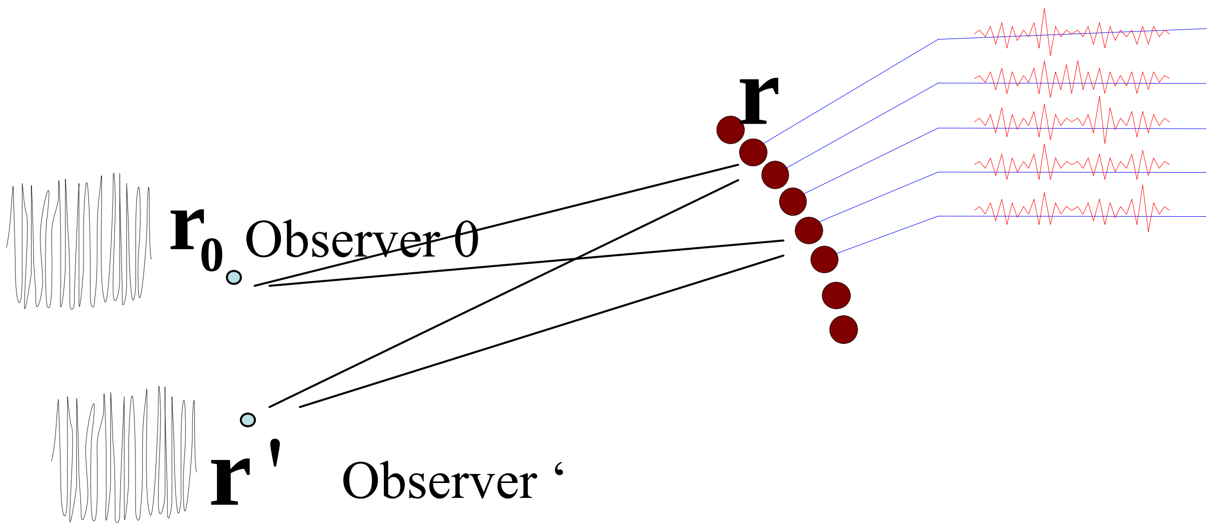
$$Corr\left(\mathbf{r_0},\mathbf{r}',t\right) \prec G\left(\mathbf{r}',\mathbf{r};t\right) \otimes G\left(\mathbf{r_0},\mathbf{r};-t\right) \otimes n\left(t\right) \otimes n\left(-t\right)$$

If $n\left(t\right) \otimes n\left(-t\right) = \delta\left(t\right)$

then the correlation of the noise between the two observers gives the same result than a time-reversal experiment conducted with a one channel TRM

If the source of the noise is spread in space, with spatial decorrelation

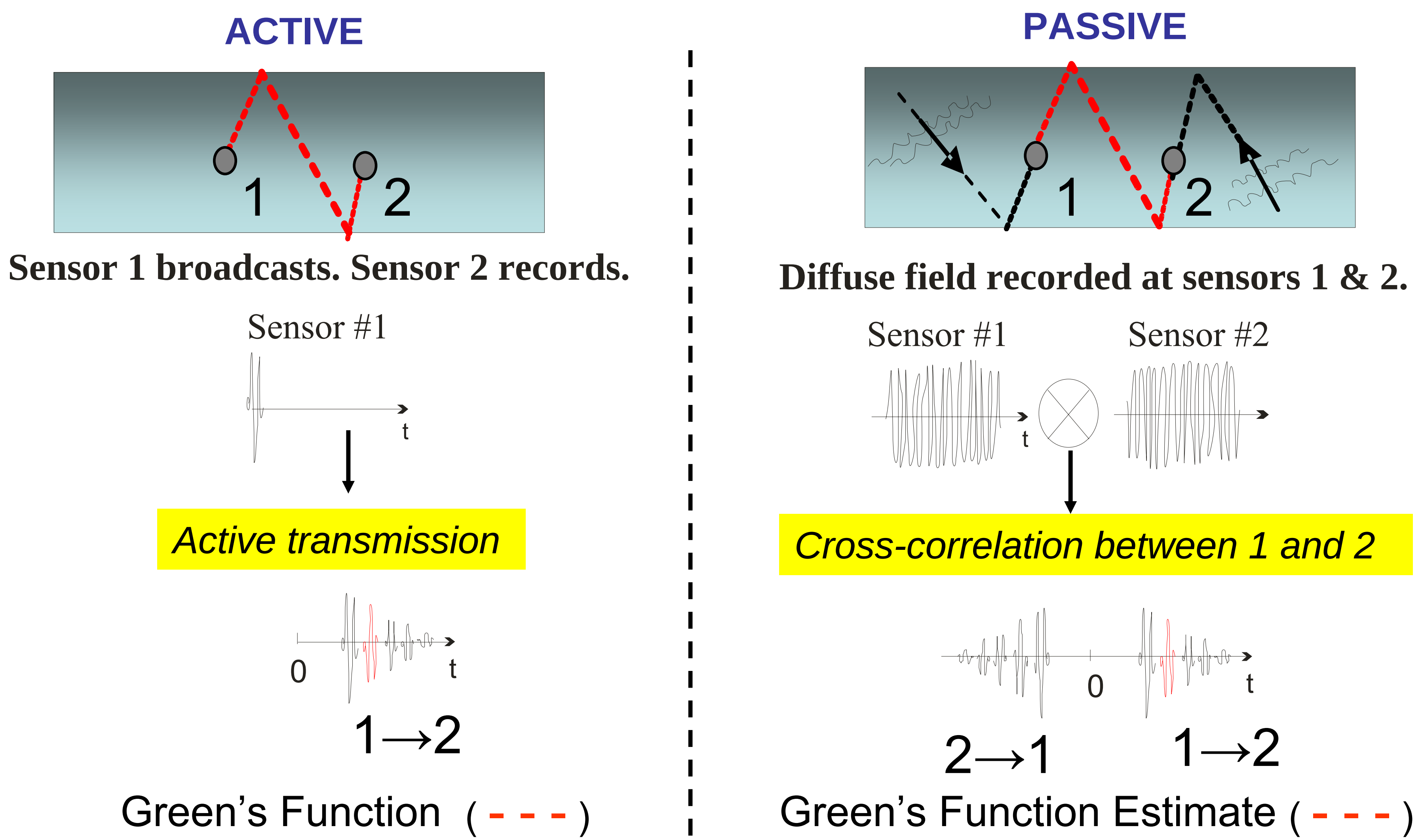
$$\left\langle n\left(r,t\right)n\left(-r,-t\right)\right\rangle = \delta\left(r,t\right)$$



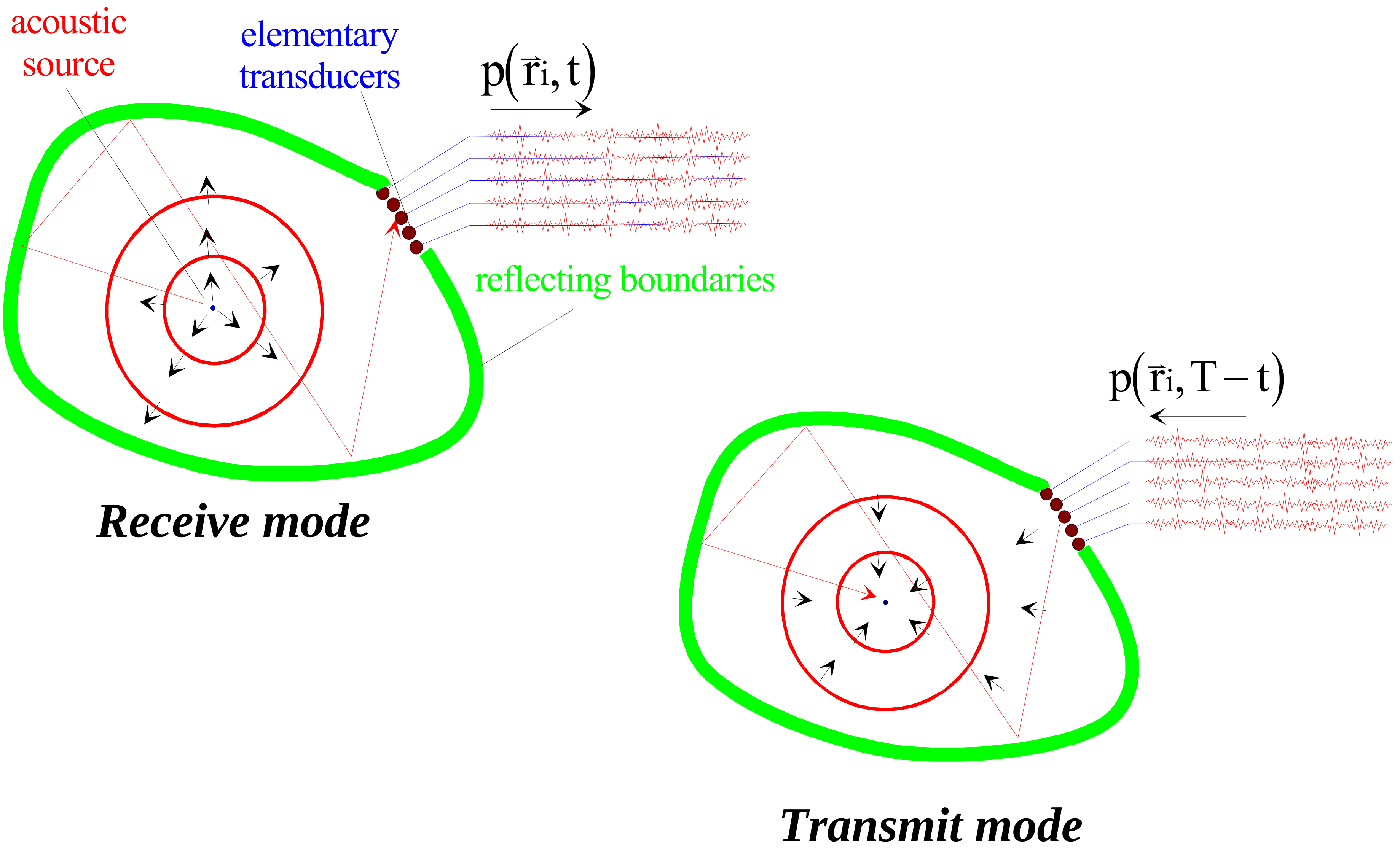
$$Corr\left(\mathbf{r_0},\mathbf{r}',t\right) \prec \iiint G\left(\mathbf{r}',\mathbf{r};t\right) \otimes G\left(\mathbf{r_0},\mathbf{r};-t\right) \otimes n\left(\mathbf{r},t\right) \otimes n\left(\mathbf{-r},-t\right)$$

$$\prec \varphi_{tr}\left(\mathbf{r}',t'\right) \prec G\left(\mathbf{r}',\mathbf{r}_0;T-t\right) - G\left(\mathbf{r}',\mathbf{r}_0;t-T\right)$$

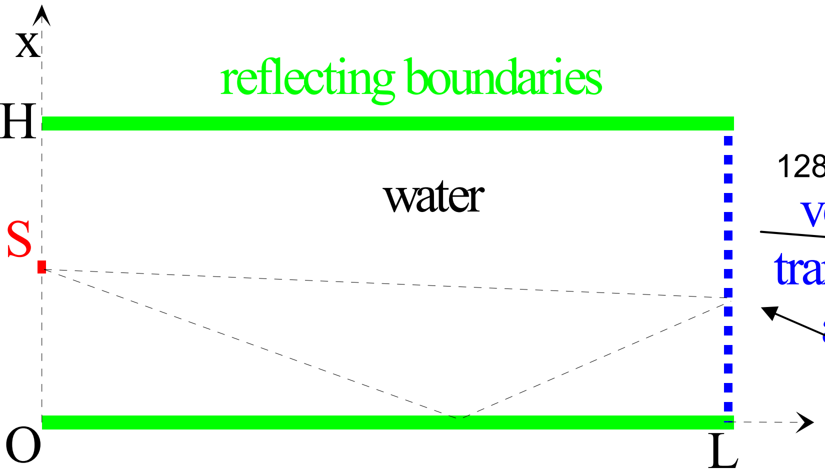
"By cross-correlating diffuse fields recorded at two locations, the Green's function between these two locations can be reconstructed". (Farrar 97, Claerbout 1999, Weaver 2001, etc...)

