Imaging and monitoring with coda waves and microseisms

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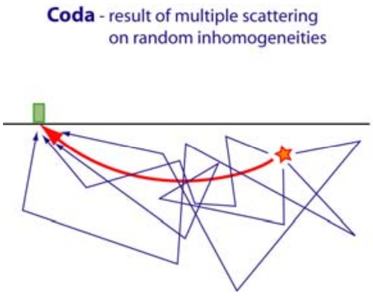
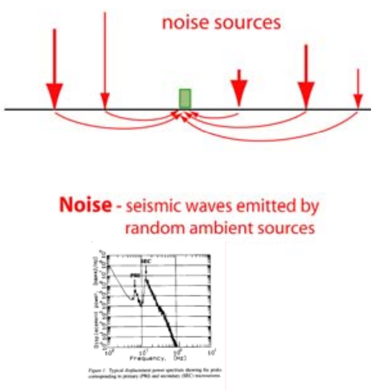
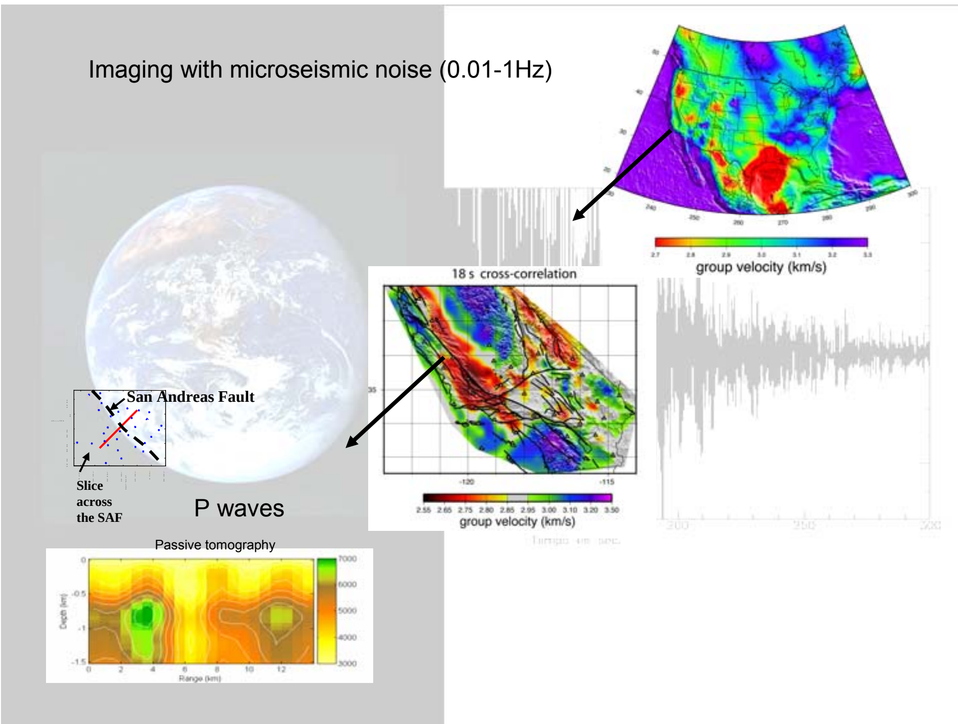
Coda and ambient noise







Mathematical (firm) ground:

The case of homogeneously distributed white noise sources in *an arbitrarily complex medium*:

$$\frac{\partial^2 u}{\partial t^2} + 2\,a\,\frac{\partial u}{\partial t} - L\,u = f$$

If $L = c^2(\vec{r})\,\Delta$, we recognize the usual wave equation.

f is a white noise distributed everywhere in the medium, such that the position and the activation time of each source are uncorrelated.

With a small damping factor *a* , it has been shown (e.g. Colin de Verdière, 2006; Gouedard et al., 2007, see also Roux et al., 2004) that:

$$\frac{d}{d\tau}\,C_{A,B}(\tau) = \frac{-\sigma^2}{4\,a}\left(G_a(\tau,\vec{r}_A,\vec{r}_B) - G_a(-\tau,\vec{r}_A,\vec{r}_B)\right)$$

Limitations: source distributions…… but complexity helps!

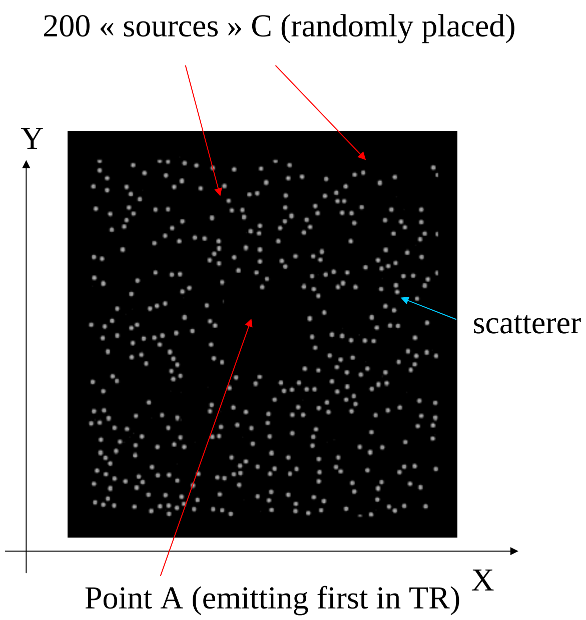
PHYSICAL INTERPRETATIONS:
(Derode et al.,2003; …..)

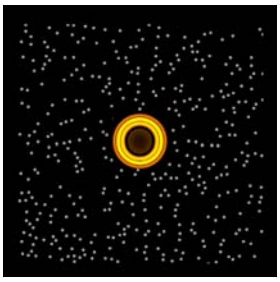
Correlation vs Time Reversal

Correlation	Time Reversal
• C source • A and B receivers	• A source • C receiver • C re-emits time-reversed field • B receiver
• Correlation :	• Convolution :

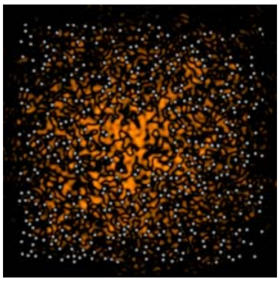
$$S_{CA}(t) \text{ x } S_{CB}(t) = S_{CA}(t) \otimes S_{CB}(-t)$$