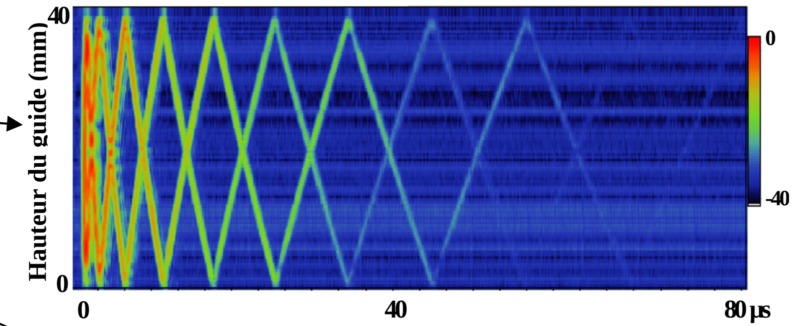


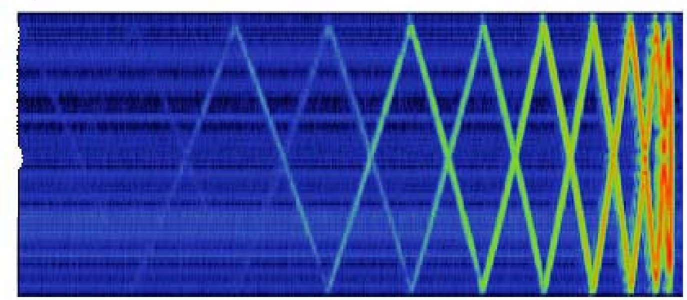
Influence of boundaries on TR

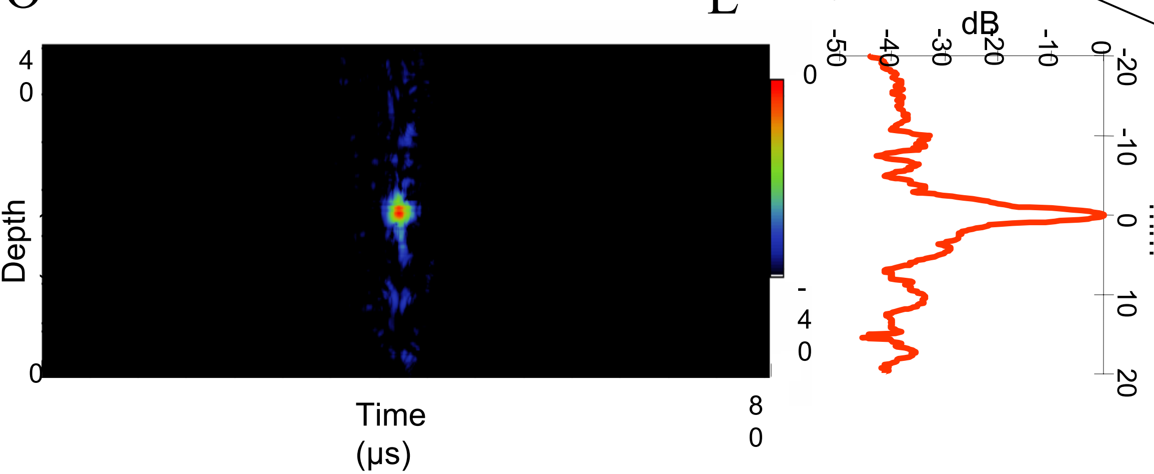


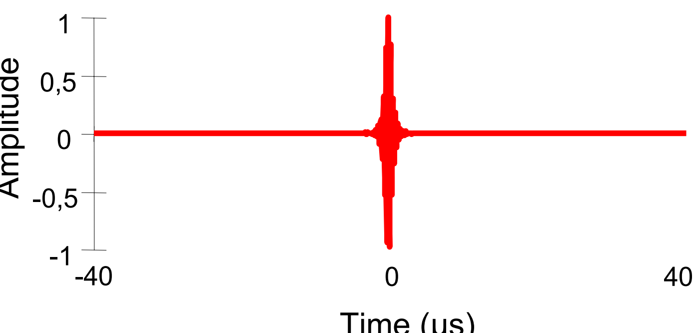
Retournement temporel dans un guide d'onde











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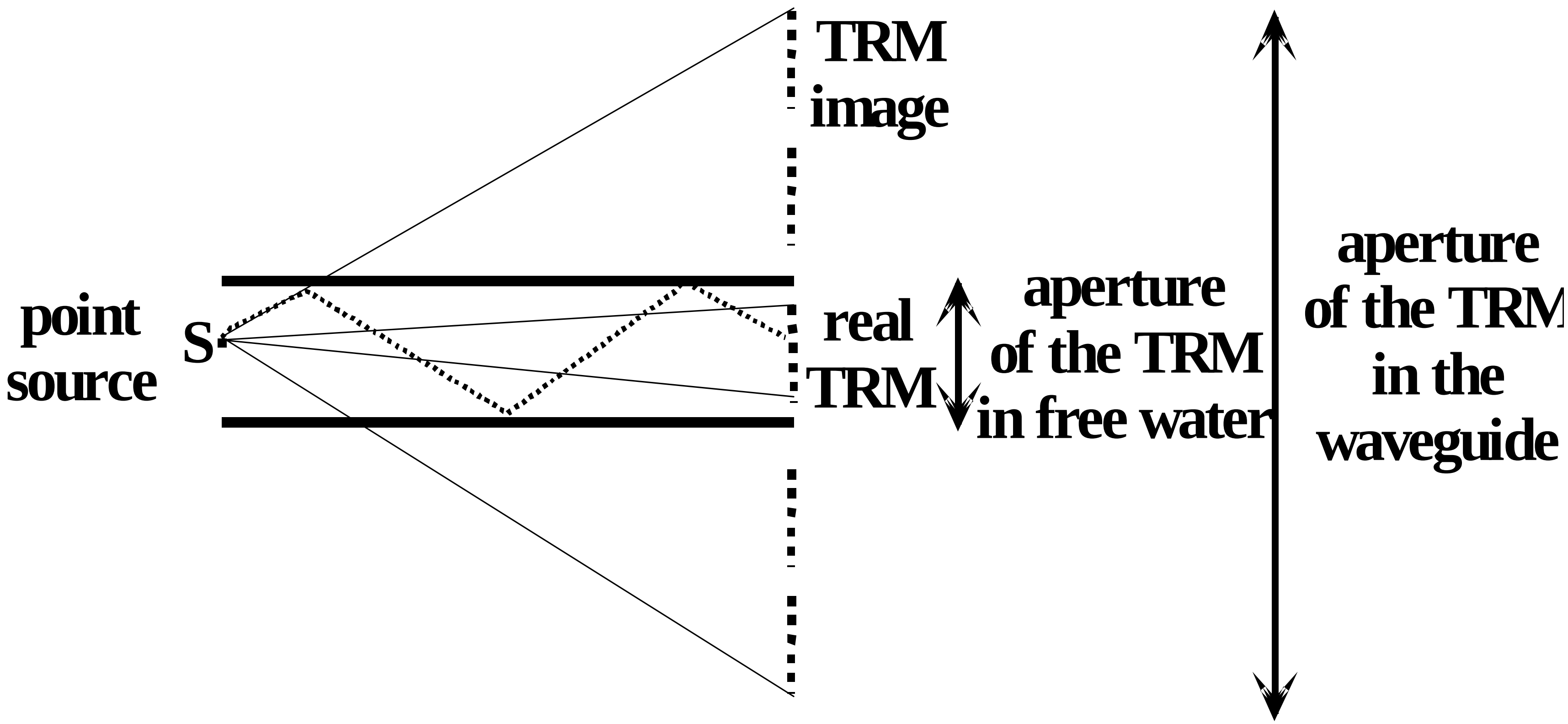
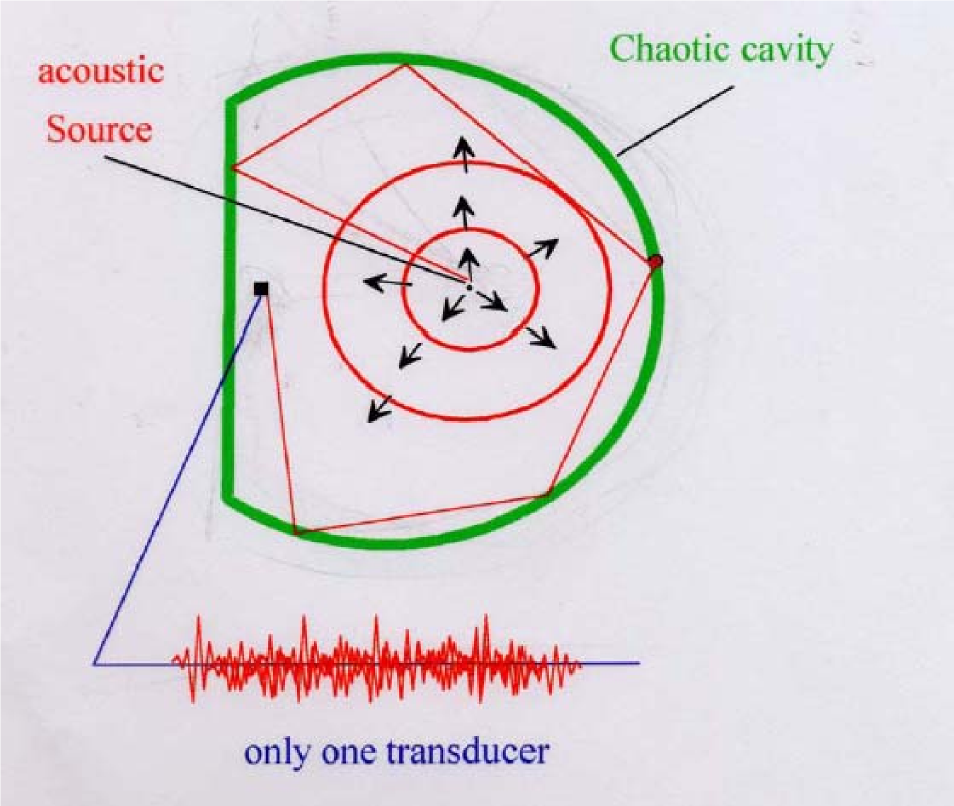
L'effet Kaléidoscope : les transducteurs virtuels