Numerical 2D FD simulation

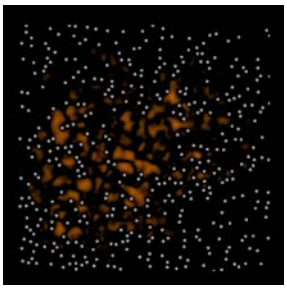




A pulse is emited in A
and recorded at point randomly distributed

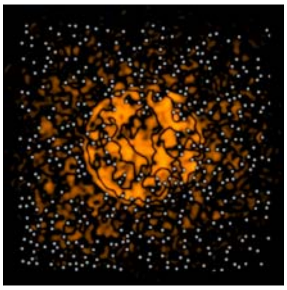


time

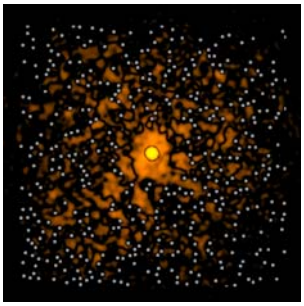


Re-emission from the points ‘C’ of the time-reversed signals

(= map of average crosscorrelations)

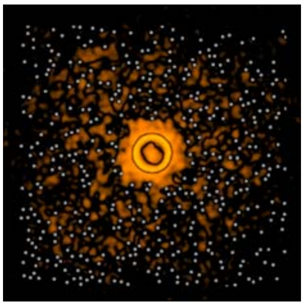


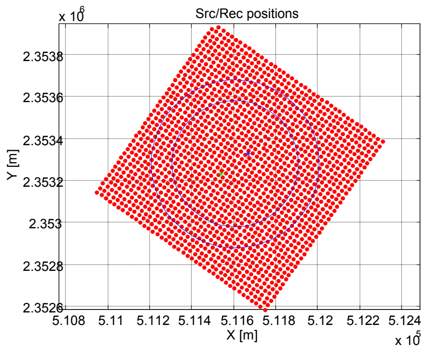
Converging field
: G(-t)



Nearly perfect refocalisation

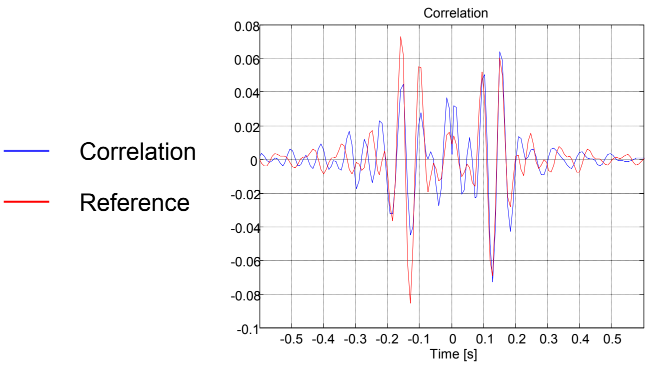
Re-emission from A :
G (t)



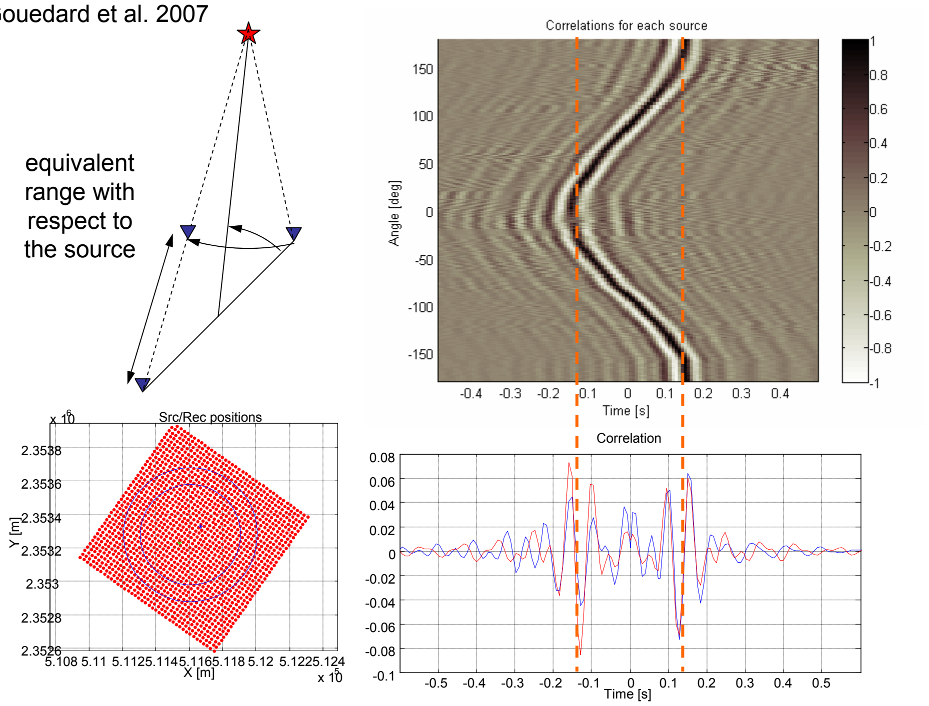


PHYSICAL INTERPRETATIONS:
Stationary phase (high frequency ray approx;

(Roux and Kuperman; Snieder,…)

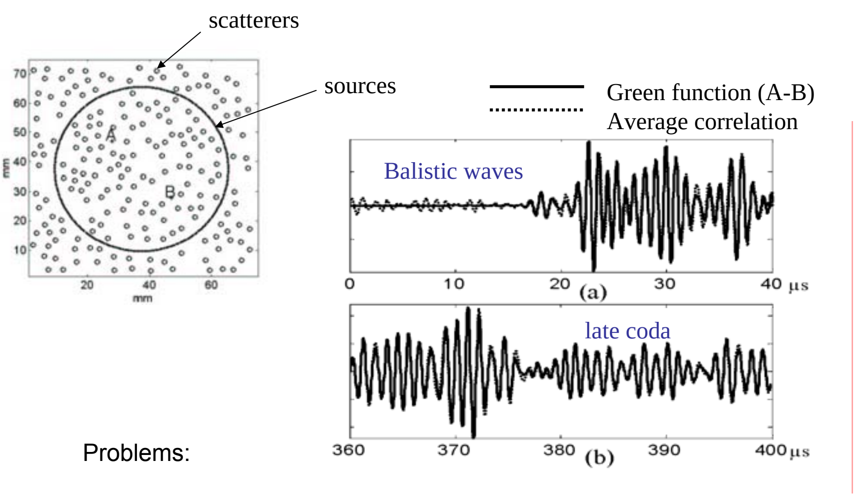


Gouedard et al. 2007



Emergence of the deterministic Green function

A numerical experiment with an open medium (absorbing boundaries):



Problems:

Representation theorem for elastic waves (stress tensor)?
Source distribution in practice ….

Derode et al., 2003

Waves in a complex medium:

Simple Representation of a diffuse field - Equipartition

Equipartion principle for a completely randomized (diffuse) wave-field: in average, all the modes of propagation are excited to equal energy.

Finite body: superposition of normal modes

$$\Psi(\vec{r},t) = \Re(\sum a_n u_n(\vec{r}) exp(i\omega_n t))$$

with

$$\langle a_n \rangle = 0;\ \langle a_n a_m^* \rangle = E(\omega_n)\delta_{nm}$$

2 point field correlation:

$$C(\vec{r},\vec{r'},t-t') = \left\langle \Psi(\vec{r},t)\Psi(\vec{r'},t') \right\rangle = \sum_n E(\omega_n) u_n(\vec{r}) u_n(\vec{r'}) cos(\omega_n(t-t'))$$

of Fourier transform:

$$C(\vec{r},\vec{r'},\omega) = \pi \sum_n E(\omega_n) u_n(\vec{r}) u_n(\vec{r'})(\delta(\omega-\omega_n)+\delta(\omega+\omega_n))$$

We can compare:

$$C(\vec{r},\vec{r'},\omega) = \pi \sum_n E(\omega_n) u_n(\vec{r}) u_n(\vec{r'})(\delta(\omega-\omega_n)+\delta(\omega+\omega_n))$$

with

$$\Im(G(\vec{r},\vec{r'},\omega)) = \pi \sum_n u_n(\vec{r}) u_n(\vec{r'})(\frac{\delta(\omega-\omega_n)+\delta(\omega+\omega_n)}{\omega})$$

Phase (derivative)
Spectrum (E, …)

Harmonic solution for the homogeneous open space:

superposition of plane waves at frequency ω

$$\Psi(\vec{r},t) = \oint a(\vec{n}) exp(i\omega t) exp(-i\vec{n}\vec{r}\omega/C) d^2\vec{n}$$

with

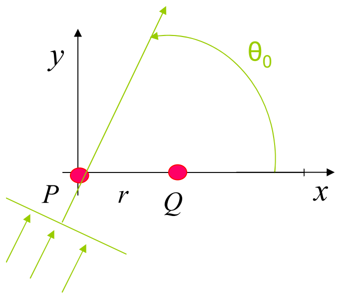
$$\langle a(\vec{n})\rangle = 0;\ \langle a(\vec{n}) a^*(\vec{n}')\rangle = A\delta^2(\vec{n}-\vec{n}')$$

Aki, 1957: *SPAC*

In this method one looks for surface waves velocities in order to find the structure. Seismic arrays are required to make the azimuthal average.

If moreover, noise is isotropic the same result can be obtained with only two stations by means of a long time average of the cross correlation.

Let's consider a plane wave

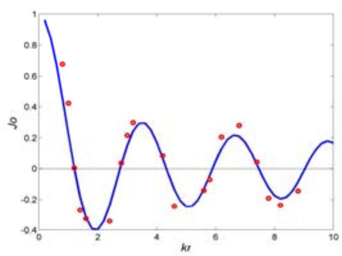


$$u(r,\theta,\omega) = F(\omega)\exp(-ikr\cos(\theta_0-\theta))$$

$$\frac{u^P u^{Q^*}}{|u^P||u^Q|} = e^{+ikr\cos\theta_0}$$

$$\langle\rho(r,\omega)\rangle = \left\langle \frac{u^P u^{Q^*}}{|u^P||u^Q|} \right\rangle = \left\langle e^{ikr\cos\theta_0} \right\rangle = \frac{1}{2\pi}\int_0^{2\pi} e^{ikr\cos\theta_0}\, d\theta_0 = J_0(kr)$$

azimuthal average of the spatial cross-correlation



$$G_{22}(r,\omega) = \frac{1}{4\mu}\left\{ -Y_0\left(\frac{\omega r}{c}\right) - iJ_0\left(\frac{\omega r}{c}\right)\right\}$$

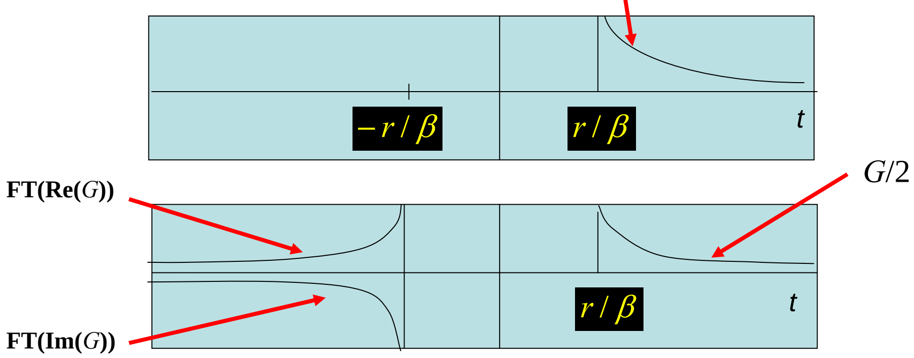
$$J_0\left(\frac{\omega r}{c(\omega)}\right) = -4\mu\,\mathrm{Im}\left(G_{22}(r,\omega)\right) \qquad r = |P,Q|$$

$$\mathrm{Im}\left(G_{22}^{PQ}\right) = \frac{-1}{4\mu}\left\langle \frac{u_2(P)u_2^*(Q)}{|u_2(P)||u_2(Q)|}\right\rangle$$

Causality

$G = \frac{1}{4i\mu} H_0^{(2)}\left(\frac{\omega r}{\beta}\right)$ in frequency domain

➔ in time domain: $TF(G) = \frac{1}{2\pi\mu}\frac{H\left(t - {r}/{\beta}\right)}{\sqrt{t^2 - {r^2}/{\beta^2}}}$



P-SV case

Green function in 2D

$$G_{ij} = \frac{i}{4\rho\omega^2}\left\{-\delta_{ij}k^2 H_0^{(2)}(kr) + \frac{\partial^2}{\partial x_i \partial x_l}\left[H_0^{(2)}(qr) - H_0^{(2)}(kr)\right]\delta_{lj}\right\}$$

$$G_{ij}(P,Q) = \frac{-\mathrm{i}}{8\rho}\left\{A\delta_{ij} - B\left(2\gamma_i\gamma_j - \delta_{ij}\right)\right\} \qquad \gamma_j = \frac{x_j - \xi_j}{r}$$

$$A = \frac{H_0^{(2)}(qr)}{\alpha^2} + \frac{H_0^{(2)}(kr)}{\beta^2} \qquad B = \frac{H_2^{(2)}(qr)}{\alpha^2} - \frac{H_2^{(2)}(kr)}{\beta^2} \qquad r = |P,Q|$$

Hankel functions

$$q = \frac{\omega}{\alpha} \qquad k = \frac{\omega}{\beta} \qquad \alpha = \sqrt{\frac{\lambda + 2\mu}{\rho}} \qquad \beta = \sqrt{\frac{\mu}{\rho}}$$