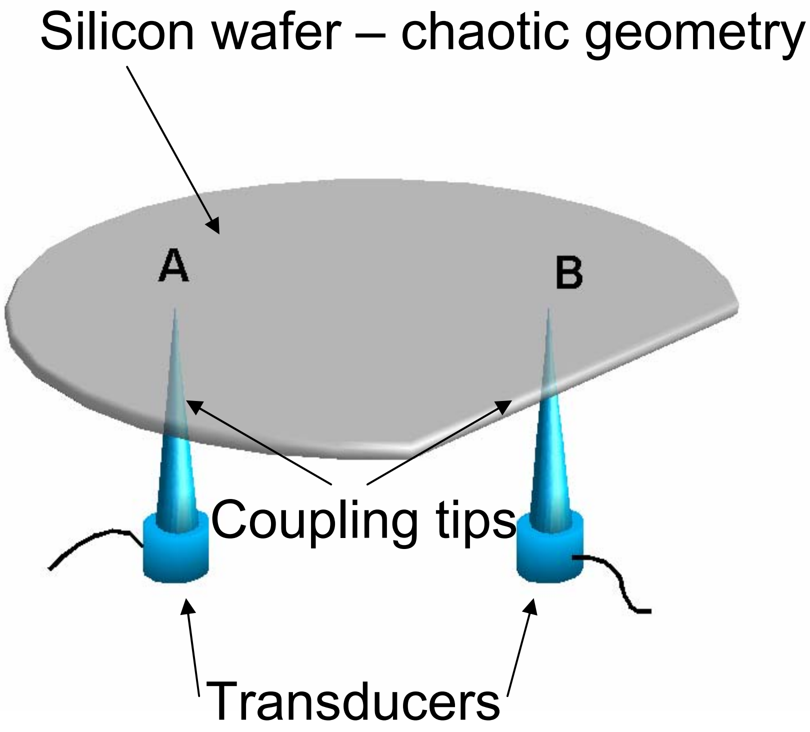


If the pitch is to large : grating lobes

Time-Reversal in a Chaotic Billiard



Ergodicity



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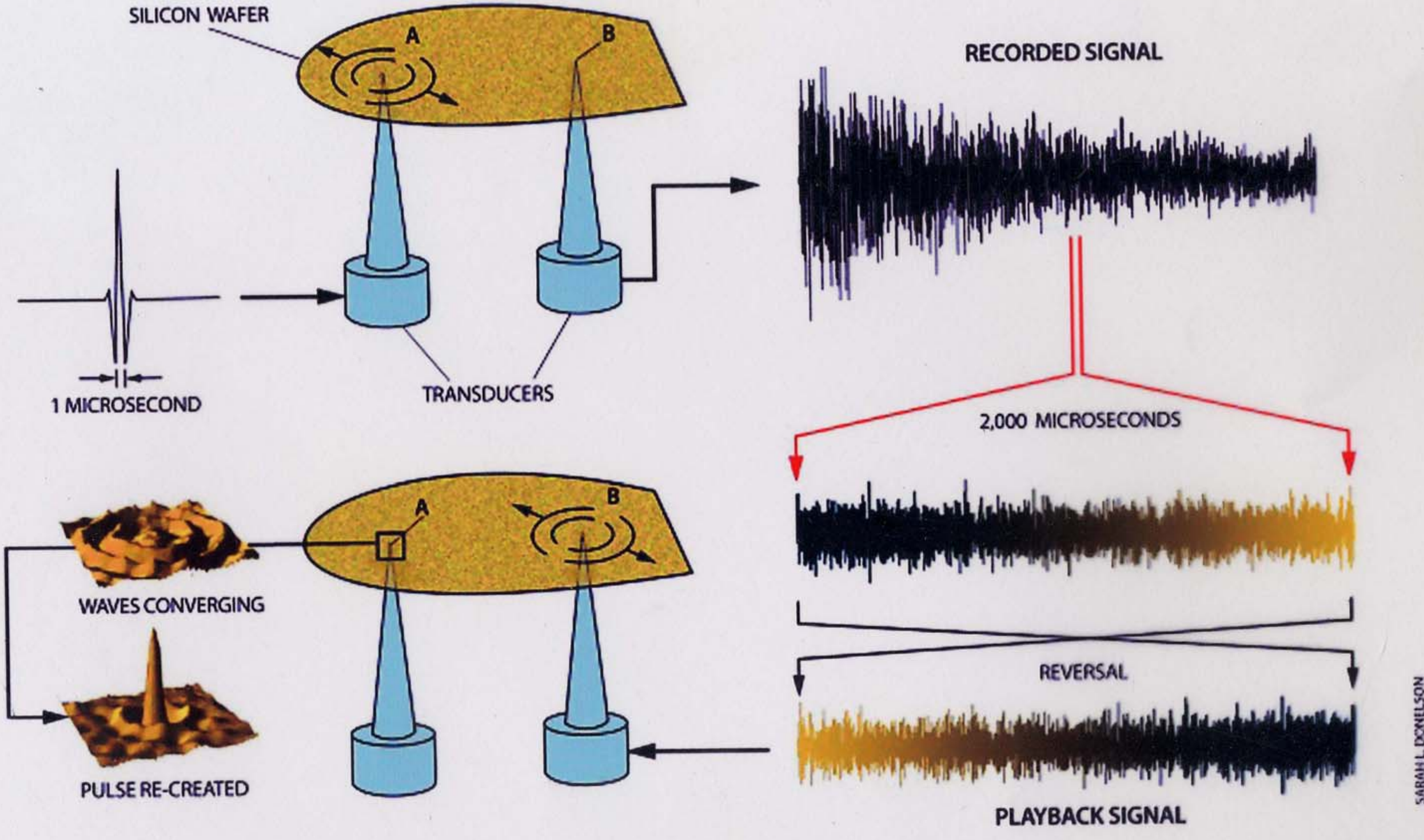
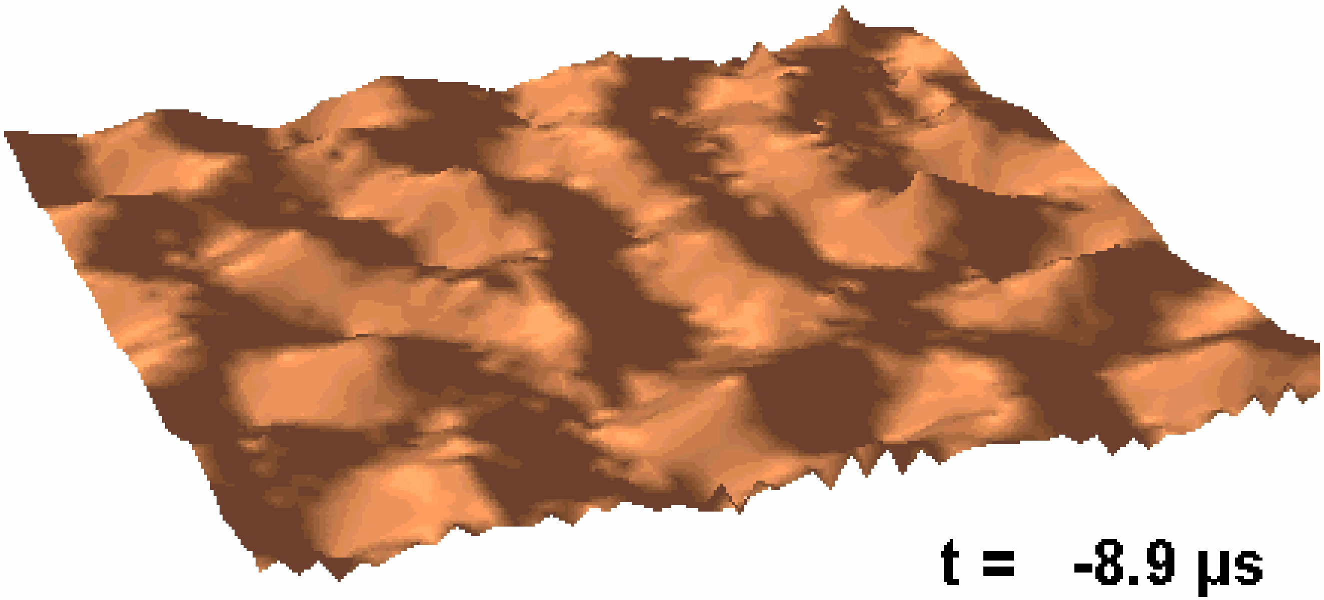
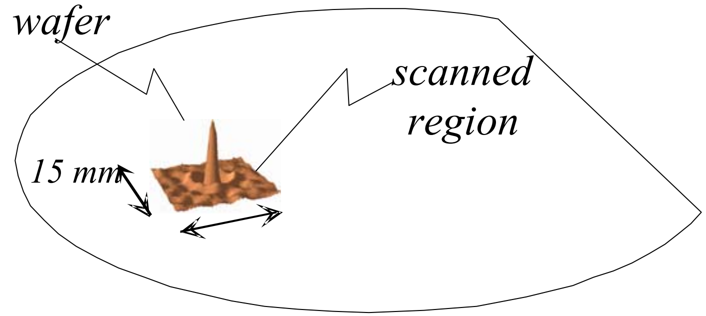


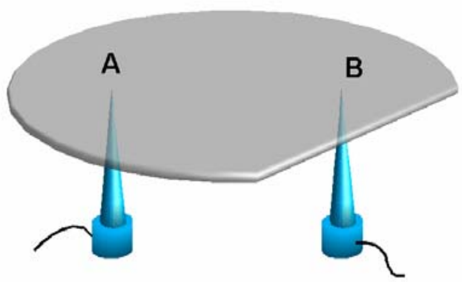
SINGLE TRANSDUCER can time-reverse a wave in an enclosed "cavity." A source transducer emits a pulse at location A on a small silicon wafer (*top*). A transducer at location B records chaotic reverberations of the pulse reflected off the wafer edges hundreds of times. The transducer at B plays back a short segment of that signal in reverse (*bottom*). After many reflections, these recombine to re-create the short pulse focused again at location A, as was revealed by imaging the waves on the wafer near A (*bottom left*).

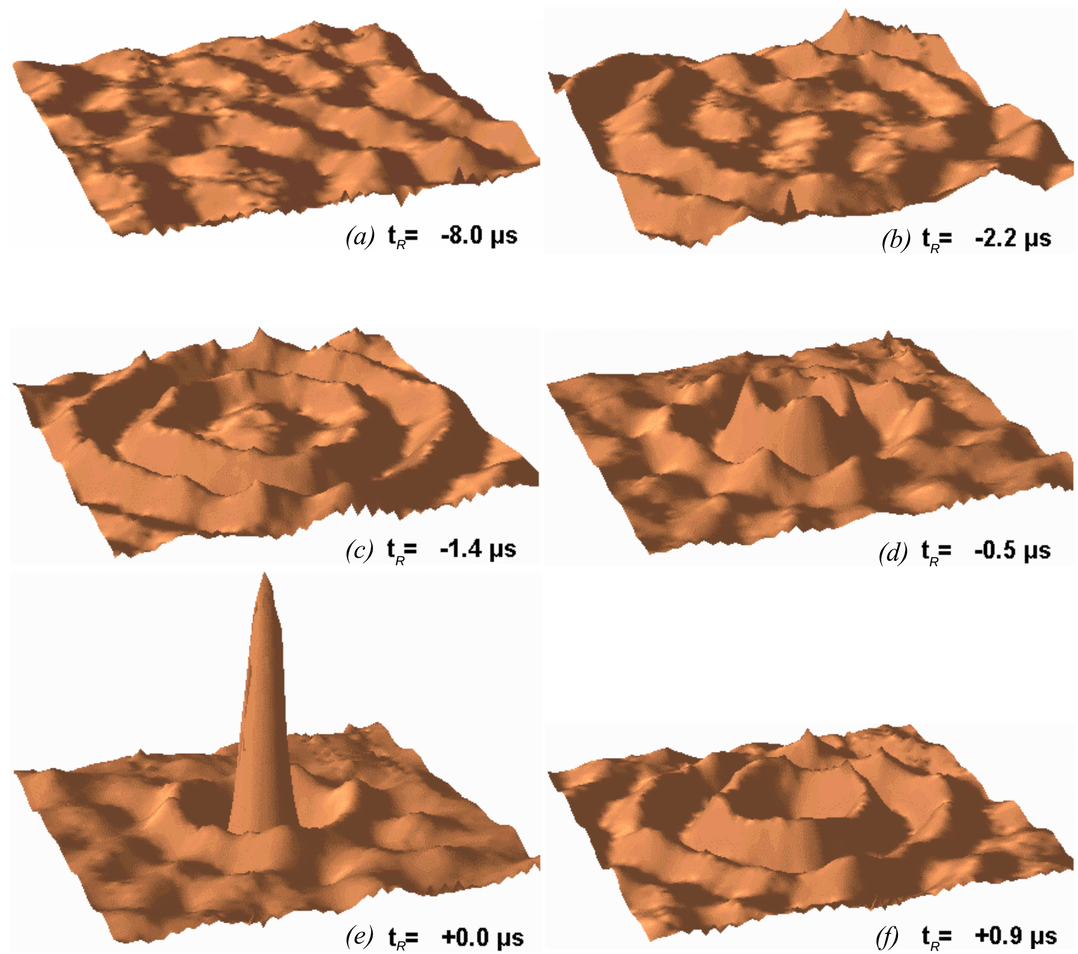


A 2ms window corresponds to the Heisenberg time of the cavity : $\tau_{Heis} = \frac{1}{\delta\omega}$

with $\delta\omega$ being the mean distance between modes



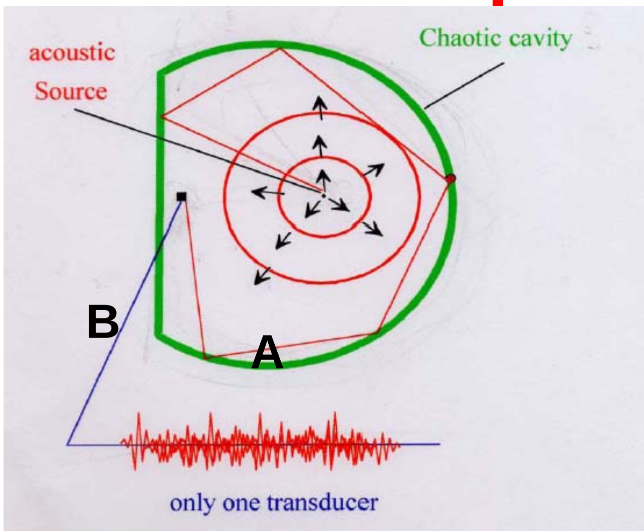




The signal to noise
Is proportional to the
bandwidth square: $\sqrt{\frac{\Delta\omega}{\delta\omega}}$

Indeed, a monochromatic
source located at B will
never focused at A

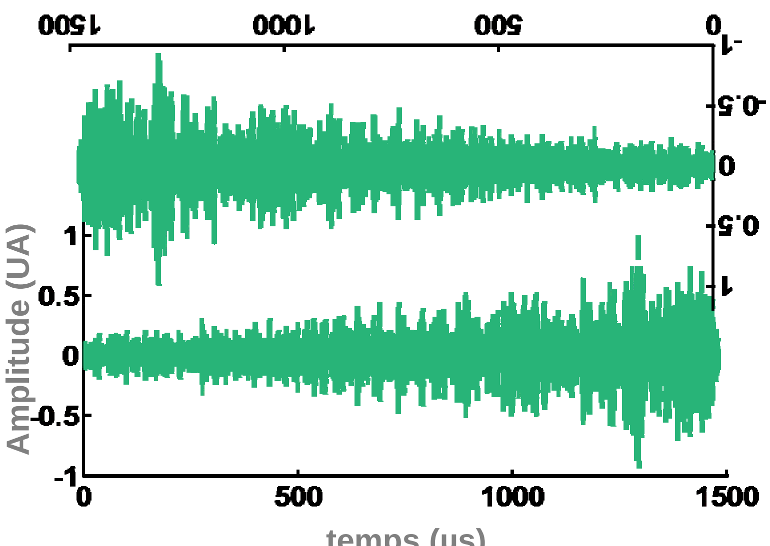
An important formula

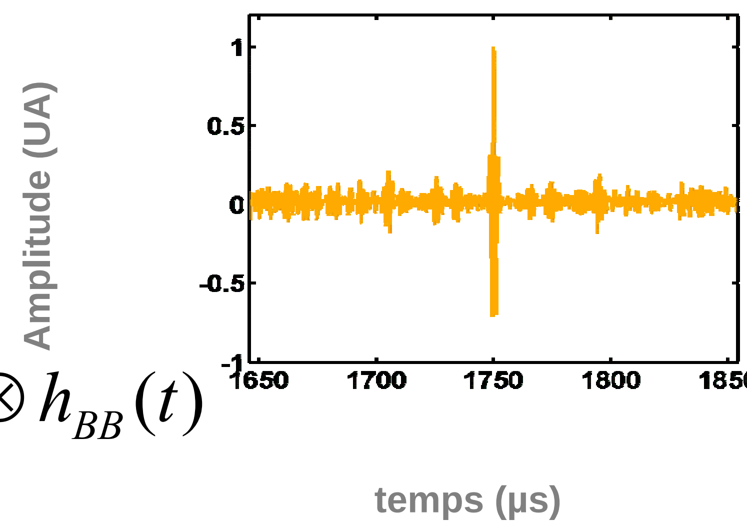


In terms of the cavity modes
A and B cannot exchange
all informations, because
A and B are always at the
antinodes of some modes