THE 2D VECTOR CASE

Summation of P and S plane waves:

$$u_l(\mathbf{x},\omega,t) = P(\omega,\phi)n_l \exp(-\mathrm{i}\frac{\omega}{\alpha}x_j n_j) + S(\omega,\psi)m_l \exp(-\mathrm{i}\frac{\omega}{\beta}x_j m_j)$$

ratio of energy α : $P^2\alpha^2 = \varepsilon S^2\beta^2$

Correlation and Azimuthal average:

$$\langle \bullet \rangle = \frac{1}{4\pi^2}\int_0^{2\pi} d\phi \int_0^{2\pi} \bullet \, d\psi$$

$$\langle u_i(\mathbf{y})u_j^*(\mathbf{x})\rangle = \frac{S^2\beta^2}{2}\left\{A\delta_{ij} - B(2\gamma_i\gamma_j - \delta_{ij})\right\}$$

$$A = \varepsilon\frac{J_0(qr)}{\alpha^2} + \frac{J_0(kr)}{\beta^2} \text{ and } B = \varepsilon\frac{J_2(qr)}{\alpha^2} - \frac{J_2(kr)}{\beta^2}$$

Ratio of P to S energy?

$$P^2\alpha^2 = \varepsilon S^2 \beta^2$$

Equipartition of diffuse fields : (*ε*=1)

$$E_S / E_P = \left(\frac{\alpha}{\beta}\right)^2$$

$$\left\langle u_i(\mathbf{y}) u_j^*(\mathbf{x}) \right\rangle = \frac{S^2\beta^2}{2}\left\{A\delta_{ij} - B(2\gamma_i\gamma_j - \delta_{ij})\right\}$$

$$A = \varepsilon\frac{J_0(qr)}{\alpha^2} + \frac{J_0(kr)}{\beta^2} \text{ and } B = \varepsilon\frac{J_2(qr)}{\alpha^2} - \frac{J_2(kr)}{\beta^2}$$

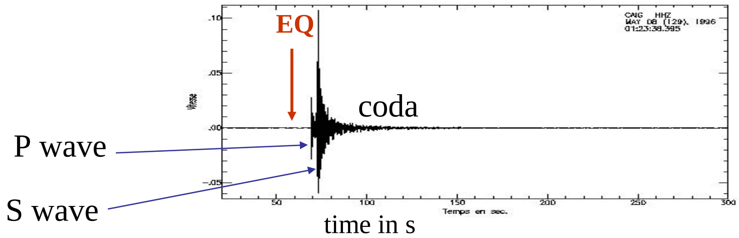
$$\left\langle u_i(\mathbf{y},\omega) u_j^*(\mathbf{x},\omega) \right\rangle = -8 E_S k^{-2} \operatorname{Im}\left[G_{ij}(\mathbf{x},\mathbf{y},\omega)\right]$$

Formally, same result in 3D (Sánchez-Sesma and Campillo, 2006)

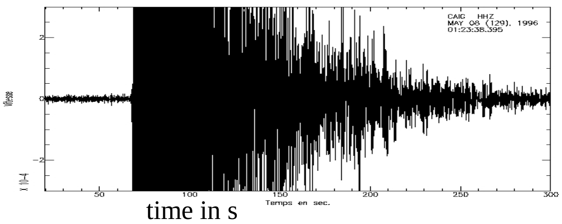
$$\left\langle u_i(\mathbf{y},\omega) u_j^*(\mathbf{x},\omega) \right\rangle = -4\pi E_S k^{-3} \operatorname{Im}\left[G_{ij}(\mathbf{x},\mathbf{y};\omega)\right]$$

Is equipartition a natural condition?

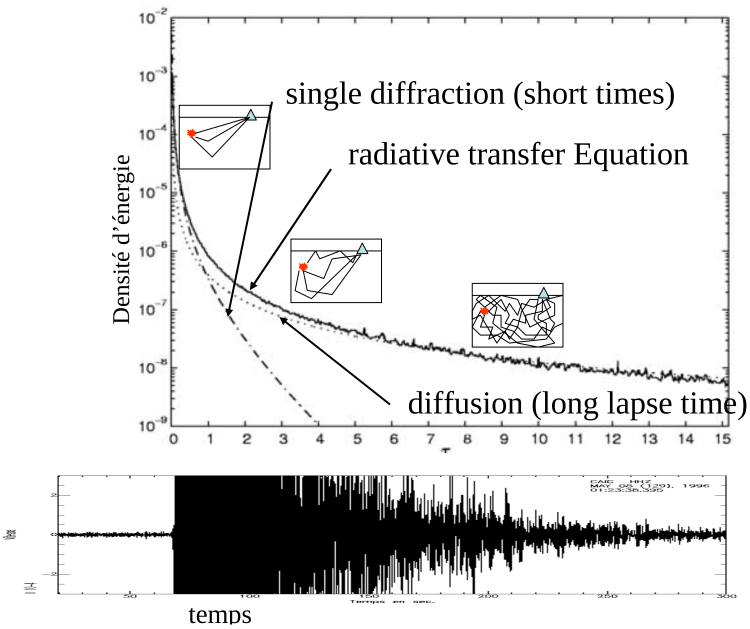
Example of a record of a local earthquake in the band .5-20Hz



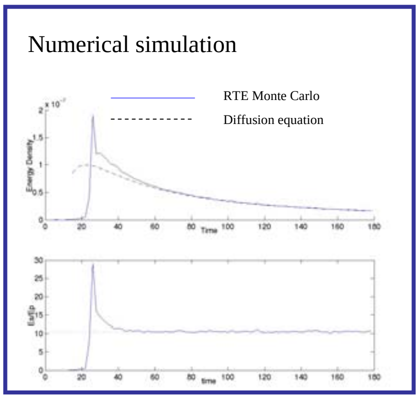
scale x 1000:

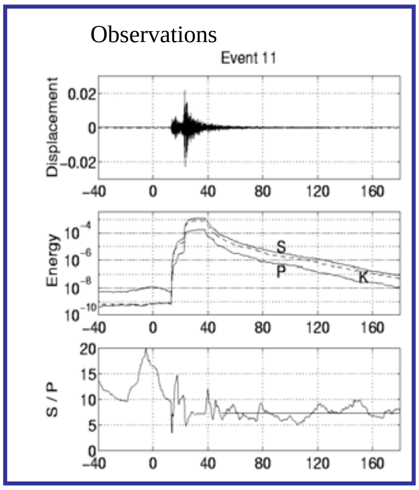


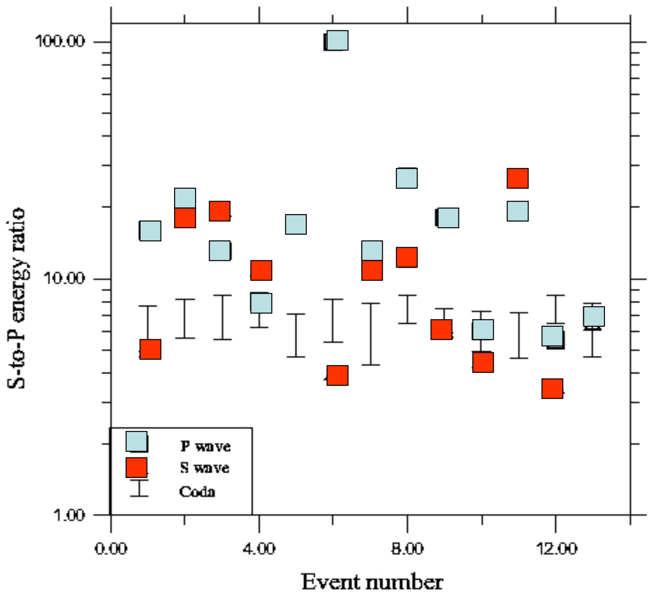
Propagation regimes and energy description



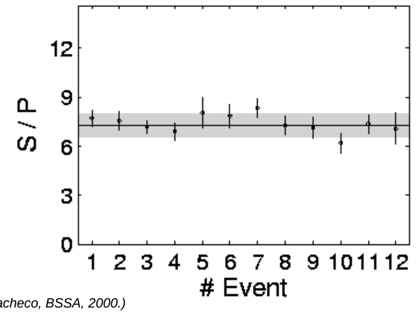
Implication of equipartition for elastic waves (Weaver, 1982, Ryzhik et al., 1996): P to S energy ratio stabilizes at a value independant of the details of scattering!







Values of stabilization for a series of events



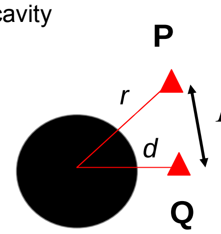
(Shapiro, Campillo, Margerin, Singh, Kostoglodov, Pacheco, BSSA, 2000.)

		Equipartition		
	OBSERVATIONS	THEORY FULL SPACE	THEORY HALF SPACE (BODY WAVES)	THEORY HALF-SPACE WITH RAYLEIGH
S/P	7.3	10.39	9.76	7.19

(Hennino et al., PRL, 2001)

Importance of Rayleigh wave contribution

Exercise: SH waves in a medium with a cylindrical cavity



$R = |P,Q|$

Analytical solution for the 2D SH Green function:

$$G_{22}(\mathbf{x},\mathbf{y};\omega) = v^0 + v^d = \frac{1}{4i\mu}\left\{ H_0^{(2)}(kR) + \sum_{n=0}^{\infty} \varepsilon_n A_n H_n^{(2)}(kd) H_n^{(2)}(kr) \cos n\theta \right\}.$$

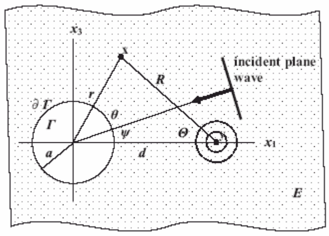
$$A_n = -\frac{J_n(ka)J_n'(qa) - \xi J_n'(ka)J_n(qa)}{H_n^{(2)}(ka)J_n'(qa) - \xi H_n^{(2)\prime}(ka)J_n(qa)}.$$

with

$$\xi = \frac{\mu_E k}{\mu_T q} = \frac{\rho_E \beta_E}{\rho_T \beta_T},$$

Consider the incidence of an isotropic distribution of plane waves :

$$v(\mathbf{x},\omega,t) = F(\omega,\psi)\, exp(-i\, kx_j n_j)\, exp(i\,\omega t),$$



Expansion (polar coordinates):

$$v^0(\mathbf{x},\omega) = F(\omega)\sum_{n=0}^{\infty}\varepsilon_n \mathrm{i}^n J_n(kr)\cos n(\psi-\theta), \text{ and}$$

and the diffracted field

$$v^d(\mathbf{x},\omega) = F(\omega)\sum_{n=0}^{\infty}\varepsilon_n \mathrm{i}^n A_n H_n^{(2)}(kr)\cos n(\psi-\theta).$$

The incident plane wave and the diffracted field reduce to

$$v(r,\theta;\psi) = F(\omega)\sum_{n=0}^{\infty} V_n(r,\omega)\cos n(\theta-\psi),$$

where

$$V_n(r,\omega) = \mathrm{i}^n \varepsilon_n\left[J_n(kr) - A_n H_n^{(2)}(kr)\right],$$

Correlation: $v(\mathbf{y},\omega)v^*(\mathbf{x},\omega) = F^2(\omega)\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} V_n(d,\omega)V_m^*(r,\omega)\cos n\psi\,\cos m(\psi-\theta)$

azimuthal average:equipartition of the incident field

$$\Rightarrow \quad \left\langle v(\mathbf{y},\omega)v^*(\mathbf{x},\omega)\right\rangle = F^2(\omega)\sum_{m=0}^{\infty}\frac{1}{\varepsilon_m}V_m(d,\omega)V_m^*(r,\omega)\,\cos m\theta$$

$$\frac{1}{\varepsilon_m}V_m(d,\omega)V_m^*(r,\omega) = \frac{\varepsilon_m}{D_m^2}\left(N_m Y_m(kd) - M_m J_m(kd)\right)\left(N_m Y_m(kr) - M_m J_m(kr)\right)$$

It leads to the exact relation:

$$\left\langle v(\mathbf{y},\omega)v^*(\mathbf{x},\omega)\right\rangle = -8E_{SH}\,k^{-2}\,\mathrm{Im}\left[G_{22}(\mathbf{x},\mathbf{y},\omega)\right].$$

(*Sanchez-Sesma et al., 2006*)
including near field, scattering, resonances,…..

Illumination by an equipartitioned field (isotropic plane waves here) is enough to retrieve the diffracted field. ***Actually a general theorem (Weaver, 2004; Colin de Verdière 2007, Gouedard et al., 2007)***

Correlation of diffuse field in acoustics

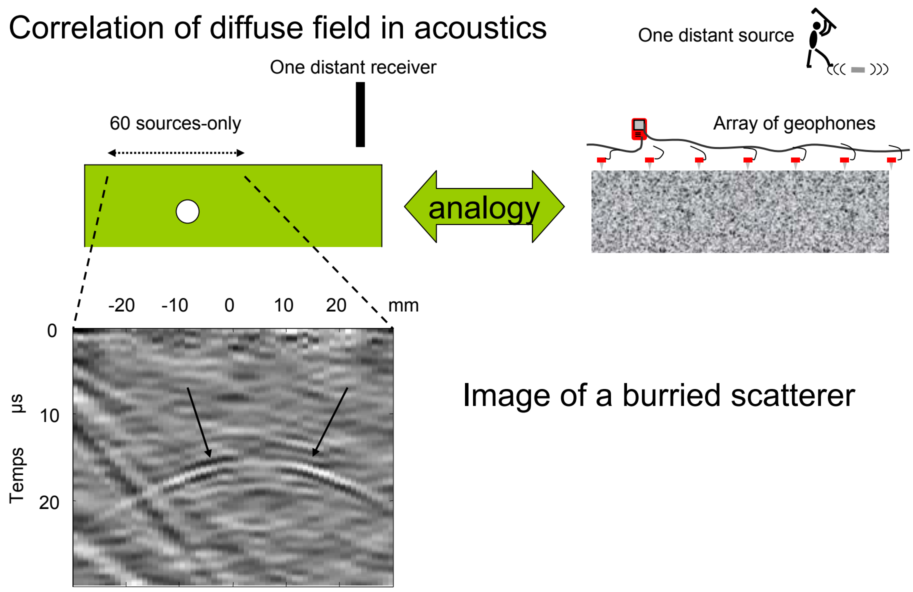


Image of a burried scatterer

Larose, Lobkis & Weaver, JASA **119** (6) p3549 (2006)