$$h_{AB}(t) = \sum_n \psi_n(A)\psi_n(B)\frac{\sin(\omega_n t)}{\omega_n}$$

ψ_n eigenmodes



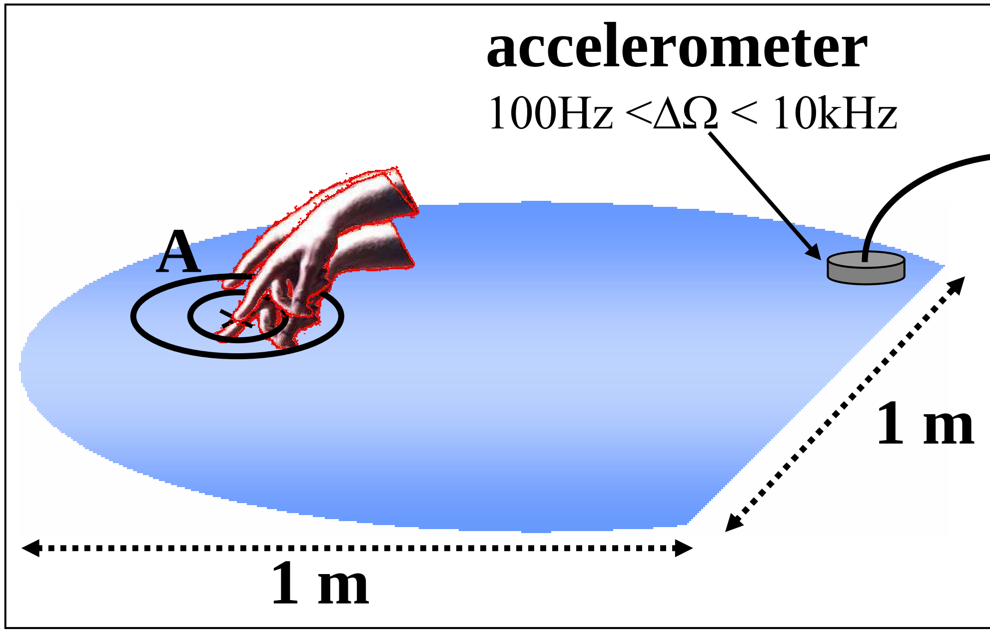


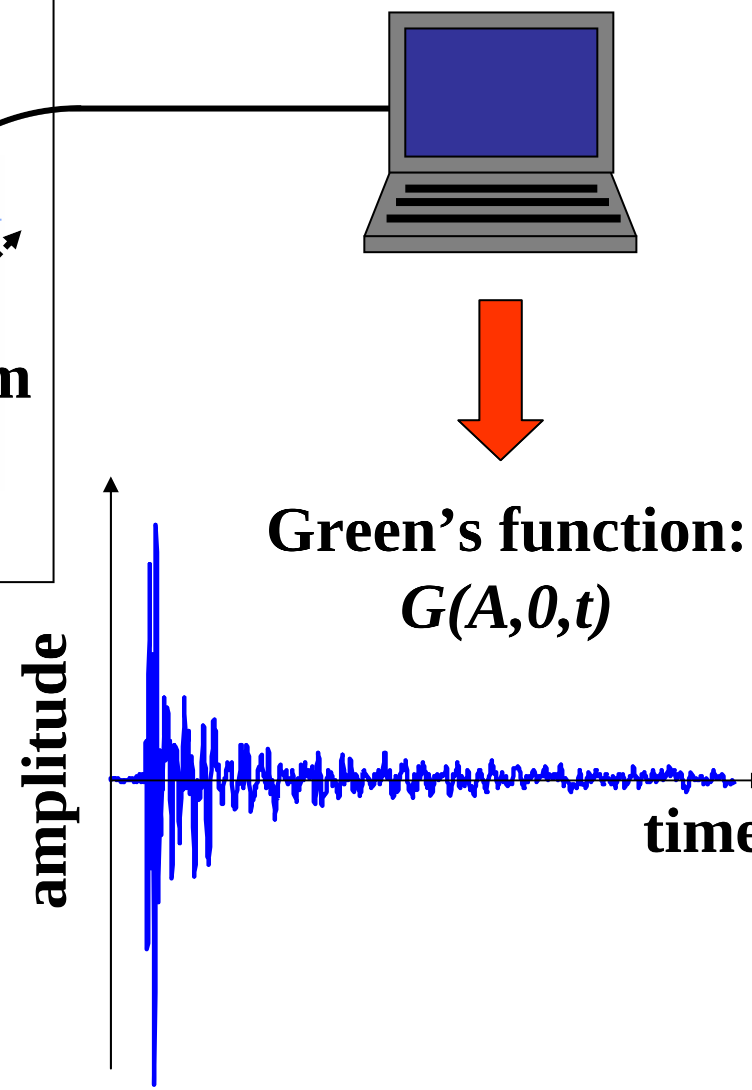
$$h_{AB}(-t) \otimes h_{BA}(t) = h_{AA}(-t) \otimes h_{BB}(t)$$

Cartsen Draeger and M Fink

A nice application : Interactive Objects

How to transform any object in a tactile screen ?





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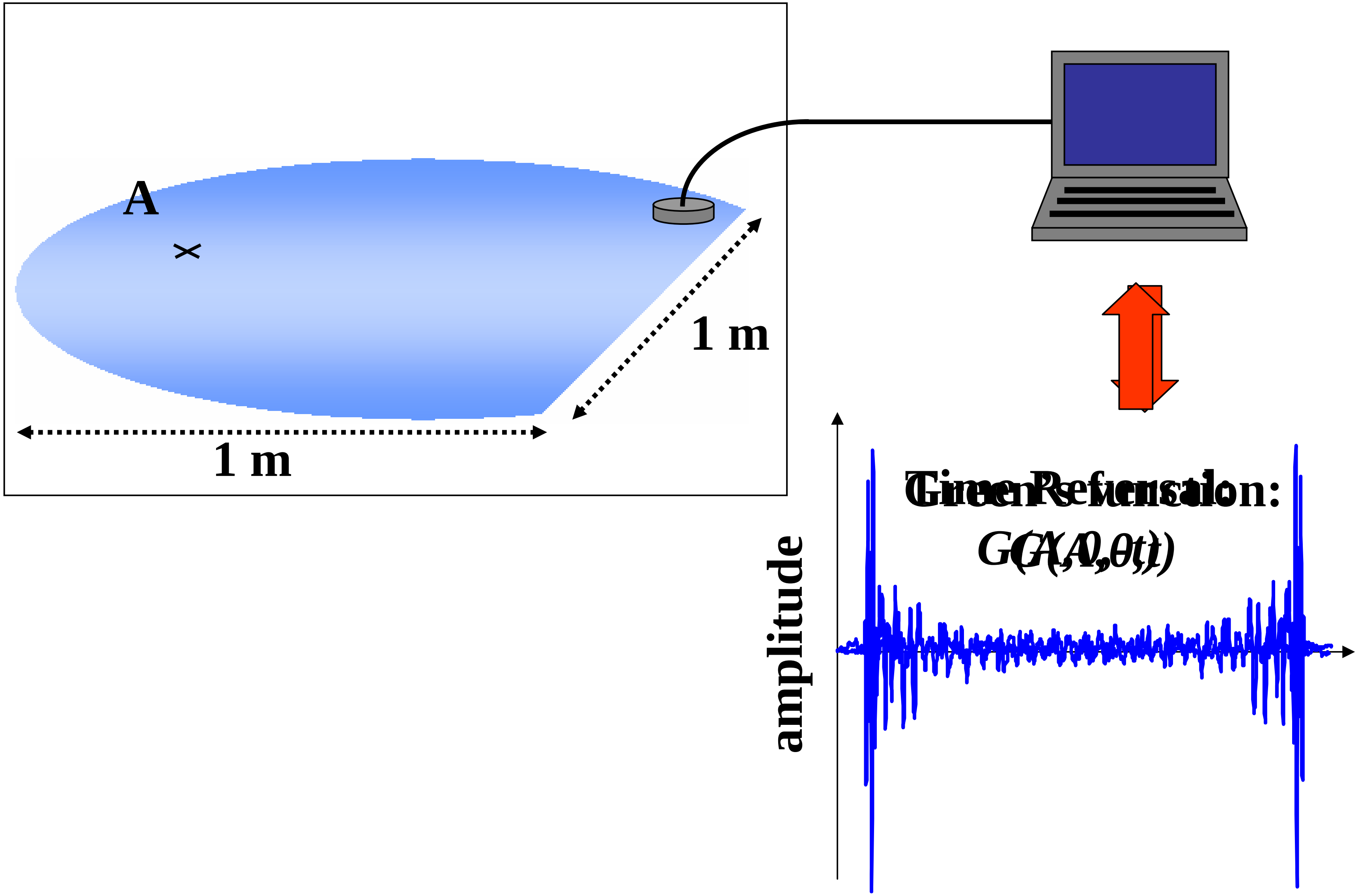
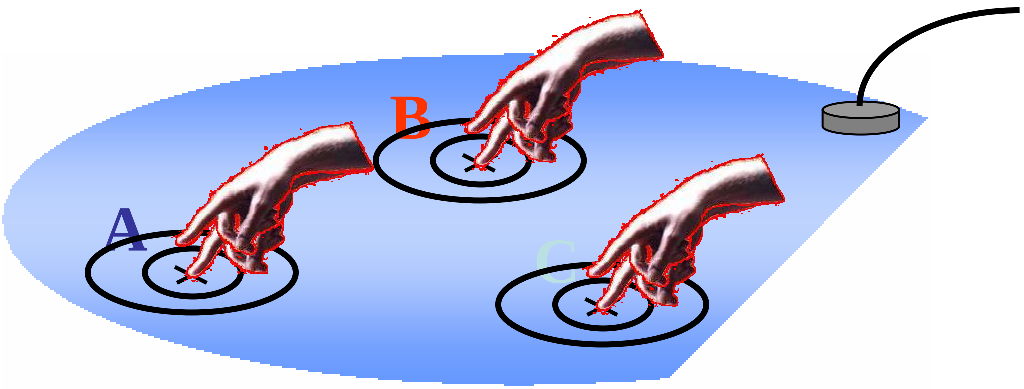
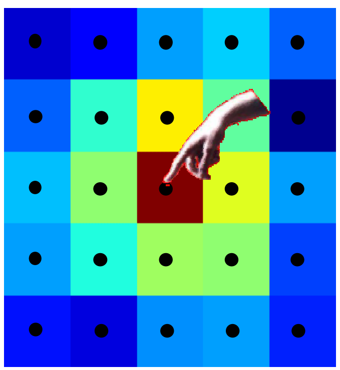
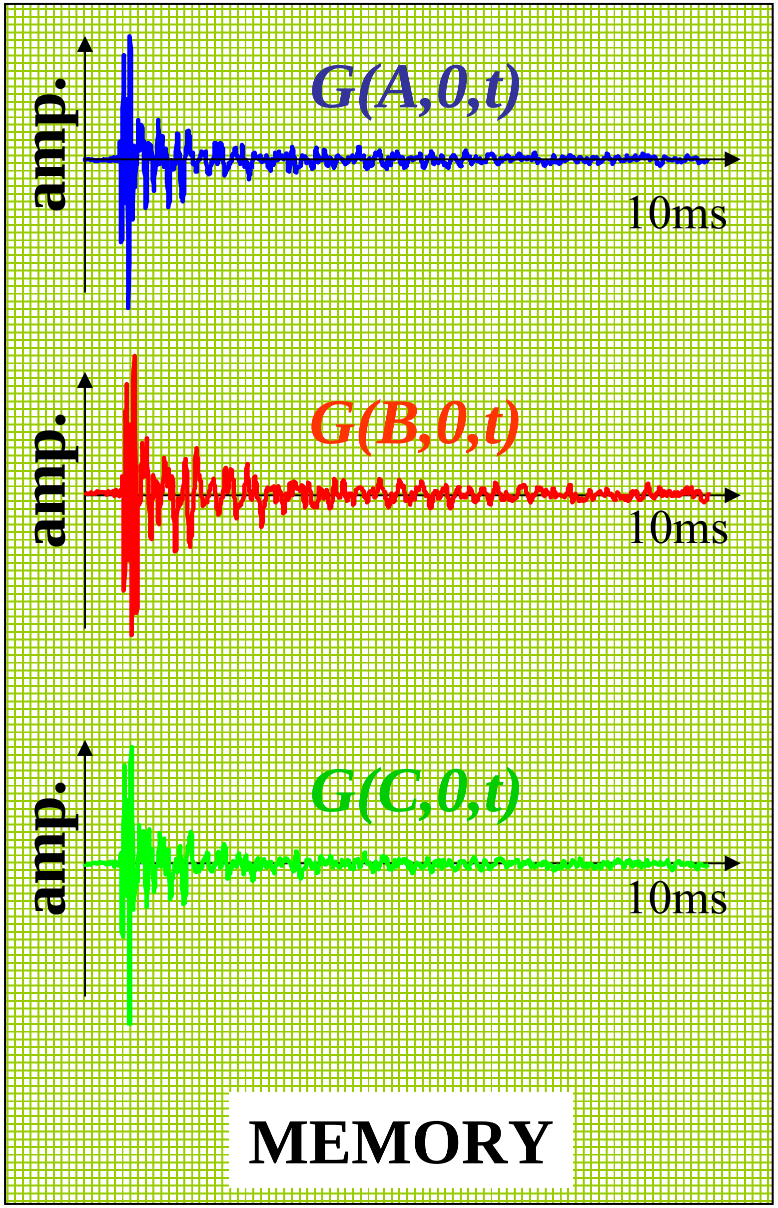

A
1 m
1 m
Time Reversal:
Green's function:
$G(A,0,t)$
amplitude

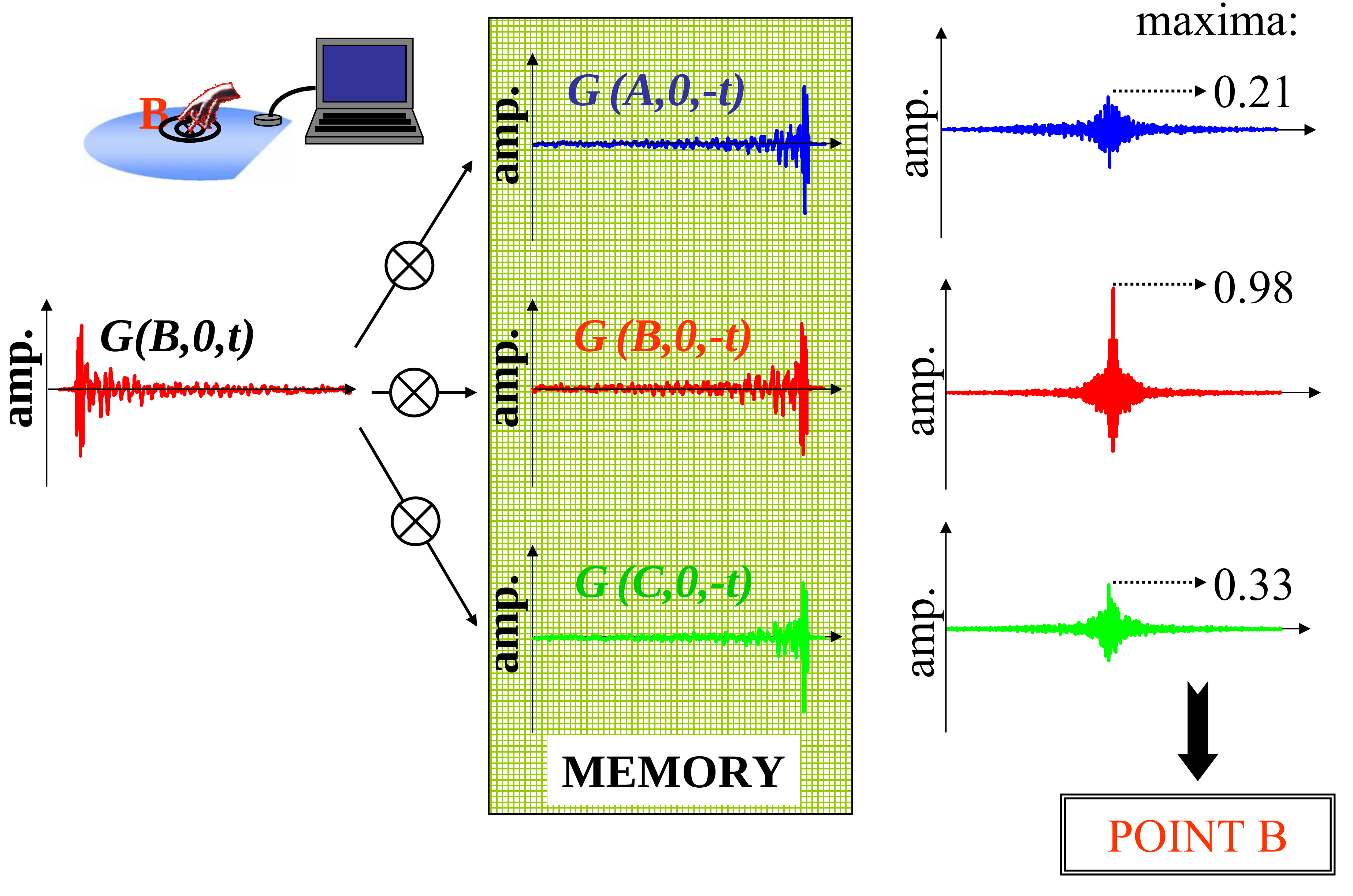
Training step: library of Green functions



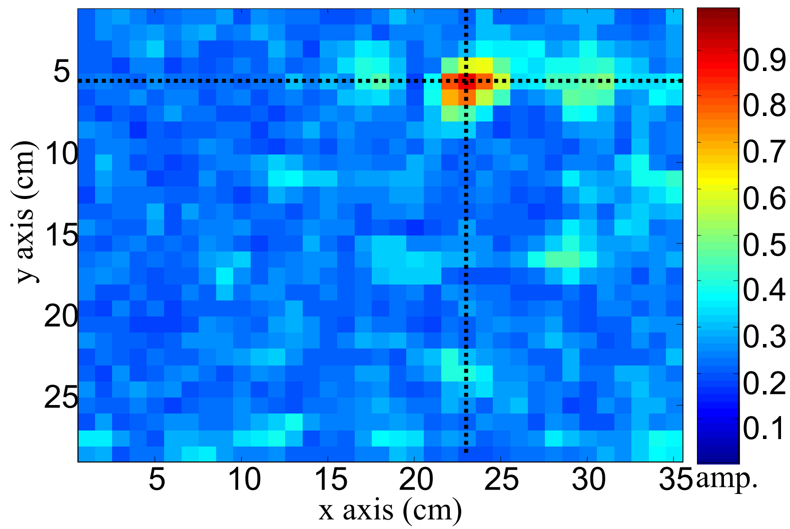


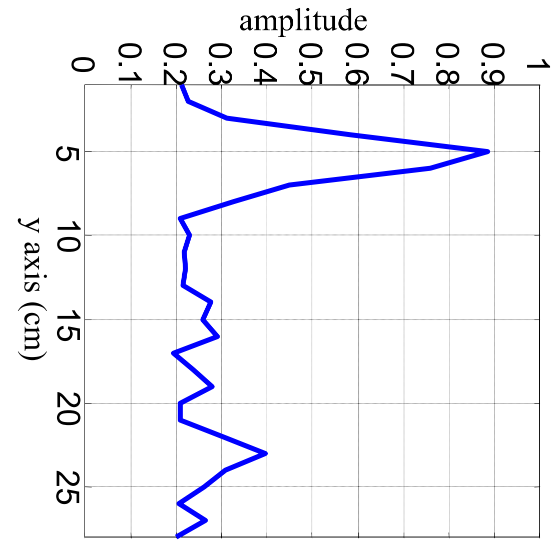


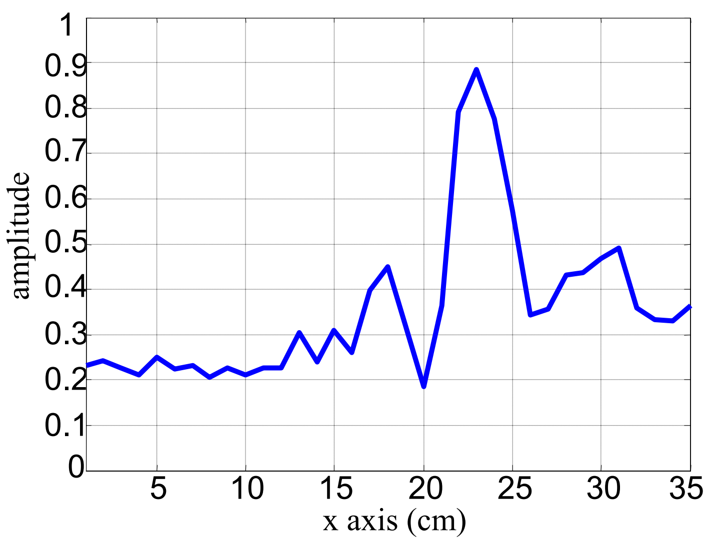
Source Localisation by cross correlation mimicking a time reversal experiment