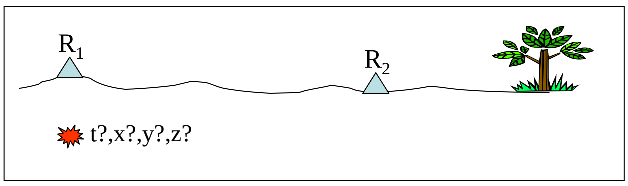
Taking advantage of the correlation properties of diffuse fields towards

Imaging without a source

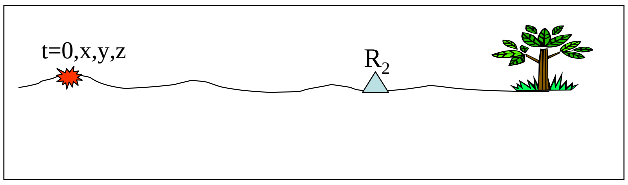
or

Empirical reconstruction of seismograms

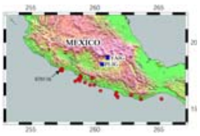
Can we with this (real) experiment:



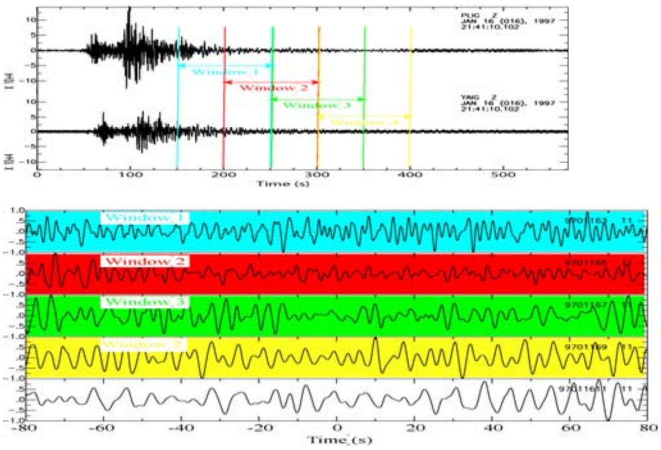
find the result of that (ideal) one?



Seismological application: coda waves

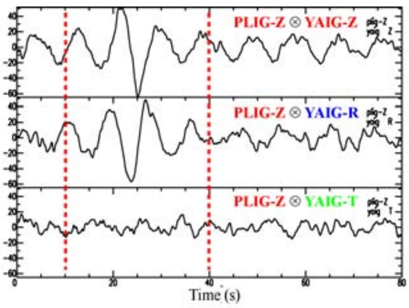


Individual cross-correlations:fluctuations dominate.



(Campillo and Paul, Science, 2003)

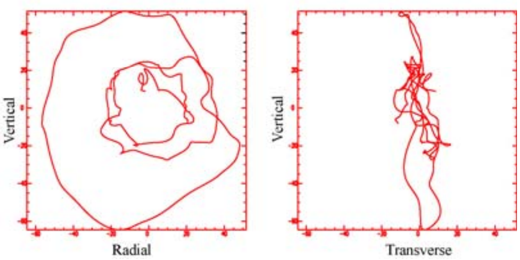
Stack of 196 windows



$C(U^1{}_Z \,|\, U^2{}_Z) \rightarrow G_{ZZ}$

$C(U^1{}_Z \,|\, U^2{}_R) \rightarrow G_{ZR}$

$C(U^1{}_Z \,|\, U^2{}_T) \rightarrow G_{ZT}$

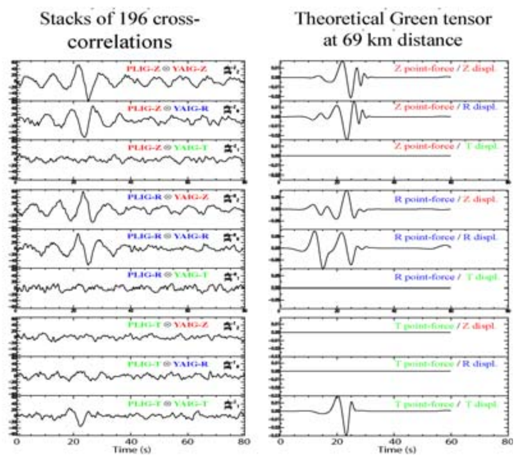


Elliptical polarisation in the plane (Z, R)

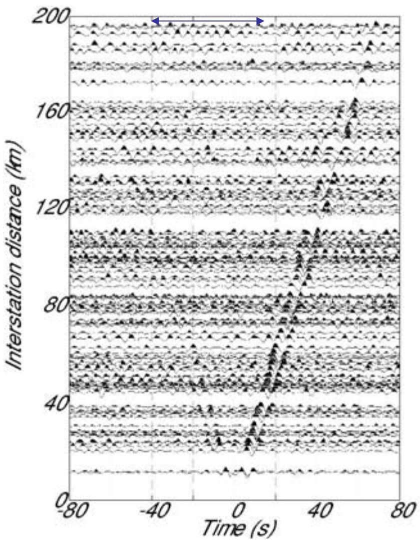
➔

Rayleigh Pulse

Emergence of the Green function

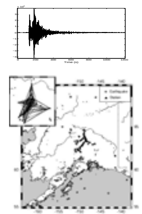


Degree of Time Symmetry: stacks of the cross-correlation of early and late coda



Positive times:
 Picking of the max: t_i
 Shift by t_i and stack
Negative times:
 Shift by $-t_i$ and stack

BEAAR

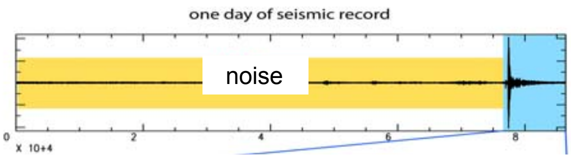


R=A+/A-

Results:

R	0.08Hz-0.15Hz	012Hz-0.3Hz
Early coda (first 300s of coda)	5.6	4.3
Late coda (after 300s)	3.9	3.1

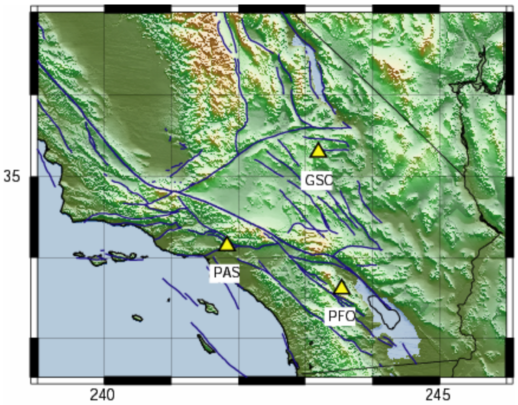
Paul et al, 2005

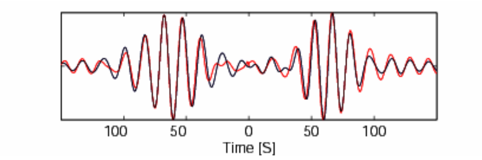


Networks with continuous recording = huge amount of noise data!