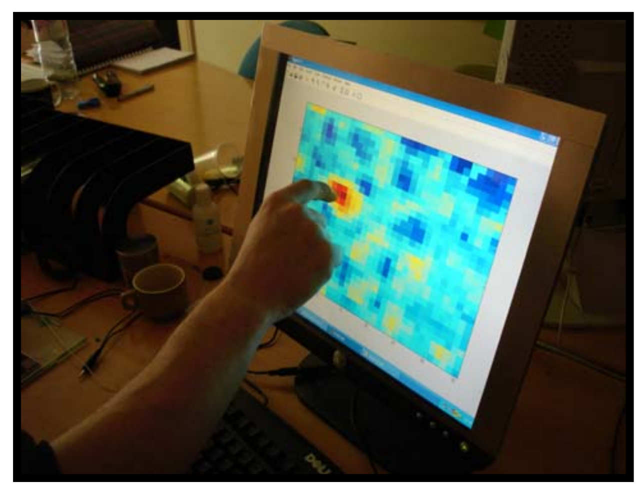


Focusing pattern



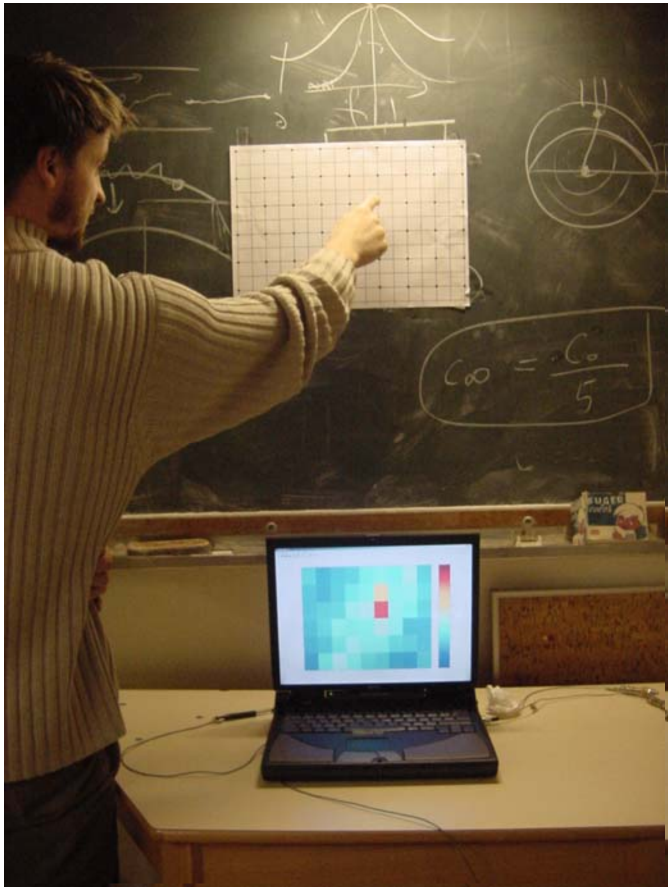




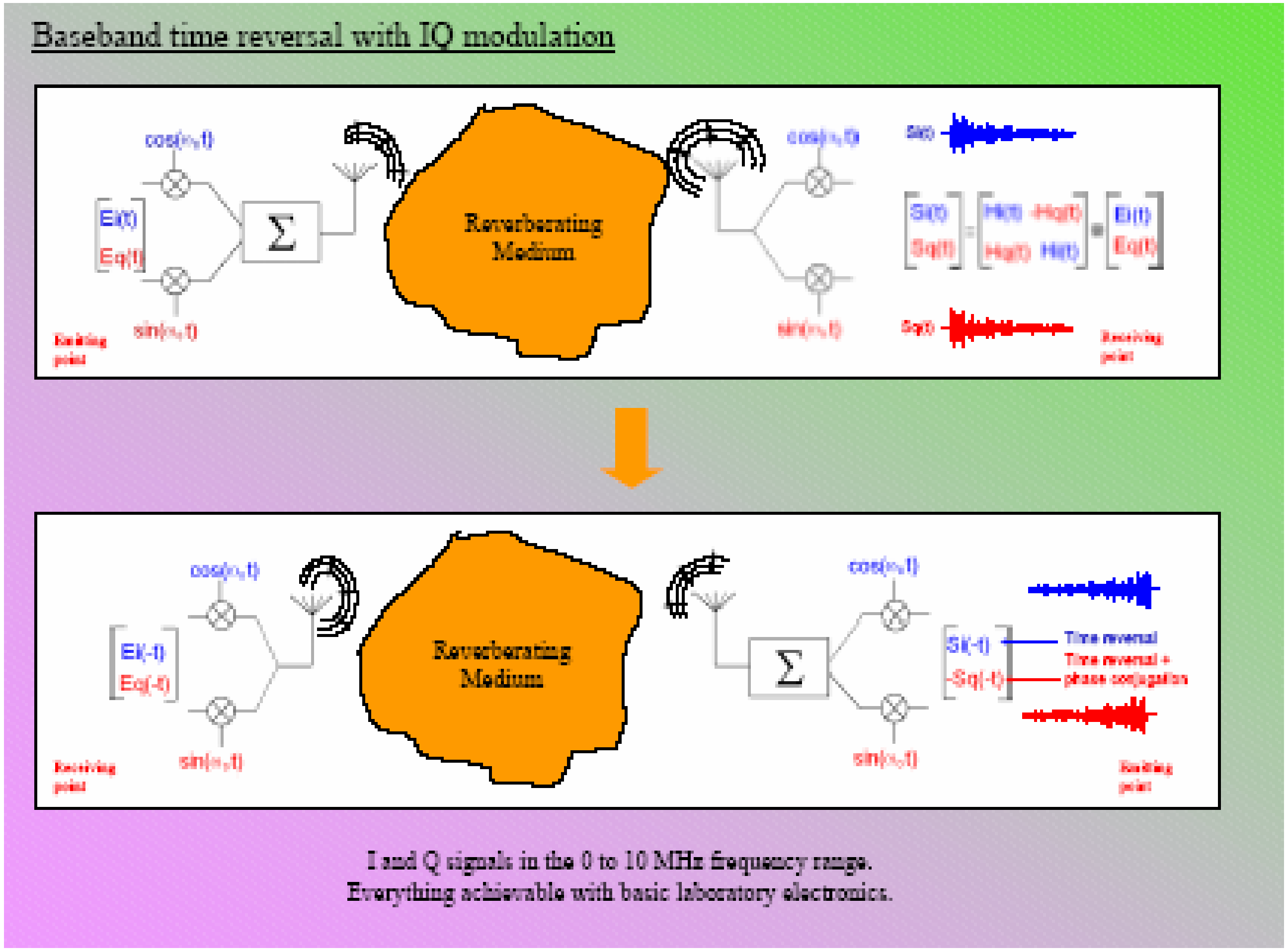


Sensitive Object Technology

Tactile Objects

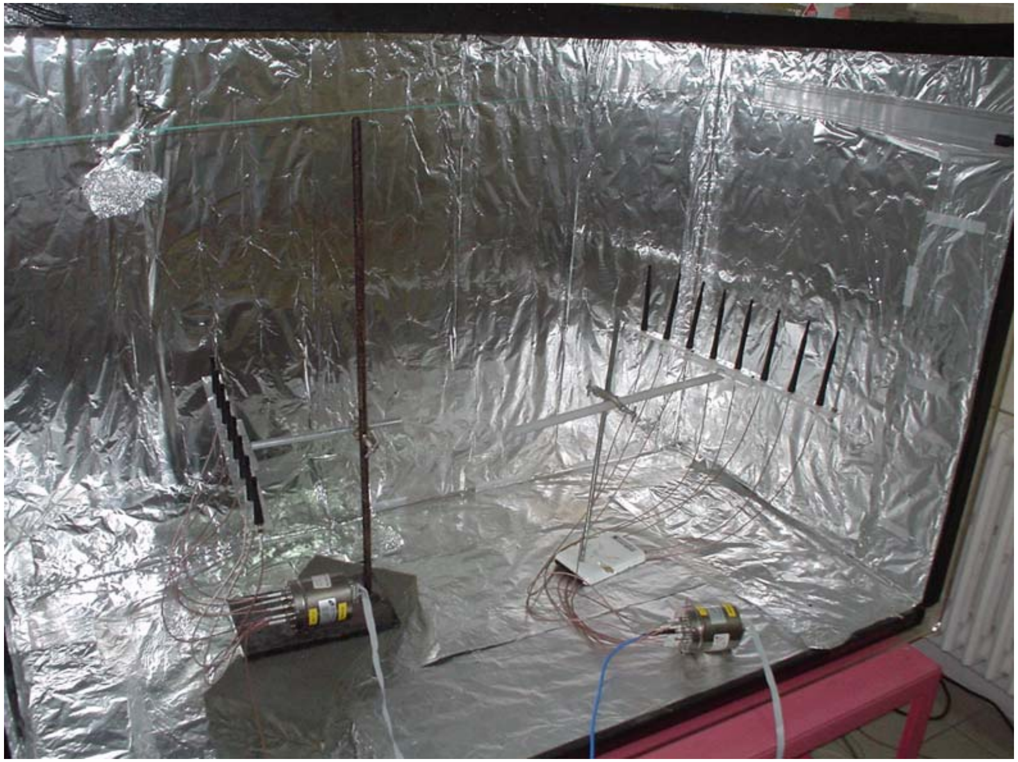


Time Reversal with Electromagnetic Waves (2.4 GHz)



G.Lerosey , J de Rosny, A Tourin, A Derode, G Montaldo M Fink,

Electromagnetic TRM

- 2 arrays of 8 antennas separated of approx 6.15 cm, i.e. half a wavelength (12.3cm @ 2.44 GHz)
- 

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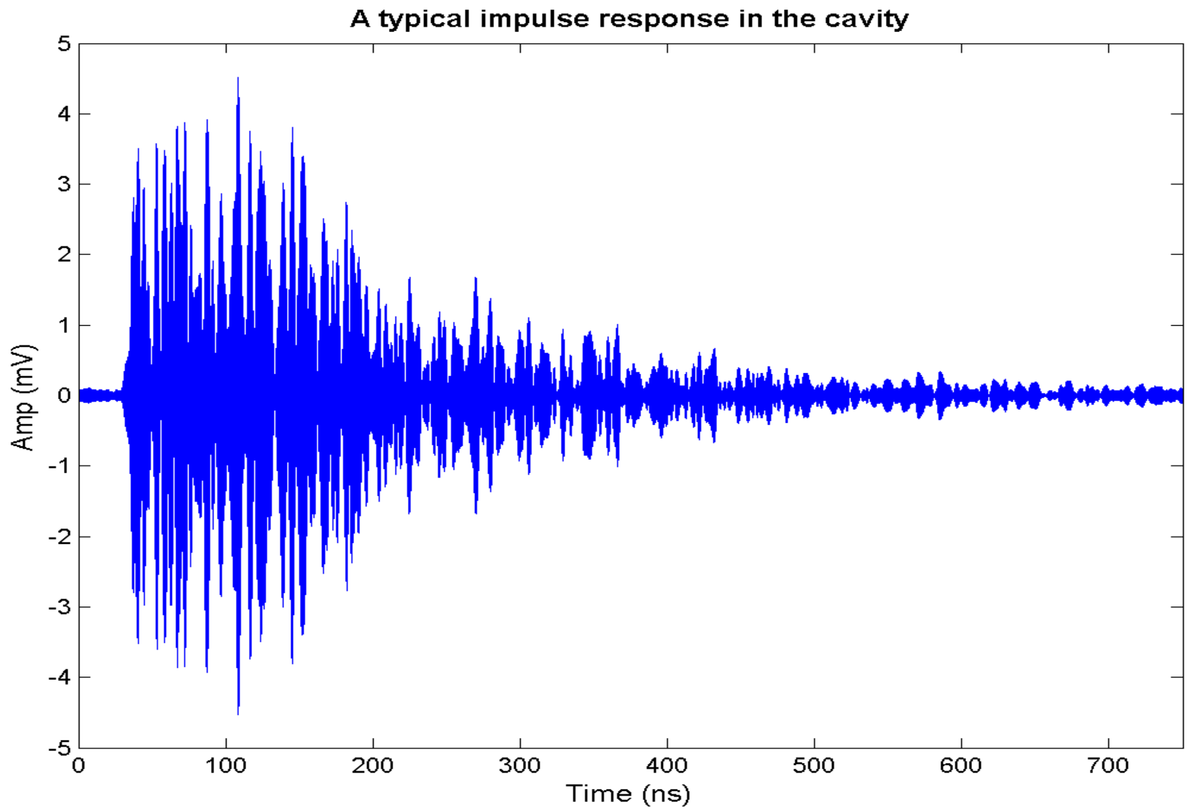
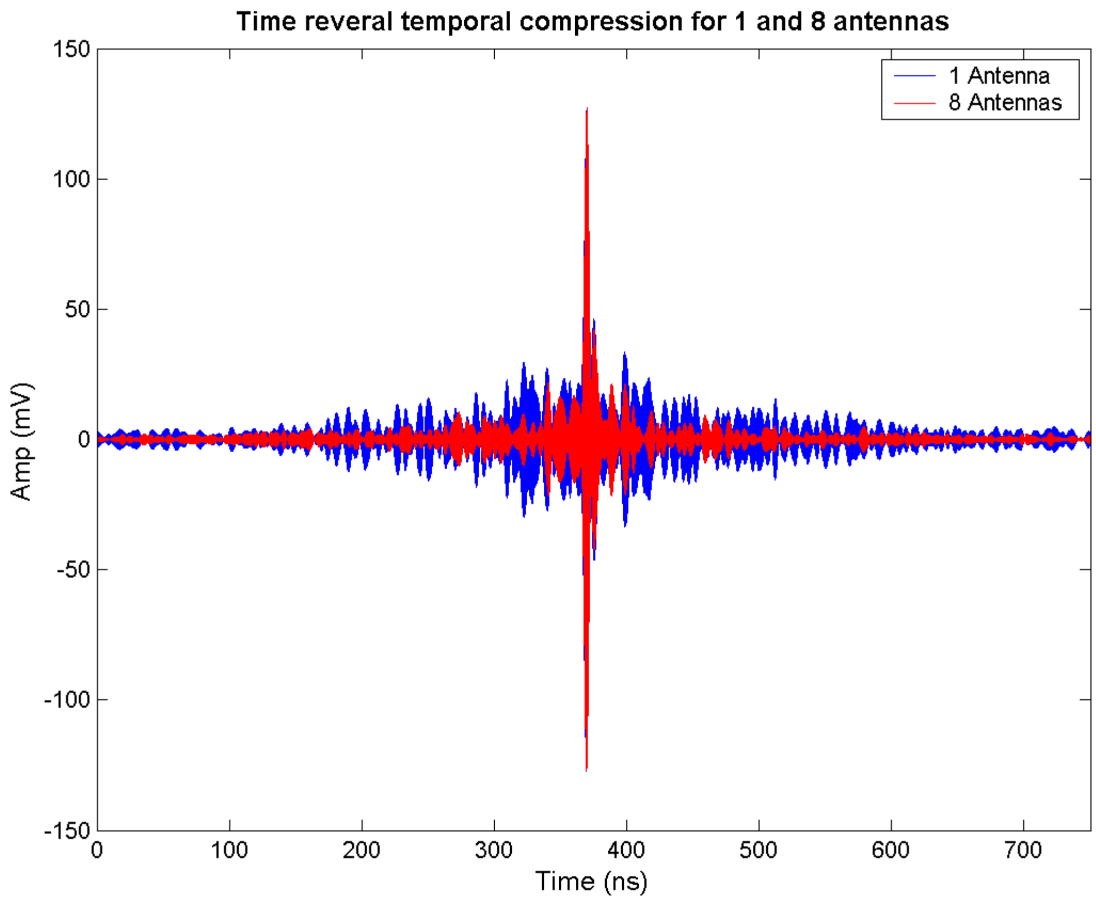
Impulse response in cavity



- Original pulse sent: gaussian, 10 ns long on the I channel.
- Estimated RMS delay spread: 160 ns. (on 64 responses).

Time reversal temporal compression

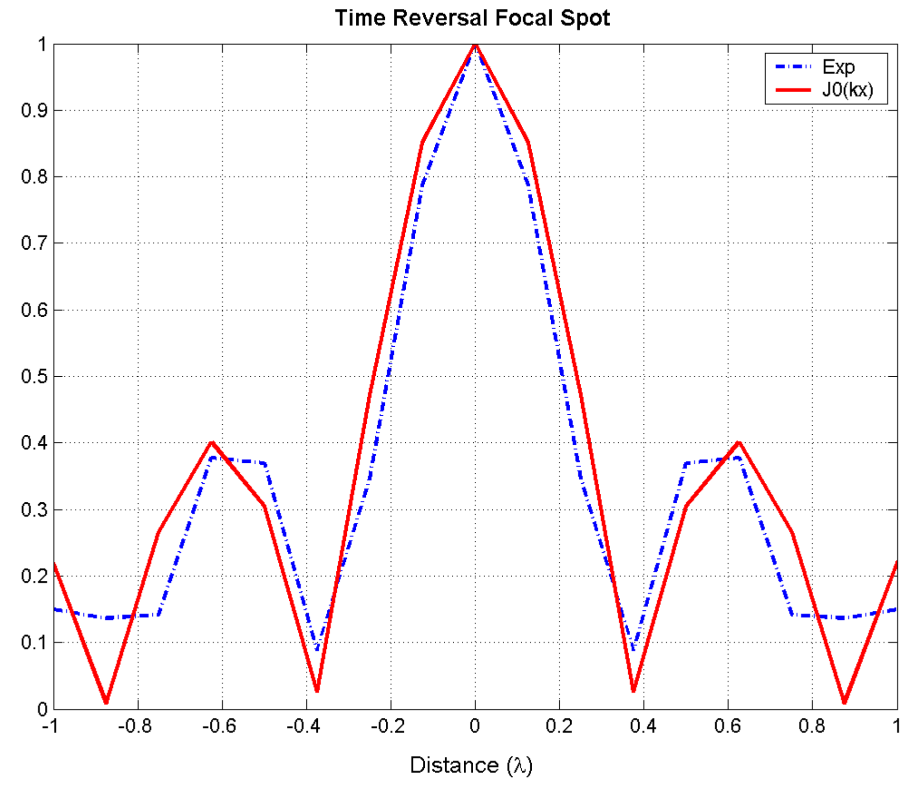


- Compression Amplitude Gain ~ τ_{RMS}
- Original pulse recovered.

Spatial Focusing

⇒ Typical λ/2 wide focal spot. Bessel like (2D approx, Jake, Clarke).

⇒ More precise (3D real study, De Doncker).

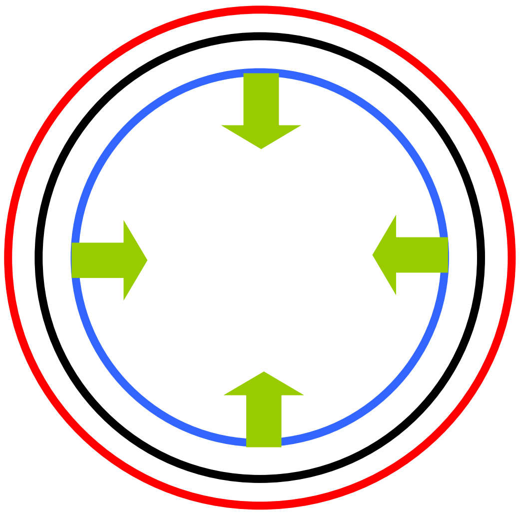


And Super Resolution !!

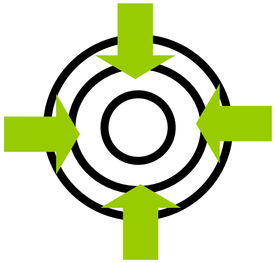
1- The acoustic sink

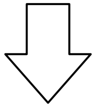
2- Media with sophisticated Green's functions

The Acoustic Sink



Converging only





$$\hat{G}^{0}_{adv}(R,\omega) \propto \frac{\exp\{j(-kR-\omega t)\}}{R}$$

with a singularity

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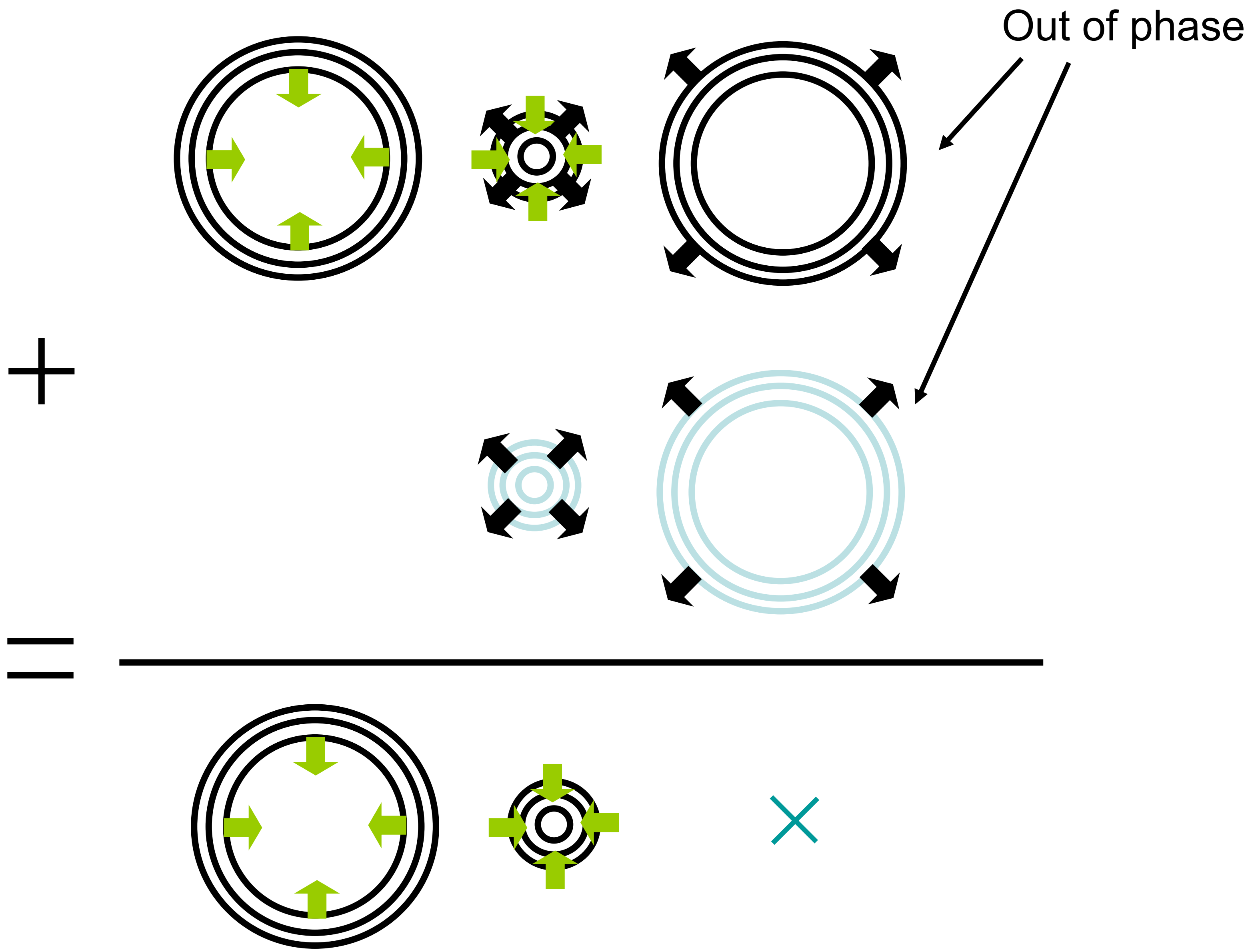
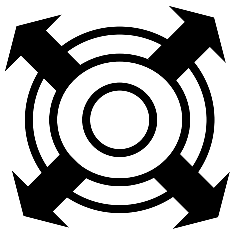
Principle of the acoustic sink



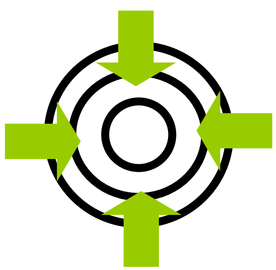
The Acoustic Sink Formalism



$$\left(\Delta - \frac{1}{c^2}\frac{\partial}{\partial t^2}\right) p(\vec{r},t) = f(t)\delta(\vec{r}-\vec{r}_0)$$

Propagating term

Point-like source

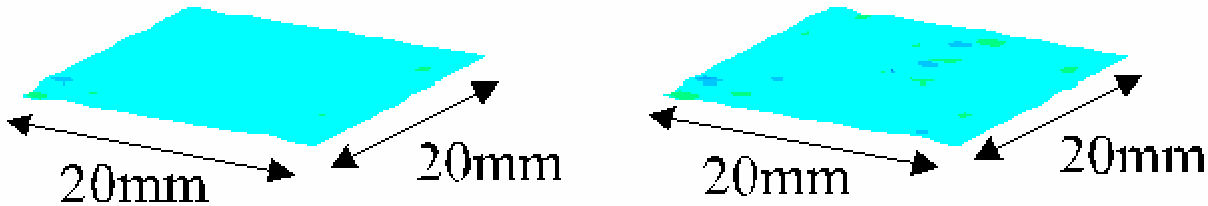


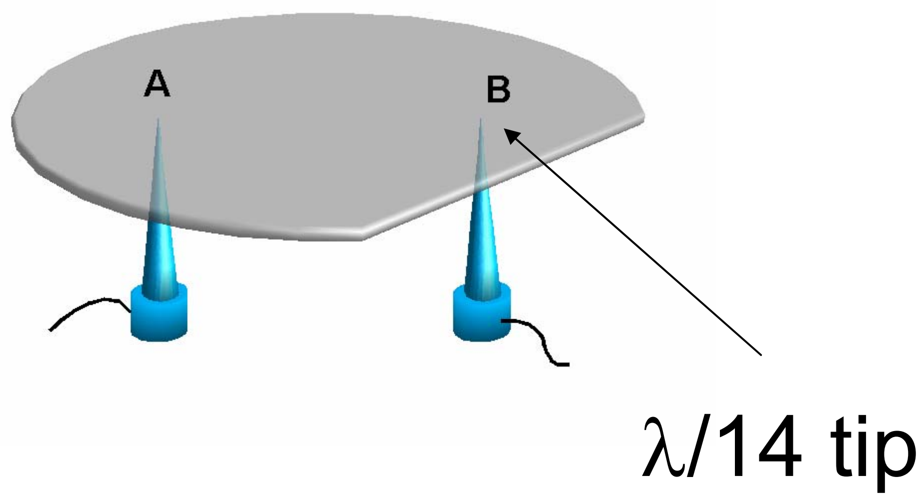
$$\left(\Delta - \frac{1}{c^2}\frac{\partial}{\partial t^2}\right) p(\vec{r},-t) = f(-t)\delta(\vec{r}-\vec{r}_0)$$

Converging wave

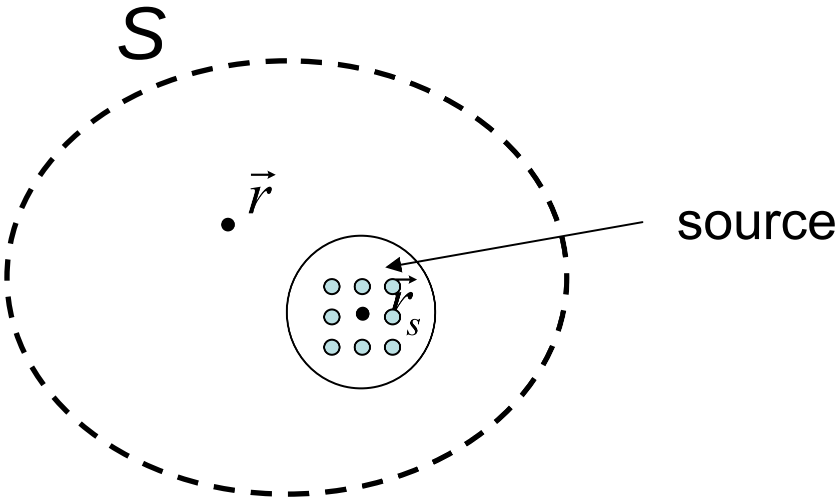
Source at r_0 excited by *f(-t)*

(TR source)


20mm
20mm
20mm
20mm


A
B
λ/14 tip

Sophisticated Green's functions



• select a medium with $\boxed{\mathrm{Im}\left\{\hat{G}_{ret}(\vec{r},\vec{r}_s,\omega)\right\}}$ that oscillate faster than the wavelength

An homogenous medium is not interesting

$$\hat{G}^0_{ret}(\vec{r},\vec{r}_s,\omega)=\frac{\exp\left(jk\left|\vec{r}-\vec{r}_s\right|\right)}{k\left|\vec{r}-\vec{r}_s\right|}$$

$$\mathrm{Im}\left\{\hat{G}^0_{ret}(\vec{r},\vec{r}_s,\omega)\right\}\prec\frac{\sin\left(k\left|\vec{r}-\vec{r}_s\right|\right)}{k\left|\vec{r}-\vec{r}_s\right|}$$

Build media with complex pattern in the near field of the source : obstacles or antenna

How to create a fast oscillating Im{G} around the source ?

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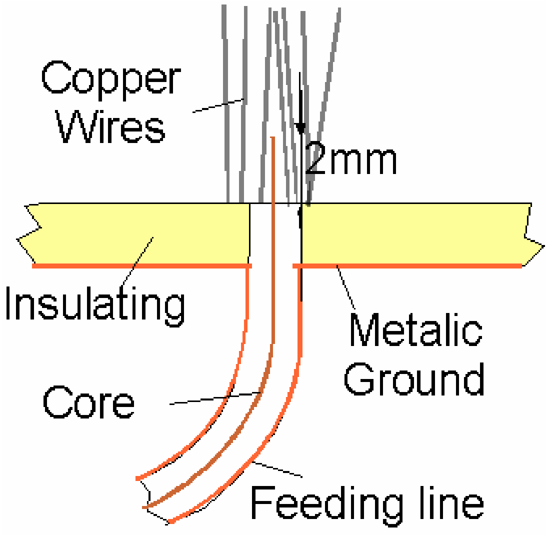
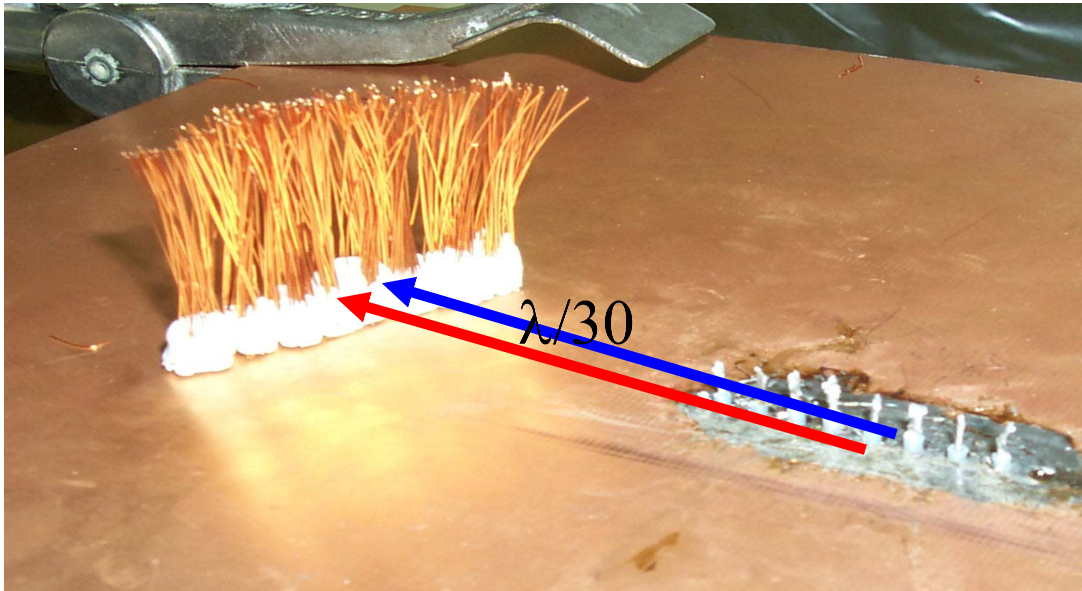
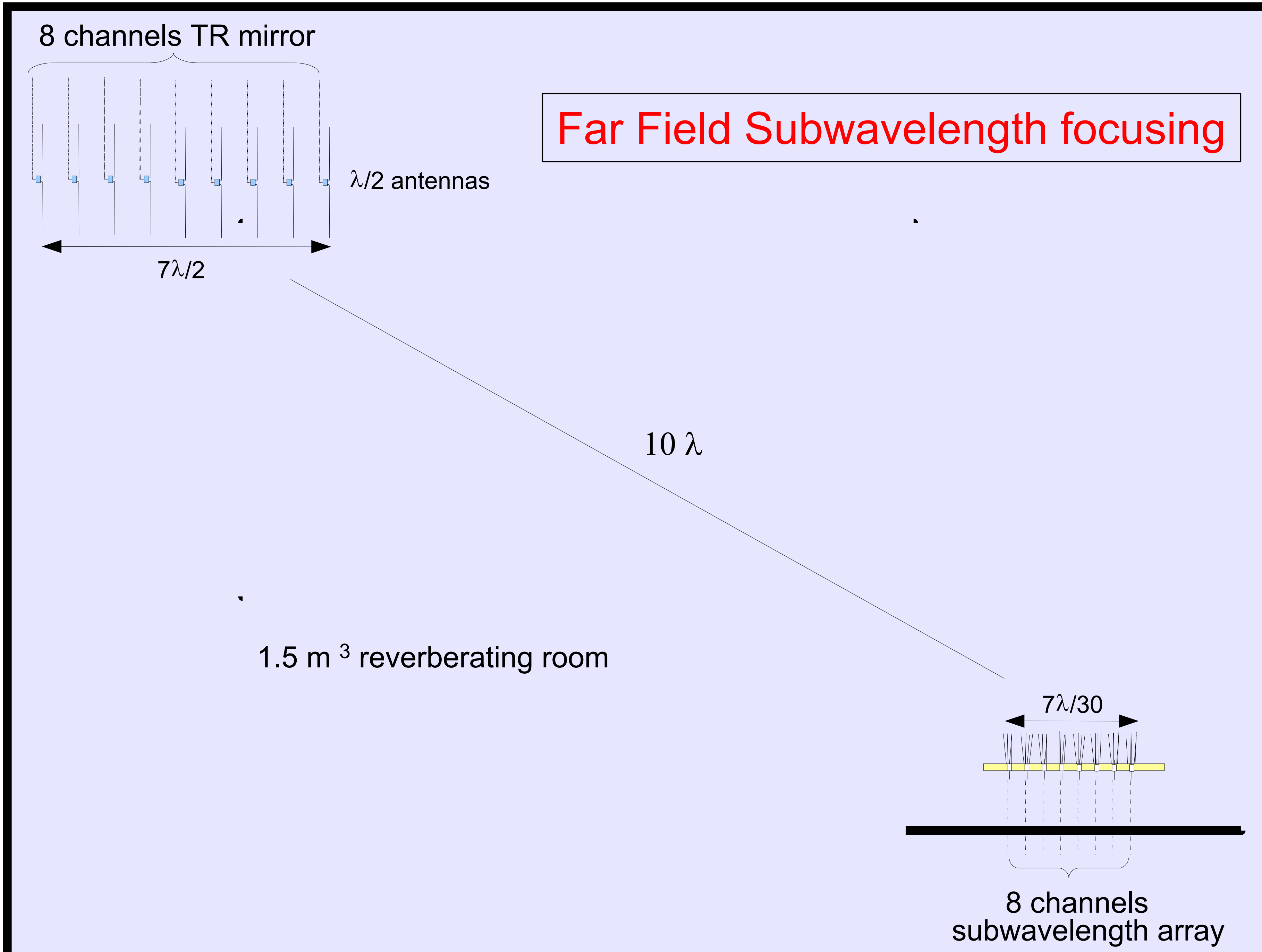
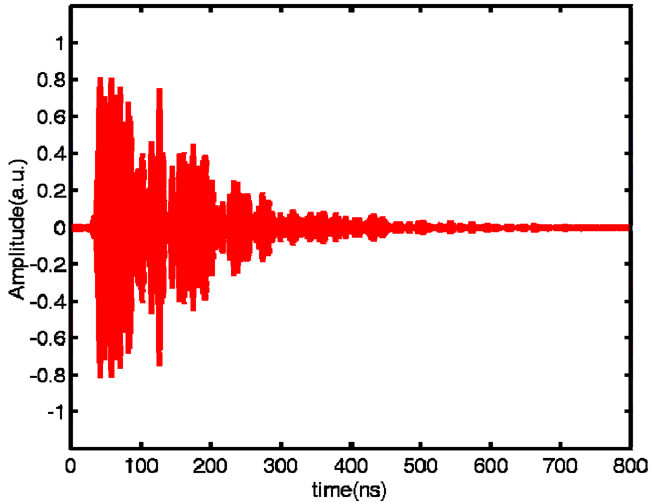
An Electromagnetic Example