Monitoring temporal changes..

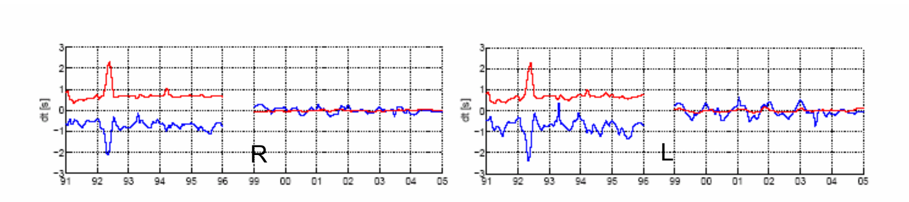
13 years of continuous recordings





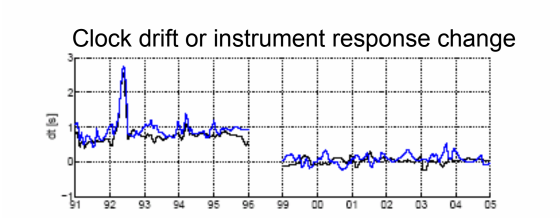
Reference Green function for GSC-PAS 10-20s (black) and the cross correlation of Jan 1991 (red).

GSC-PAS 5-10s



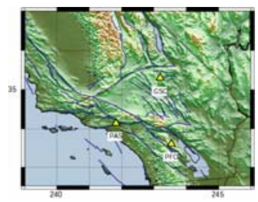
‘Causal’ and ‘anticausal’ delays with respect to the reference GF

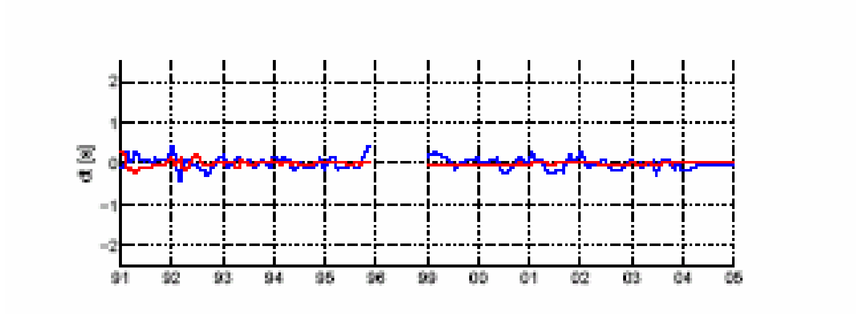
Using the principle of time symmetry of waves, we identify instrumental errors:



Stehly, Campillo and Shapiro, GJI 2007

Causal and anticausal time delays

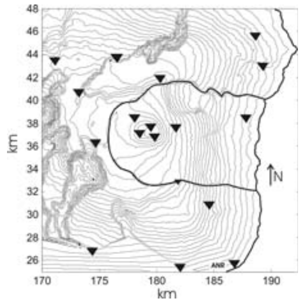




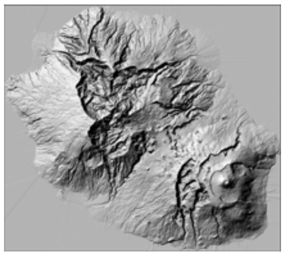
<1/1000 travel time ➔ <1/1000 dV/V

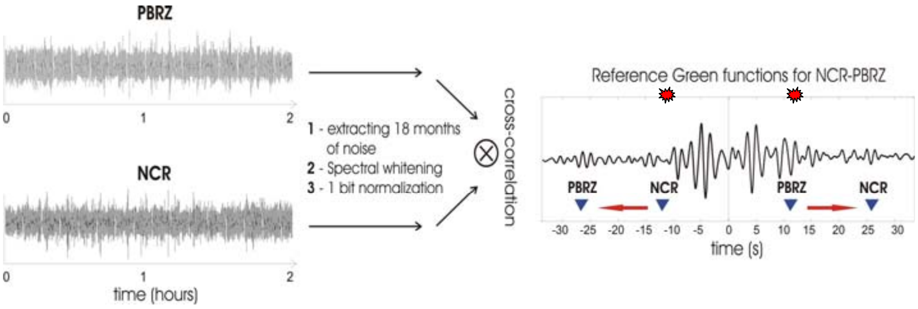
Stehly, Campillo and Shapiro, GJI 2007

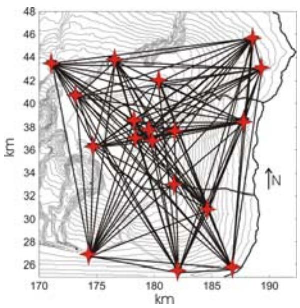
Piton de la Fournaise

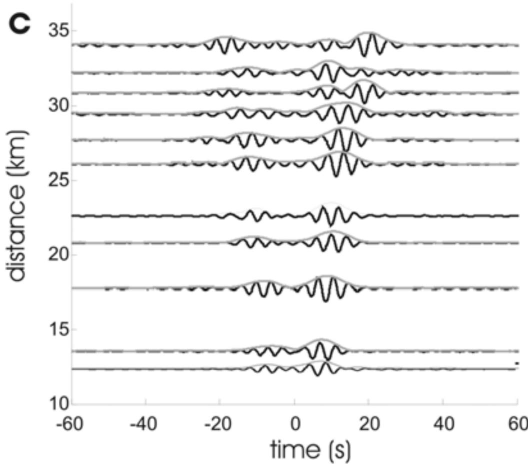


La Réunion



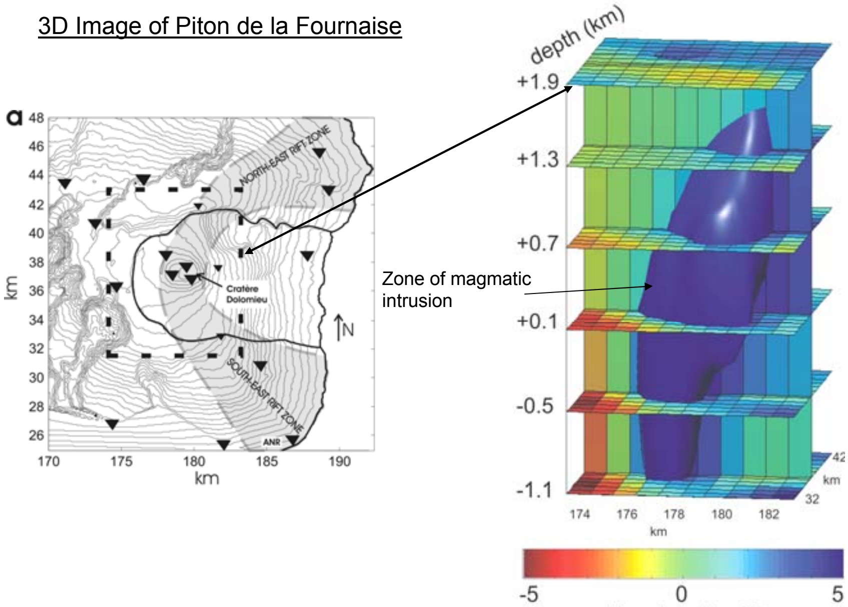




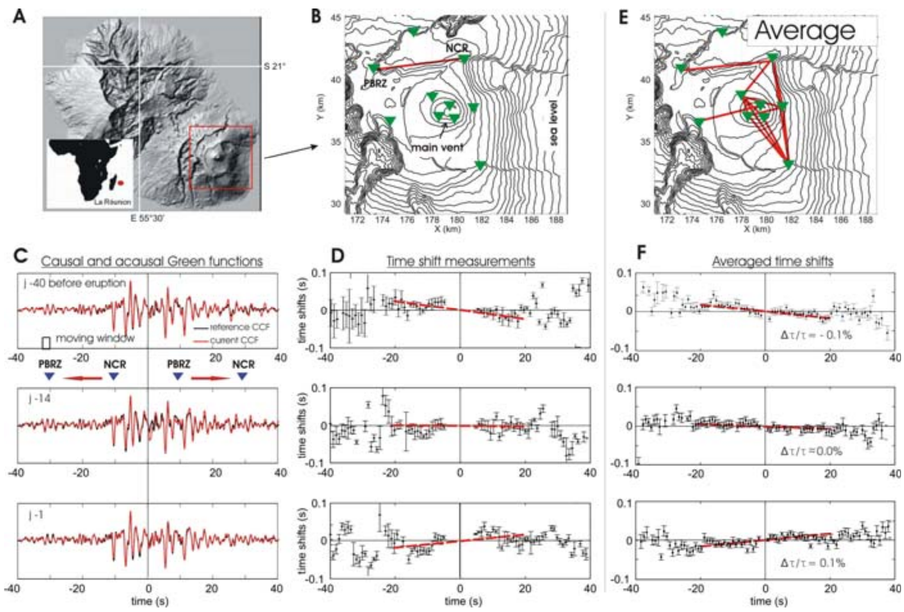


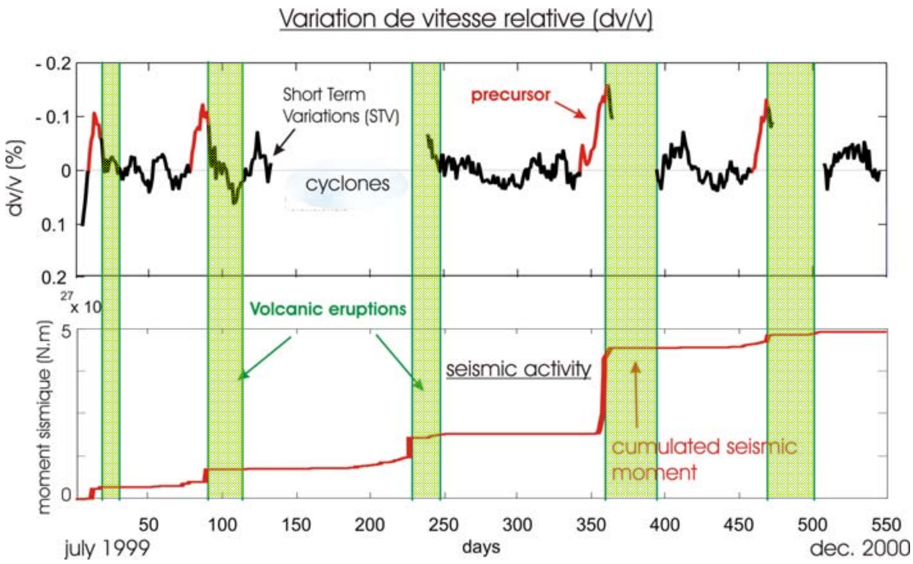
(Brenguier et al, 2007a,b)

3D Image of Piton de la Fournaise



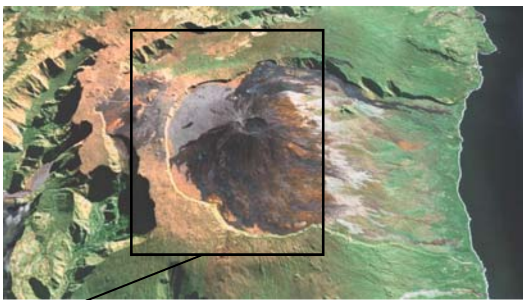
Brenguier et al. *GRL*, 2007

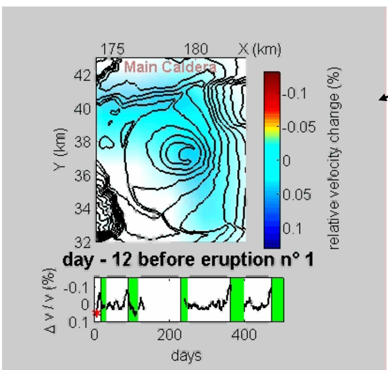


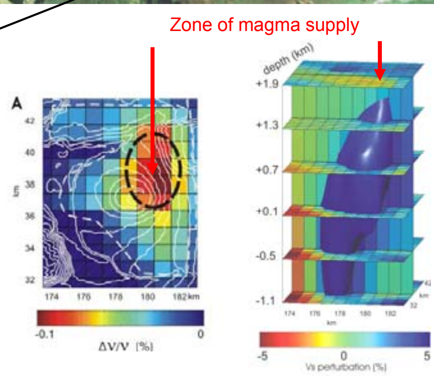


Brenguier et al. *submitted*, 2007

Temporal changes : regionalization







Some conclusions:

Possibility of 'empirical synthesis' of seismograms from long range average correlation.

Partial but 'true' Green function

Station to station: Increase of resolution for surface wave imaging

Applicability in various environments and frequency ranges

Precision and stability

Original applications (time symmetry, reciprocity….)

Questions:

What part of the wavefields can be reconstructed in practice?

Structure of the 'noise' wavefield (sources, propagation regime…)?

…….