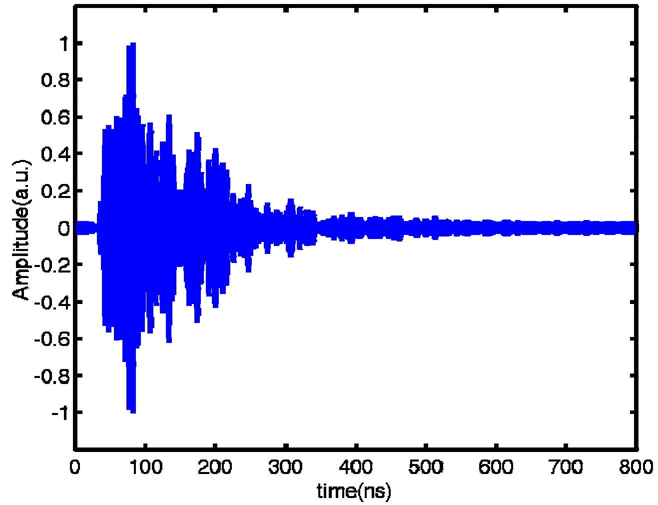




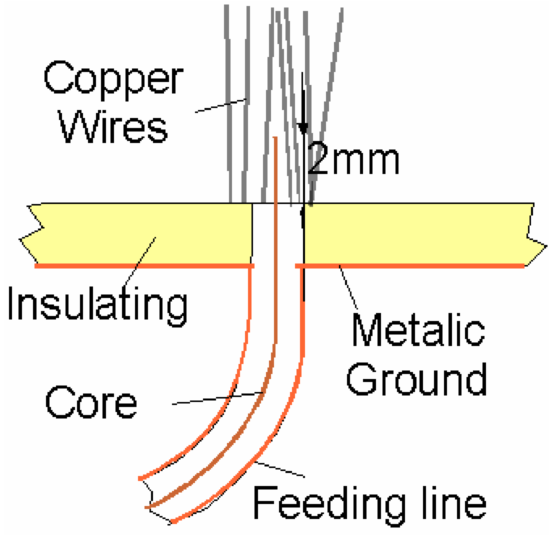
Far Field Subwavelength focusing

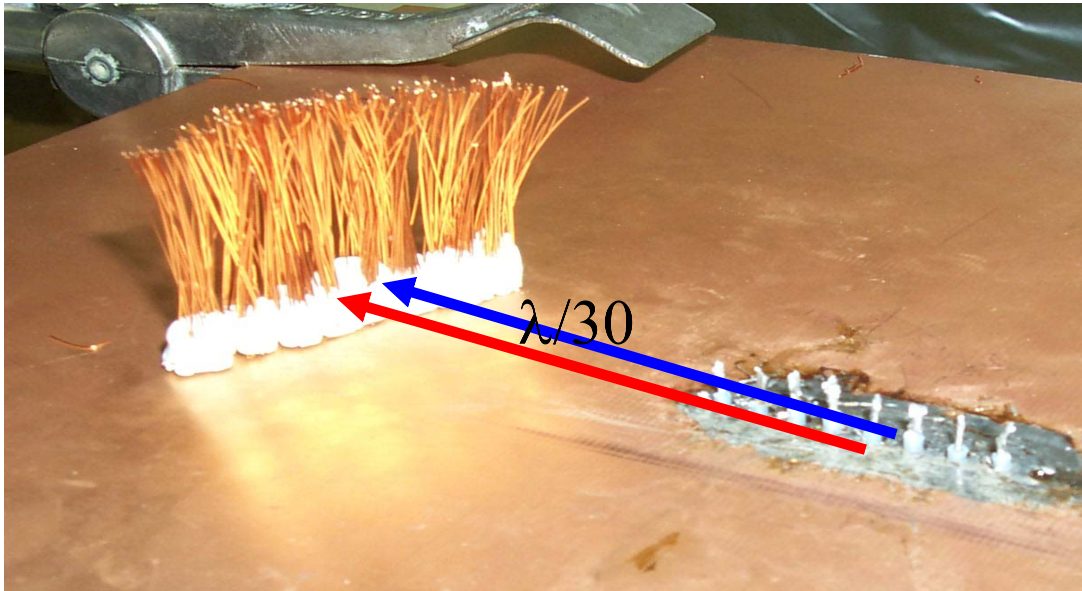




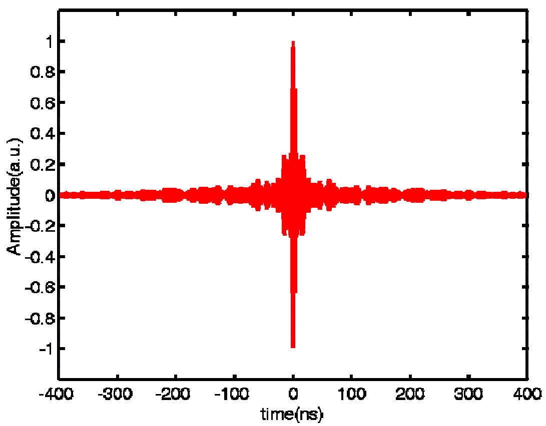


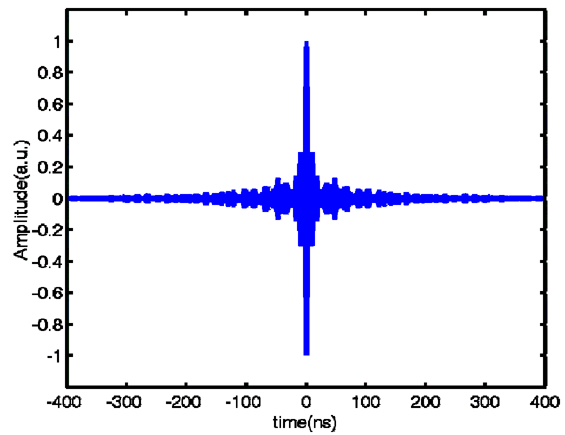
Signals received by one channel of the time reversal mirror



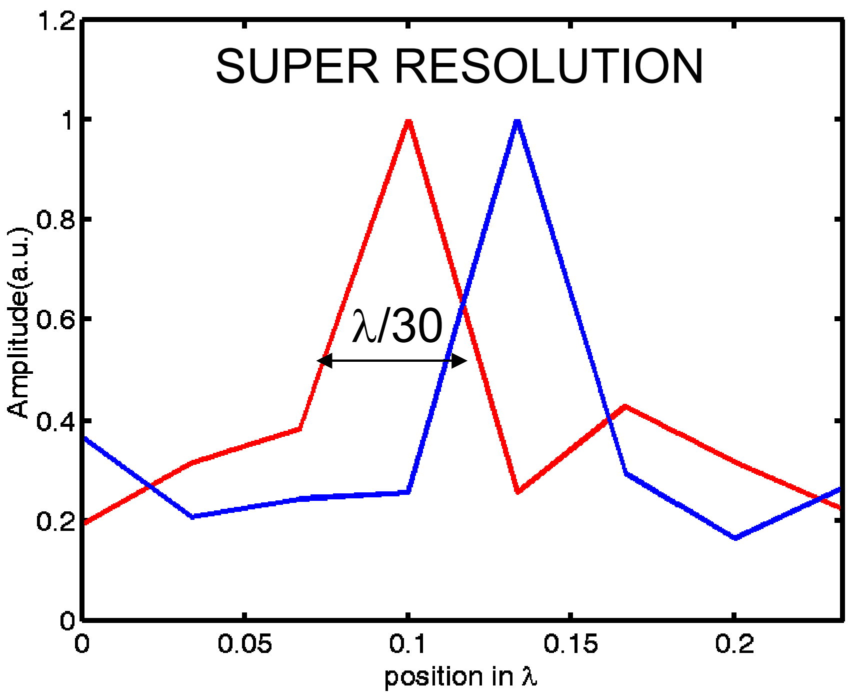


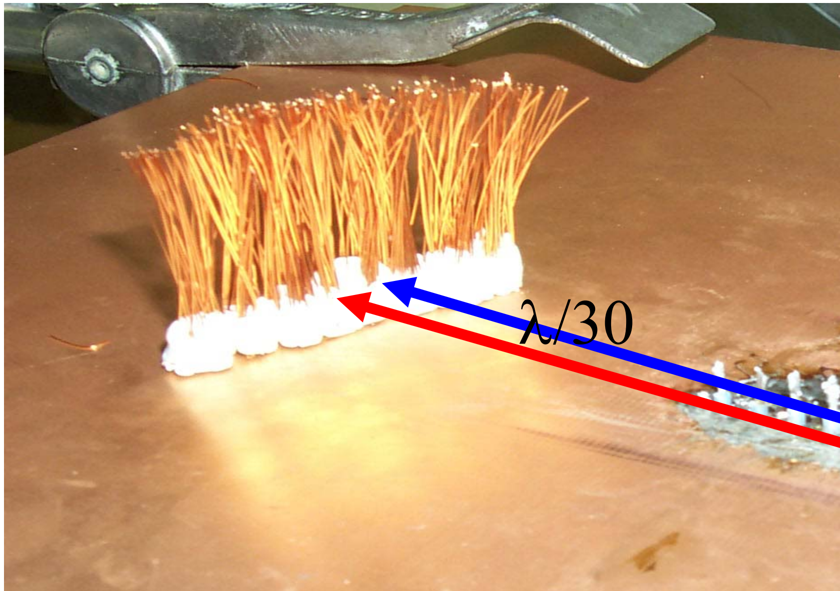
FAR FIELD TR SUBWAVELENGTH FOCUSING





Time reversed signals observed on the red and the blue antenna





Telecommunications

3 bitstreams (RGB) with a data rate of 50 Mbits/s each.
The intended global data rate is thus 150 Mbits/s.

The TRM is made of 3 antenna
2.45 GHz central frequency
180 MHz bandwidth